Bubble Dynamics - the Moody Linearized Equation

The linearize buble equation

$$\frac{d^3}{dt^3}R_1(t) + a\cdot\frac{d^2}{dt^2}R_1(t) + b\cdot\frac{d}{dt}R_1(t) + c\cdot R_1(t) = 0$$

$$\frac{d^3}{dt^3}R_1(t) + a\left(\frac{d^2}{dt^2}R_1(t)\right) + b\left(\frac{d}{dt}R_1(t)\right) + c\,R_1(t) = 0 \quad \textbf{(1)}$$

For critical dampinng

$$evalf\left(H = \frac{2\cdot 101300\cdot \mathrm{sqrt}\left(\frac{1\cdot 101300}{1000}\right)}{3\cdot(1.4-1)\cdot 273}\right)$$

$$H = 6224.43979 \quad \textbf{(2)}$$

$$subs\left(k = 1.4,\ R_\infty = 0.001,\ \textbf{(2)},\ subs\left(V_b = \frac{4}{3}\pi R_\infty{}^3,\ R_\infty = 1e{-}3,\ \rho = 1.18,\ M = \rho\cdot V_b\right),\ R_g = 287,\ a = \frac{4\cdot\pi\cdot(k-1)\cdot R_\infty{}^2 H}{3\cdot M\cdot R_g}\right)$$

$$a = 7351.84527 \quad \textbf{(3)}$$

$$subs\left(g_0 = 1,\ \rho = 1000,\ k = 1.4,\ P_\infty = 101300,\ R_\infty = 0.001,\ b = 3\cdot g_0\cdot k\cdot\frac{P_\infty}{\rho\cdot R_\infty{}^2}\right)$$

$$b = 4.25460\ 10^8 \quad \textbf{(4)}$$

$$subs\left(P_\infty = 101300,\ g_0 = 1,\ R_\infty = 0.001,\ \rho = 1000,\ c = \frac{rhs(\textbf{(3)})\cdot 3\cdot P_\infty\cdot g_0}{\rho\cdot R_\infty{}^2}\right)$$

$$c = 2.23423\ 10^{12} \quad \textbf{(5)}$$

$$subs(\textbf{(3)},\ \textbf{(4)},\ \textbf{(5)},\ \textbf{(1)})$$

$$\frac{d^3}{dt^3}R_1(t) + 7351.84527\,\frac{d^2}{dt^2}R_1(t) + 4.25460\ 10^8\,\frac{d}{dt}R_1(t) + 2.23423\ 10^{12}\,R_1(t) = 0 \quad \textbf{(6)}$$

$$sol := dsolve\left(\left\{\textbf{(6)},\ R_1(0) = 1,\ D(R_1)(0) = 0,\ D(D(R_1))(0) = -\frac{3\cdot 1\cdot 101300}{1000\cdot 0.001^2}\right\},\ numeric\right)$$

$$sol := \textbf{proc}(x\_rkf45)\ \ldots\ \textbf{end proc} \quad \textbf{(7)}$$

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with(plots):
odeplot(sol, 0..0.004)
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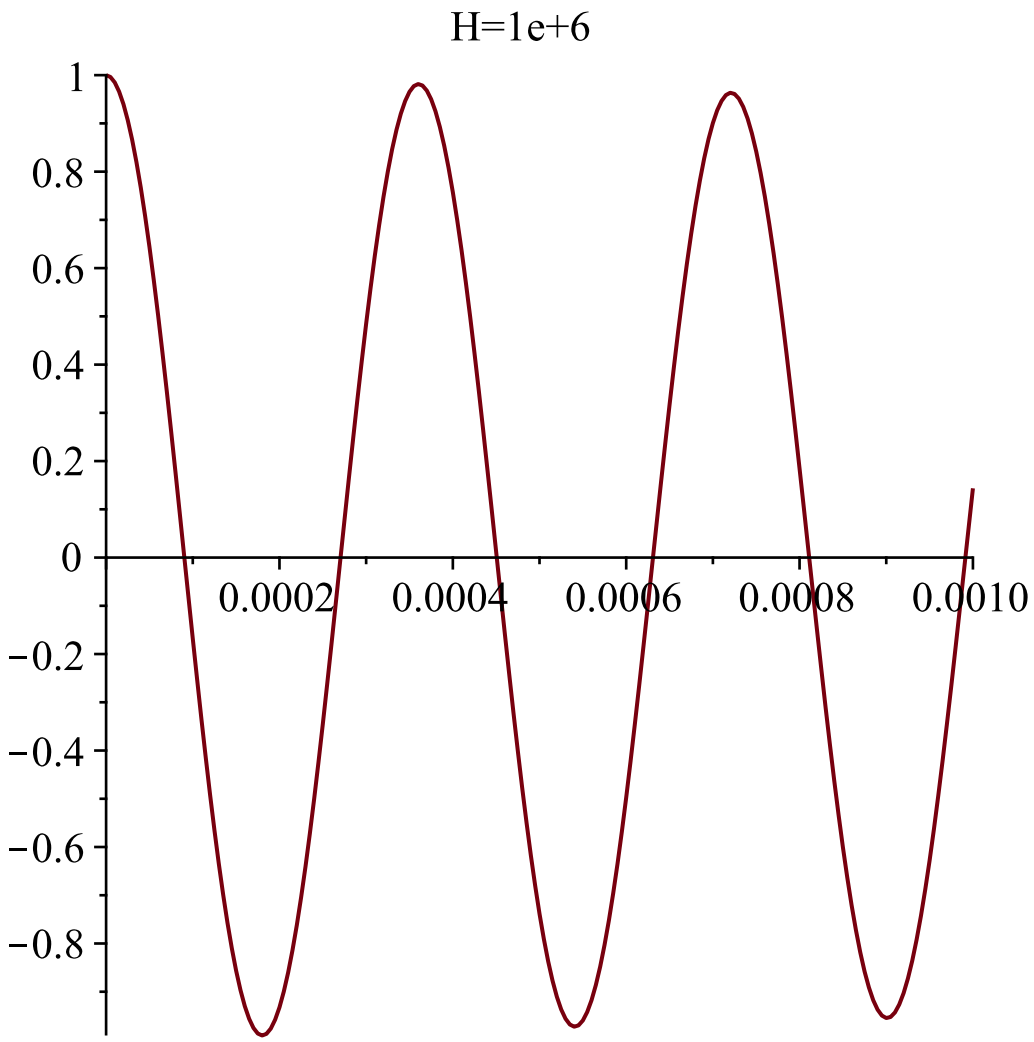
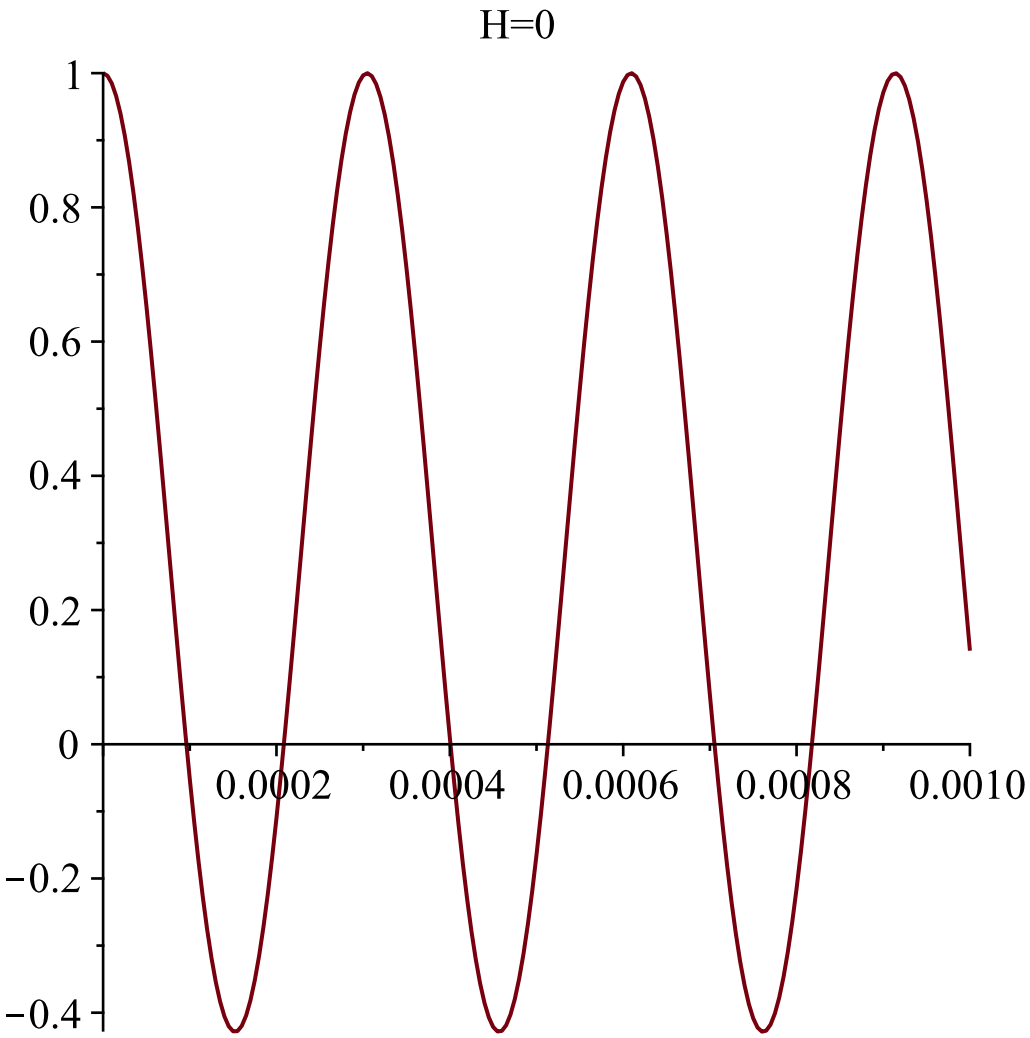
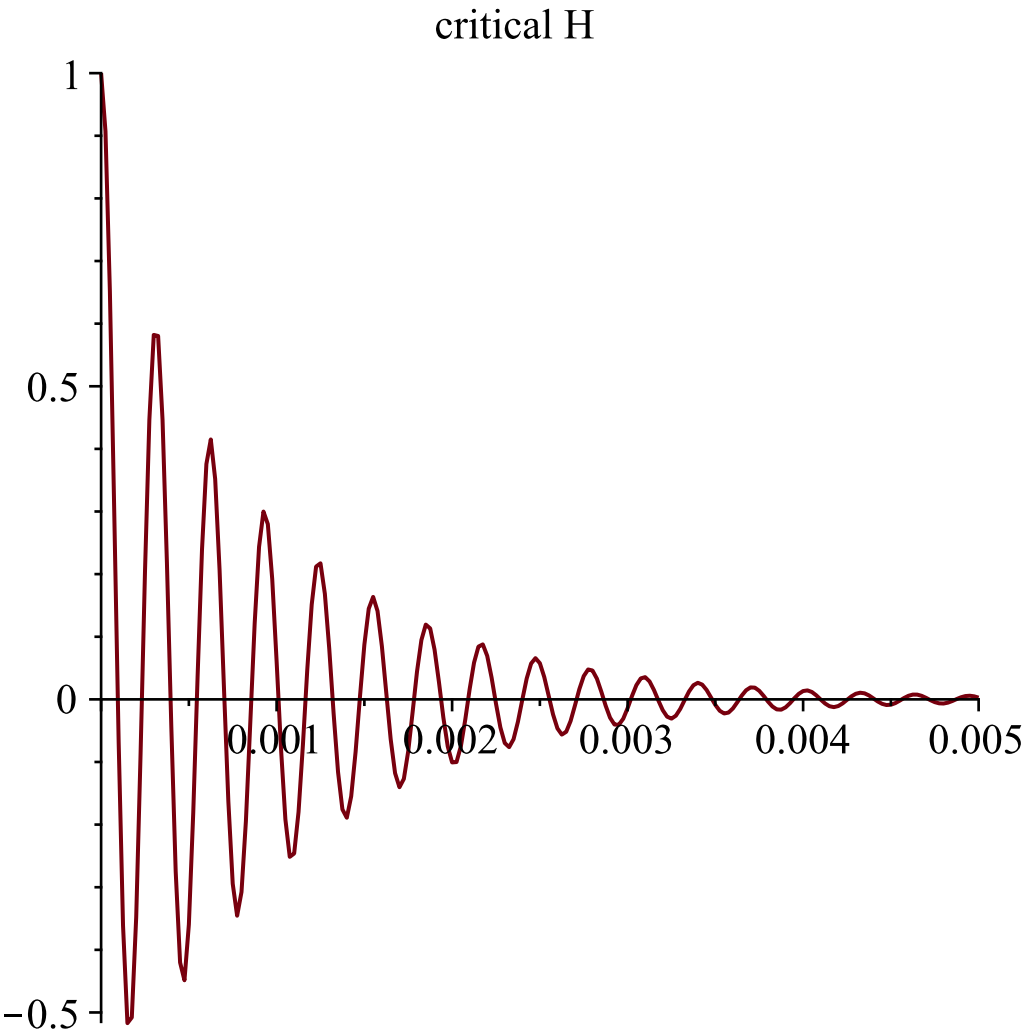

H=1e+6
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0.0002
0.0004
0.0006
0.0008
0.0010


H=0
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0.0002
0.0004
0.0006
0.0008
0.0010


critical H
1
0.5
0
−0.5
0.001
0.002
0.003
0.004
0.005