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Support vector machines for face authentication

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Abstract

We present an extensive study of the support vector machine (SVM) sensitivity to various processing steps in the context of face authentication. In particular, we evaluate the impact of the representation space and photometric normalisation technique on the SVM performance. Our study supports the hypothesis that the SVM approach is able to extract the relevant discriminatory information from the training data. We believe that this is the main reason for its superior performance over benchmark methods (e.g. the eigenface technique). However, when the representation space already captures and emphasises the discriminatory information content (e.g. the fisherface method), the SVMs cease to be superior to the benchmark techniques. The SVM performance evaluation is carried out on a large face database containing 295 subjects. © 2002 Published by Elsevier Science B.V.

Keywords: Face verification; Support vector machines; Principal component analysis; Linear discriminant analysis

1. Introduction

High-security verification systems based on biometric modalities such as the iris, retina and fingerprints have been commercially available for some time. However, one of the most attractive sources of biometric information is the human face since highly discriminatory measurements can be acquired without user interaction. The recognition of faces is a well-established field of research and a large number of algorithms have been proposed in the literature. Popular approaches include the ones based on eigenfaces [16], dynamic link matching [10], and active appearance models [4]. These techniques vary in recognition performance and computational complexity, and the choice of algorithm is typically dependent on the specific application. The verification problem on the other hand is less explored. Recent examples include Ref. [9] in which a robust form of correlation is applied to face authentication.

The aim of the paper is to evaluate the effectiveness of the SVM approach to face authentication. SVMs provide a powerful tool for machine learning [5]. They have been used successfully in many object recognition applications [1]. SVMs are known to generalise well even in high dimensional spaces under small training sample conditions. This property is particularly attractive in the context of face verification and recognition where such conditions are typically encountered. An earlier study of SVMs in face verification has been reported by Phillips [5]. An SVM verification system design was compared with a standard principal component (PC) analysis face authentication method and the former was found to be significantly better. In this approach, a single SVM was trained to distinguish between the populations of within-client and between client difference images, respectively, as originally proposed by Moghaddam [4]. This method gives client non-specific support vectors.

In our approach, we adopt a client-specific solution which requires learning client-specific support vectors. However, this is not the main distinguishing feature of our work, it merely reflects our focus on authentication as opposed to recognition which is of concern in Ref. [15]. Our primary motivation for carrying out a similar study was to establish why the performance of the SVM approach is superior. We want to investigate the inherent potential of SVMs to extract the relevant discriminatory information from the training data irrespective of representation and pre-processing. In order to achieve this objective we have designed experiments in which faces are represented in both PC and linear discriminant (LD) subspaces. The latter basis (fisherfaces) is used as an example of a face representation with focus on discriminatory feature extraction while the former achieves simply data compression. We also study the effect of image photometric normalisation on the performance of the SVM method.

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A number of criteria have been considered as a basis for the SVM approach evaluation, using other baseline techniques as a benchmark. We have included as benchmark verification methods not only the classical PC variants with the L^2 norm and correlation coefficient, respectively, but also the LD space with the same two decision schemes. As criteria for evaluating SVMs in relation to the benchmark methods, we have concentrated on the following: ability to extract discriminatory information, robustness (sensitivity to input data conditioning) and the merits of non-linear boundaries. These criteria are expressed quantitatively in terms of the false client rejection and impostor acceptance rates.

The findings of our study strongly support the hypothesis that the SVM approach is powerful in the sense of being able to extract the relevant discriminatory information from the training data. This is the main reason for the large difference between the observed performance of the classical eigenface classification methods and SVMs. However, when the representation space already captures and emphasises the discriminatory information content as in the case of LD bases, SVMs cease to be superior to the simple Euclidean distance or correlation decision rules. The results also show that the absolute performance of the SVMs is relatively insensitive to the representation space and preprocessing technique.

The paper is organised as follows. In Section 2, we introduce the two face representation spaces used in our study, namely eigenfaces and fisherfaces and also overview the SVM approach to face identity verification and summarise the benchmark classification methods. Section 3 introduces the face database used in experimentation and describes the experiments carried out, their objectives and the results obtained. Finally, conclusions are drawn in Section 4.

2. Face authentication

Any authentication process involves two basic computational stages. In the first stage, a suitable representation is derived with the multiple objective of making the subsequent, decision making stage, computationally feasible, immune to environmental changes during the biometric data acquisition, and effective by providing it only with information which is pertinent to the authentication task. The purpose of the second stage is to accept or reject the identity claim corresponding to a probe biometric measurement. This is basically a two-class pattern recognition problem. In the following subsections, we introduce the methods adopted for the design of each of these two stages in the context of the face authentication study pursued in this paper.

2.1. Representation of faces

The first step in the face representation process involves image pre-processing in order to establish correspondence between face images to be compared. Once an image is registered, it can further be normalised photometrically. In our study, we set out to investigate the sensitivity of different decision making methods to the normalisation technique and thus this step was applied only in a subset of experiments. In the final step of processing, the image is projected into a coordinate system which facilitates the decision making process computationally and possibly emphasises the important attributes for face verification.

Geometric normalisation. As the focus of the paper is on the decision making aspects of face authentication we have tried to eliminate the dependency of our experiments on processes which may lack robustness. For this reason, we have performed face registration semi-automatically. The procedure is based on manually localised eye positions. Four parameters computed from the eye coordinates (rotation, scaling and translation in the horizontal and vertical directions) are used to crop the face part from the original image and scale it to any desired resolution.

Photometric normalisation. When applied, the photometric normalisation consisted of either shifting/scaling or equalising the intensity distribution. In the first case, we subtract the mean of the registered image and scale the pixel values by their standard deviation, estimated over the whole cropped image. In the second case, we apply standard histogram equalisation.

Image projection. Suppose that we have c clients and M training face images x_i, $i = 1, \ldots, M$, $x_i \in R^{\mathrm{D}}$ each belonging to one of the client classes $\{C_1, C_2, \ldots, C_c\}$. Then we can define the following second-order statistics:

- Between-class scatter matrix:

$$S_{\mathrm{B}} = \frac{1}{c} \sum_{k=1}^{c} (\mu_k - \mu)(\mu_k - \mu)^{\mathrm{T}} \tag{1}$$

- Within-class scatter matrix:

$$S_{\mathrm{W}} = \frac{1}{M} \sum_{k=1}^{c} \sum_{i|x_i \in C_k} (x_i - \mu_k)(x_i - \mu_k)^{\mathrm{T}} \tag{2}$$

- Total scatter matrix:

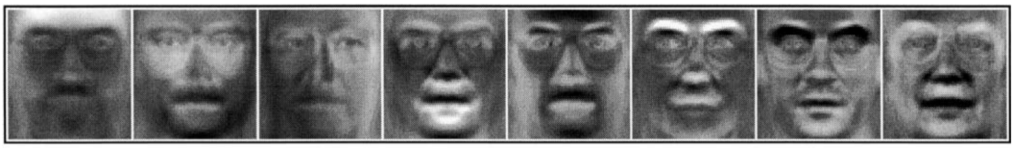
$$S_{\mathrm{T}} = S_{\mathrm{W}} + S_{\mathrm{B}} \tag{3}$$

where μ is the grand mean and μ_k is the mean of class C_k.

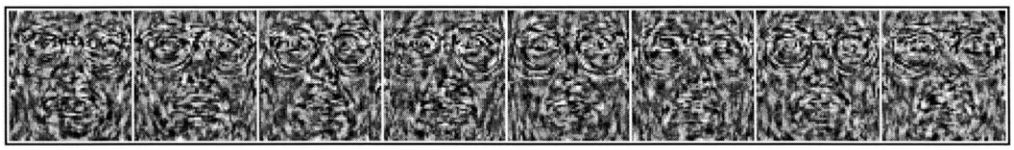
The aim of the PC analysis is to identify the subspace of the image space spanned by the training face image data and to decorrelate the pixel values. This can be achieved by finding the eigenvectors W_{PC} of matrix S_{T} associated with non-zero eigenvalues Λ by solving

$$S_{\mathrm{T}} W_{\mathrm{PC}} - W_{\mathrm{PC}} \Lambda = 0 \tag{4}$$

In the context of face processing, these eigenvectors are typically referred to as *eigenfaces*. The classical representation



(a) PC basis vectors (eigenfaces)



(b) LD basis vectors (fisherfaces)

Fig. 1. The first eight basis vectors of the (a) PC and (b) LD subspaces computed from the training set. The photometric normalisation is histogram equalisation. The basis vectors are ordered according to the magnitude of their corresponding eigenvalues starting with the most significant basis vector to the far left.

of a face image is obtained by projecting it to the coordinate system defined by the eigenfaces.

The projection of face images into the PC (eigenface) subspace achieves information compression, decorrelation and dimensionality reduction to facilitate decision making. If one is also interested in identifying important attributes (features) for face authentication, one can adopt a feature extraction mapping. A popular technique is to find the Fisher linear discriminants (or *fisherfaces*) by solving

$$S_B W_{LD} - S_W W_{LD} \Lambda = 0 \tag{5}$$

The projection of a face image into the system of fisherfaces associated with non-zero eigenvalues will yield a representation, which emphasises the discriminatory content of the image. The solution of the generalised eigenvalue problem in Eq. (5) is known, but due to the high dimensionality many standard methods fail and the choice of a stable numerical algorithm is non-trivial [11]. Fig. 1 shows the first few PC and LD basis images.

In Section 3, we perform experiments with a different number of basis vectors taken from either the eigenface or fisherface systems. In the following, for the sake of notational simplicity, we shall not distinguish between the two different basis systems, nor shall we explicitly denote the dimensionality of the representation space. The actual representation used will be clear from the experiment description. Thus in general, in each experiment we shall work with some transformation matrix W. A sample face image $\mathbf{y}$ will then be represented by a projection $\mathbf{x}$ obtained as $\mathbf{x} = W^T\mathbf{y}$. Similarly, the client model μ_k will be projected into a vector w_k in the appropriate representation space.

2.2. Classification

Support vector machines. The main decision making tool investigated in this paper is the support vector machine (SVM). Below, we give a brief presentation of the basic theory. The reader is referred to Ref. [3] for a more comprehensive introduction. SVMs are based on the principle of structural risk minimisation. The aim is to minimise the upper bound on the expected (or actual) risk defined as[1]

$$R(\alpha) = \int \frac{1}{2}|z - f(\mathbf{x}, \alpha)| \mathrm{d}P(\mathbf{x}, z) \tag{6}$$

where α is a set of parameters defining the trained machine, z a class label associated with a training sample $\mathbf{x}$, $f(\boldsymbol{x}, \alpha)$ a function providing a mapping from training samples to class labels, and $P(\boldsymbol{x}, z)$ the unknown probability distribution associating a class label with each training sample. Let l denote the number of training samples and choose some η such that $0 \le \eta \le 1$. Then, with probability $1 - \eta$, the following bound on the expected risk holds

$$R(\alpha) \le R_{\text{emp}}(\alpha) + \sqrt{\frac{h(\log(2l/h) + 1) - \log(\eta/4)}{l}} \tag{7}$$

where $R_{\text{emp}}(\alpha)$ is the empirical risk as measured on the training set and h is the Vapnik Chervonenkis (VC) dimension. The second term on the right hand side is called the VC confidence. There are two strategies for minimising the upper bound. The first one is to keep the VC confidence fixed and to minimise the empirical risk and the second one to fix the empirical risk (to a small value) and minimise the VC confidence. The latter approach is the basis for SVMs and below we will briefly outline this procedure.

First, consider the linear separable case. We are looking for the optimal hyperplane in the set of hyperplanes separating the given training samples. This hyperplane minimises the VC confidence and provides the best generalisation capabilities. Giving a geometric interpretation, the optimal hyperplane maximises the sum of the distances to the closest positive and negative training samples. This sum is referred to as the *margin* of the separating hyperplane. It can be

[1] The notation is similar to the one in Ref. [3].

Table 1
Verification performance as function of decision rule and photometric normalisation, PC subspace. The verification threshold (Thr), equal error rate (EER), false acceptance (FA), false rejection (FR) and weighted error rate (WE, in these experiments simply the mean of FA and FR) are shown for different classifiers (Cls) and normalisation techniques (Nrm). The classifiers are the Euclidean distance (EDs), normalised correlation (NCr) and support vector machines (SVM). The photometric normalisation techniques are no normalisation (×), zero-mean and unit variance (ZMn), and histogram equalisation (HEq). The SVM kernel is the RBF function ($\gamma = 0.01$)

Cls	Nrm	Thr	EER	FA	FR	WE
EDs	×	0.63	9.92	9.82	6.08	7.95
NCr		3.91	7.68	8.32	6.50	7.41
SVM		4.34	3.00	3.65	1.29	2.47
EDs	ZMn	0.40	8.45	9.39	4.50	6.95
NCr		4.13	6.60	7.39	4.25	5.82
SVM		4.40	3.00	3.64	2.00	2.82
EDs	HEq	0.09	5.36	6.46	4.00	5.23
NCr		4.45	4.32	5.01	3.75	4.38
SVM		4.79	1.50	2.19	1.50	1.85

shown that the optimal hyperplane $\mathbf{w}\cdot\mathbf{x} + b = 0$ (where $\mathbf{w}$ is normal to the hyperplane) is obtained by minimising $\|\mathbf{w}\|^2$ subject to a set of constraints. This is a quadratic optimisation problem.

These concepts can be extended to the non-separable and non-linear case. The separability problem is solved by adding a term to the expression subject to minimisation. This term is the sum of the deviations of the non-separable training samples from the boundary of the margin. This sum is weighted using a parameter controlling the cost of misclassification. The second problem is how to handle the non-linear decision boundaries. This is solved by mapping the training samples to a high dimensional feature space using kernel functions. In this space, the decision boundary is linear and the techniques outlined earlier can be directly applied. The kernel function used in the experiments reported in Section 3 is the radial basis function (RBF) defined as

$$K(\mathbf{x}_i, \mathbf{x}_j) = \mathrm{e}^{-\gamma\|\mathbf{x}_i - \mathbf{x}_j\|^2} \tag{8}$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ denote two samples. The γ-value is a user-controlled parameter which determines the width of the Gaussian kernel. The reader is referred to Refs. [7,8] for an evaluation of the relative performances of different kernel functions.

In addition to the SVMs, we have also implemented the following standard classification rules as baselines for experimental comparison: nearest mean classifier using the Euclidean metric, and the normalised correlation that currently represents the state of the art benchmark on the face database used in the experiments presented in Section 3, as shown in the comparative study reported in Ref. [12].

Euclidean distance. The most commonly used decision rule is based on the Euclidean distance between the sample projection $\mathbf{x}$ and the projection of the kth client mean w_k, i.e.

$$d_{\mathrm{E}}(\mathbf{x}, w_k) = \sqrt{(\mathbf{x} - w_k)^{\mathrm{T}}(\mathbf{x} - w_k)} \tag{9}$$

The claimed client identity is accepted if $d_{\mathrm{E}}(\mathbf{x}, w_k)$ is below a threshold $\tau_{\mathrm{E}k}$. Otherwise it is rejected.

Normalised correlation. Alternatively, the decision can be based on the correlation score

$$d_{\mathrm{C}}(\mathbf{x}, w_k) = \frac{|\mathbf{x}^{\mathrm{T}} w_k|}{|\mathbf{x}||w_k|} \tag{10}$$

In the case of the correlation measure, the claimed identity is accepted if $d_{\mathrm{C}}(\mathbf{x}, w_k)$ exceeds a pre-specified threshold $\tau_{\mathrm{C}k}$.

Client-specific thresholding. The client-specific threshold τ_k is determined using an independent evaluation set. Given the mean μ_k and the standard deviation σ_k of the impostor distances in the evaluation set, the threshold is computed as

$$\tau_k = \mu_k + \tau\cdot\sigma_k \tag{11}$$

where τ is a global threshold.

3. Experimental results

The experiments summarised below were all performed on frontal-face images from the extended M2VTS multimodal database [13]. This publicly available database contains face images and speech recordings of 295 persons. The subjects were recorded in four separate sessions uniformly distributed over a period of five months, and within each session a number of shots were taken including both frontal-view and rotation sequences. In the frontal-view sequences, the subjects read a specific text (providing synchronised image and speech data), and in the rotation sequences the head was moved vertically and horizontally (providing information useful for 3D surface modelling of the head).

The experiments were conducted according to the Lausanwe evaluation protocol [14].[2] This protocol provides a framework within which the performance of vision- and speech-based person authentication systems running on the extended M2VTS database can be measured. The protocol specifies a partitioning of the database into three disjoint sets: a training set (200 clients), an evaluation set (200 clients and 25 impostors) and a test set (200 clients and 70 impostors). The training set is used to build client models, the evaluation set to establish verification thresholds, and the test set to obtain a reliable estimate of the true verification performance on independent data. The client training and evaluation sets are further partitioned.

3.1. Results of face authentication

We have performed a number of experiments with the

[2] The experiments reported here were all performed according to the second configuration of the protocol.

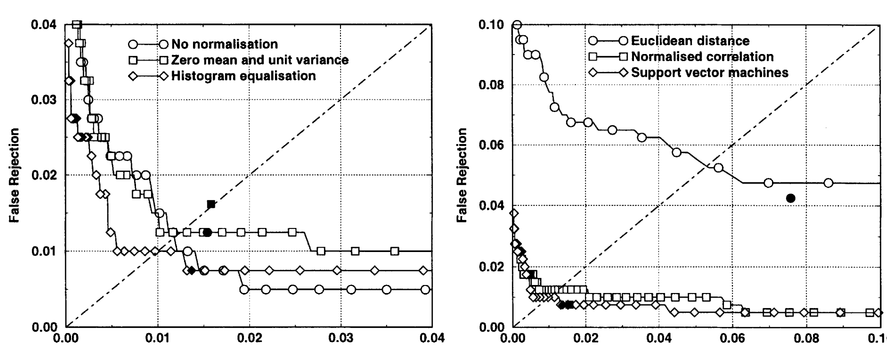
Table 2
Verification performance as function of decision rule and photometric normalisation, LD subspace. The verification threshold (Thr), equal error rate (EER), false acceptance (FA), false rejection (FR) and weighted error rate (WE, in these experiments simply the mean of FA and FR) are shown for different classifiers (Cls) and normalisation techniques (Nrm). The classifiers are the Euclidean distance (EDs), normalised correlation (NCr) and support vector machines (SVM). The photometric normalisation techniques are no normalisation (×), zero-mean and unit variance (ZMn), and histogram equalisation (HEq). The SVM kernel is the RBF function ($\gamma = 0.01$)

Cls	Nrm	Thr	EER	FA	FR	WE
EDs	×	0.66	9.47	10.29	6.22	8.26
NCr		5.44	1.69	2.23	1.25	1.74
SVM		5.07	1.18	1.54	1.25	1.40
EDs	ZMn	−0.92	1.72	1.96	1.00	1.48
NCr		5.55	1.50	1.95	1.25	1.60
SVM		5.03	1.25	1.58	1.62	1.60
EDs	HEq	0.34	5.30	7.58	4.25	5.92
NCr		5.66	1.25	1.56	0.75	1.15
SVM		5.07	1.00	1.37	0.75	1.06

different decision rules (Euclidean distance, normalised correlation and SVMs), subspaces (PC and LD) and photometric normalisation techniques (no normalisation, zero-mean and unit variance, and histogram equalisation). The results of these experiments are presented in Tables 1 and 2 for the PC and LD subspaces, respectively. Looking at the impact of photometric normalisation in the PC subspace, we see that the verification performance of the baseline techniques (Euclidean distance and normalised correlation) increases monotonically with the data quality (no normalisation, zero-mean and unit variance, and histogram equalisation). In the case of the SVMs, the performance is very similar on unnormalised images and images which have been processed with the zero-mean method. However, when histogram equalisation is applied, the error rates drop significantly. Similar results have been reported in the context of face detection [6] and recognition [15]. In the LD space, we can observe similar relationships for the normalised correlation and the SVMs but the performance of the Euclidean distance does not improve for histogram equalisation. In fact, the error rates are much worse than for zero-mean normalisation which is somewhat surprising. In general, the Euclidean distance is highly sensitive to deviations from the implicit model underlying the approach, e.g. client clusters being compact and roughly spherical. The correlation coefficient can cope better with deviations from the sphericity. However, once the data is of that form, as in the case of the LD subspace with zero-mean normalised data, the inherent flexibility of the normalised correlation results in a slightly worse performance.

We can also make a number of interesting observations regarding the relative performances of the different classifiers. In the PC subspace, the verification performance increases monotonically with the complexity of the description rule (Euclidean distance, normalised correlation and SVMs). The SVMs clearly outperform the baseline methods independent of the photometric normalisation technique. In the LD subspace, we observe the same relationship on unnormalised data. However, the SVMs loose their superiority in comparison with the baseline techniques on zero-mean normalised images. In fact, in some cases the Euclidean distance is the best performing classifier. These results suggest that, when the representation space already captures and emphasises the discriminatory information content (as in the case of the fisherface bases), the SVMs do not provide any additional performance advantages in comparison with standard techniques. In fact, since the SVMs require extensive training, the less sophisticated classification methods may prove more suitable in some applications.

Looking at the absolute performance of the SVMs, we see that the error rates are fairly similar in the two subspaces. The SVMs are doing surprisingly well on the PC representation: the error rates for the low-quality data (unnormalised images represented in the PC subspace) are not far behind those of the other extreme (images normalised with histogram equalisation and represented in the LD subspace).



(a) Normalisation

(b) Decision rule

Fig. 2. Verification performance as function of (a) decision rule and (b) photometric normalisation, LD subspace. The photometric normalisation technique and decision rule used in (a) and (b) are histogram equalisation and SVMs, respectively. The SVM kernel function is the RBF function ($\gamma = 0.01$). The error rates on the evaluation and test sets are shown with opaque and filled markers, respectively.

(a) Original (b) PC (c) LD (d) SVM

Fig. 3. Reconstructions from subspace representations: (a) original image, reconstruction from (b) PC (191 coefficients), (c) LD (199 coefficients) and (d) SVM (37 coefficients) subspaces. All subspaces were obtained from unnormalised data.

These results show that the SVMs are robust and relatively insensitive to the representation space and pre-processing technique. The error rates listed in Tables 1 and 2 correspond to single points in the receiver operating characteristics (ROC). By varying the verification threshold, we obtain a set of points showing the trade-off between the false rejection and acceptance rates. This is shown in Fig. 2 for the LD subspace. In the case of the test set, we only plot the single point corresponding to the EER threshold obtained on the evaluation set. In Fig. 2(a), we show the impact of the photometric normalisation technique on the SVM performance. On the other hand, in Fig. 2(b) we contrast the different decision rules in the LD subspace, keeping the photometric normalisation fixed to histogram equalisation.

Note also the difference in the information used by the various decision making schemes which can be gleaned from Fig. 3. In this figure, we show the original representation of a probe image, its PC reconstruction in Fig. 3(b), its LD 'reconstruction' in Fig. 3(c) and finally its SVM reconstruction in Fig. 3(d). The latter two reconstructions have been produced in an analogical way to the PC reconstruction approach. The classical Euclidean distance and correlation methods use the standard PC reconstruction of the probe image shown in Fig. 3(b). The SVM using the PC coefficients work with a similar source of information but some regions in the image are weighted more heavily. Thus, SVMs seem to be capable of performing client-dependent feature extraction. The fisherface reconstruction uses the global mean image as a starting point. The bright areas indicate the increased weighting applied to some pixels in the image. This weighting is client-dependent and is a function of the probe image projection into the LD space.

4. Concluding remarks

We presented an extensive study of the SVM sensitivity to various processes in the context of face authentication. In particular, we evaluated the impact of the representation space and photometric normalisation technique on the SVM performance. Our study supports the hypothesis that the SVM approach is able to extract the relevant discriminatory information from the training data. This is the main reason for the large difference between the observed performance of the classical eigenface method used as a benchmark and the SVMs. When the representation space already captures and emphasises the discriminatory information content as in the case of the LD bases, the SVMs cease to be superior to the simple Euclidean distance or correlation decision rules. The results also indicated that the absolute performance of the SVMs is relatively insensitive to the representation space and pre-processing technique. The SVM performance evaluation was carried out on a large face database containing 295 subjects.

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