Mathmatics

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Abstract

This report introduces some mathematics in games, including Cartesian coordinate system, vectors and scalars, points, dot product and cross product in Algebra and Geometry, matrix with its transpose and multiplication.

Keywords: Coordinator system, vector, point, product, matrix

1 Introduction

Almost all games in all categories has their own mechanics and logic which make them much more attractive and joyful. It is the game mathematics behind the vivid graphics that make all these things possible. So mathematics is one of the most important skill in game engineering. In the meantime, it covers a wide range of knowledge and is hard for people to master it. [Buchanan 2001]

2 Coordinator system

3D math is all about measuring locations, distances, and angles precisely and mathematically in 3D space. The most frequently used framework to perform such calculations using a computer is called the Cartesian coordinate system. There are two types of coordinates in 3D: left-handed coordinates and right-handed coordinates. In right-handed coordinates, thumb stands for x-axis, index finger is y-axis and z-axis goes "out from" screen.

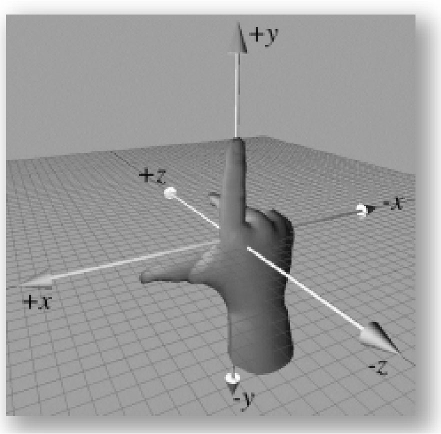

Figure 1: *The right-handed coordinates*

3 Point and Vector

A vector is a list of scalars in square brackets. eg. [1,2,3]. A point is a position in Cartesian space. The difference between points and vectors is points are measured relative to the origin while vectors are intrinsically relative to everything. So a vector can be used to represent a point, but vectors don't have a location.
Vectors has many operations, they can addition, subtraction, negation, scaling and more. The two important operations are dot product and cross product.

- Dot product: in Algebra, it distributes over vector addition and subtraction.

$$a \cdot b = \sum_{i=1}^{n} a_i b_i \tag{1}$$

in Geometry, it is the magnitude of the projection of one vector onto another.

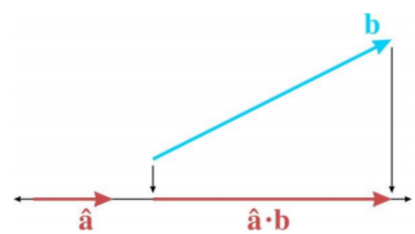

Figure 2: *Dot Product: Geometry*

- Cross product: in Algebra, it is anticommutative.

$$a \times b = -(b \times a) \tag{2}$$

in geometry, the cross product of 2 nonzero vectors a, b is a vector perpendicular to the plane of a and b.

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \|\mathbf{b}\| \sin \theta$$

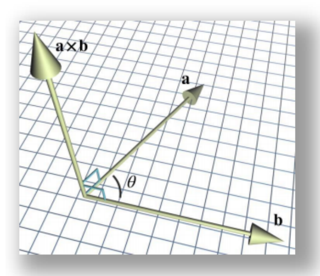

Figure 3: *Cross Product: Geometry*

4 Matrix

A matrix is a table of scalars in square brackets. Matrix dimension is the width and height of the table, w x h. In 2D space, we use 2x2 dimensions, in 3D use 3x3.

- Square Matrices: they have same number as rows as columns. Entries are called the diagonal entries. The others are called nondiagonal entries.
- Diagonal Matrices: is a square matrix whose non-diagonal elements are zero.

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Figure 4: *Square Matrices*

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Figure 5: *Diagonal Matrices*

Transpose Transpose a matrix is important and useful to calculate positions. The transpose of an $r \times x$ matrix M is a $c \times r$ matrix called M^T. What need to do is just take every row and rewrite it as a column.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Figure 6: *Transpose of a Matrix*

Transformation In linear algebra, linear transformations can be represented by matrices. If T is a linear transformation mapping R^n to R^m and $\vec{x}$ is a column vector with n entries, then for some $m \times n$ matrix A, called the transformation matrix of T.[Wikipedia]

References

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WIKIPEDIA. Transformation matrix. https://en.wikipedia.org/wiki/Transformation_matrix. Accessed: 2016-03-11. 2