Exercise (2.2.9).

Compute the homology groups of the following 2-complexes:

1. *The quotient of S^2 obtained by indentifying north and south poles to a point.*
2. $S^1 \times (S^1 \vee S^1)$.
3. *The space obtained from D^2 by first deleting the interios of two disjoint subdisks in the interior of D^2 and then identifying all three resulting boundary circles together via homeomorphisms preserving clockwise orientations of these circles.*
4. *The quotient space of $S^1 \times S^1$ obtained by identifying points in the circles $S^1 \times \{x_0\}$ that differ by $2\pi/m$ rotation and identifying points in the circle $\{x_0\} \times S^1$ that differ by $2\pi/n$ rotation.*

Solution.

TODO

Exercise (2.2.10).

Let X be the quotient space of S^2 under the identifications $x \sim -x$ for x in the equator S^1. Compute the homology groups $H_i(X)$. Do the same for S^3 with antipodal points of the equatorial $S^2 \subset S^3$ identified.

Solution.

TODO

Exercise (2.2.12).

Show that the quotient map $S^1 \times S^1 \to S^2$ collapsing the subspace $S^1 \vee S^1$ to a point is not nullhomotopic by showing that it induces an isomorphism on H_2. On the other hand, show via covering spaces that any map $S^2 \to S^1 \times S^1$ is nullhomotopic.

Exercise (2.2.14).

A map $f : S^n \to S^n$ satisfying $f(x) = f(-x)$ for all x is called an even map. show that an even map $S^n \to S^n$ must have even degree and that the degree must in fact be zero when n is even. When n is odd, show that there exist even maps of any given even degree.

Solution.

TODO

Exercise (2.2.23).

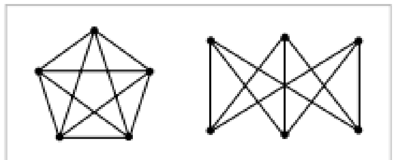
Show that if the closed orientable surgace M_g of genus g is a covering space of M_k, then $g = n(h-1)+1$ for some n, namely, n is the number of sheets in the covering.

Solution.

TODO

Exercise (2.2.24).

Suppose we build S^2 from a finite collextion of polygons by identifying edges in pairs. Show that in the resulting CW structure on S^2 the 1-skeleton cannot be either of the two graphs shown, with five and six vertices.



Solution.

TODO

Exercise (2.2.28).

1. *Use the Mayer-Vietoris sequence to compute the homology groups of the space obtained from a torus $S^1 \times S^1$ by attaching a Möbius band via a homeomorphism from the boundary circle of the Möbius band to the circle $S^1 \times \{x_0\}$ in the torus.*
2. *Do the same for the space obtained by attaching a Möbius band to $\mathbb{R}P^2$ via a homeomorphism of its boundary circle to the standard $\mathbb{R}P^1 \subset \mathbb{R}P^2$.*

Solution.

TODO

Exercise (2.2.31).

Use the Mayer-Vietoris sequence to show there are isomorphisms $\tilde{H}_n(X \vee Y) \sim \tilde{}H_n(X) \oplus \tilde{H}_n(Y)$ if the basepoints of X and Y that are identified in $X \vee Y$ are deformation retracts of $U \subset X$ and $V \subset Y$.

Solution.

TODO

Exercise (2.2.32).

For SX the suspension of X, show by a Mayer-Vietoris sequence applied to $X \cup CA$, where CA is the cone on A.

Solution.

TODO

Exercise (2.2.41).

For a finite CW complex and F a field, show that the Euler characterisitic $\chi(X)$ can also be compute by the formula $\chi(X) = \sum_n (-1)^n \dim H_n(X; F)$ the alternating sum of the dimensions of the vector spaces $H_n(X; F)$.

Solution.

TODO

Problem A1.

Use Euler characteristic to determine which orientable surface results from identifying opposite edges of a $2n$-gon.

Solution.

TODO

Problem A2.

The degree of a homeomorphism $f : \mathbb{R}6n \to \mathbb{R}^n$ can be defined as the degree of the extension of f to a homeomorphism of the one-point compactification S^n. Using this *notion, fill in the details of the following argument which shows that $\mathbb{R}^n$ is not homeomorphic to a product $X \times X$ if n is odd. Assuming $\mathbb{R}^n = X \times X$, consider the homeomorphism $f : \mathbb{R}^n \times \mathbb{R}^n \to X \times X \times X \times X$ that cyclically permutes the factors $f(x_1, x_2, x_3, x_4) = (x_2, x_3, x_4, x_1)$. Then f^2 switches the two factors of $\mathbb{R}^n \times \mathbb{R}^n$, so f^2 has degree -1 if n is odd. But $\deg(f^2) = \deg(f)^2 = 1$.*

Solution.

TODO

Problem A3.

Show that if $f : \Delta^n \to \Delta^n$ is a map that takes each $(n-1)$-dimensional face of Δ^n to itself, then f is surjective.

Solution.

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Exercise (A4).

Show that the spaces $S^1 \times S^2$ and $S^1 \vee S^2 \vee S^3$ have isomorphic homology and fundamental groups but are not homotopy equivalent.

Solution.

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