Advanced Game Engineering

Zhu Yucheng
13211149
Zhejiang Normal University
Advanced Game Engineering

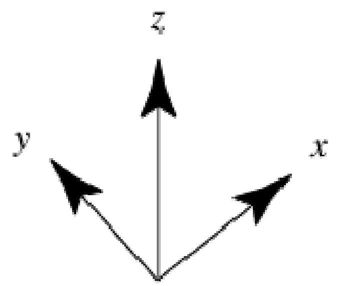
What is the difference between a point and a vector?

The difference is precisely that between *location* and *displacement*.

- Points are **locations in space**.
- Vectors are **displacements in space**.

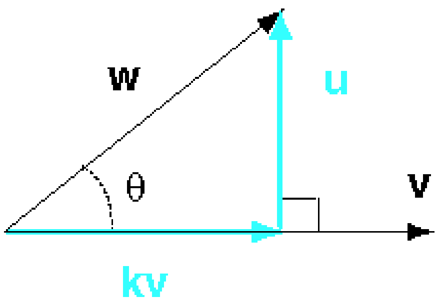
What is a right handed coordinate system?

A three-dimensional coordinate system in which the axes satisfy the right-hand rule.



right-handed coordinate system

How do you compute the projection of one vector onto another?



n the diagram w and v are any two vectors. We want a vector u that is orthogonal to v. And we want scalar k so that:

w = kv + u

Then kv is called the projection of w onto v.

What are the implicit and parametric forms of a line? Why are they named implicit and parametric?

A technique is presented for line art rendering of scenes composed of freeform surfaces. The line art that is created for parametric surfaces is practically intrinsic and is globally invariant to changes in the surface parameterization. This method is equally applicable for line art rendering of implicit forms, creating a unified line art rendering method for both parametric and implicit forms. This added flexibility exposes a new horizon of special, parameterization independent, line art effects. Moreover, the production of the line art illustrations can be combined with traditional rendering techniques such as transparency and texture mapping. Examples that demonstrate the capabilities of the proposed approach are presented for both the parametric and implicit forms.

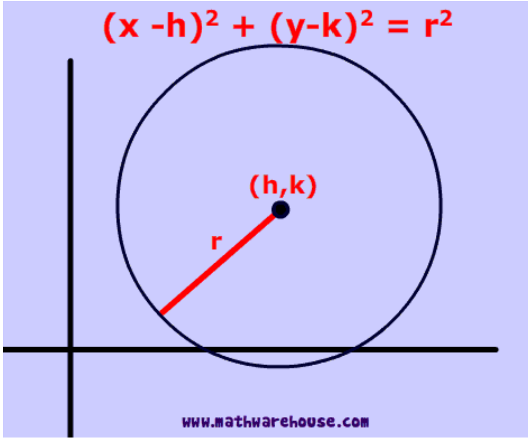
What is the implicit form of a circle centered at (xc, yc) with radius r?

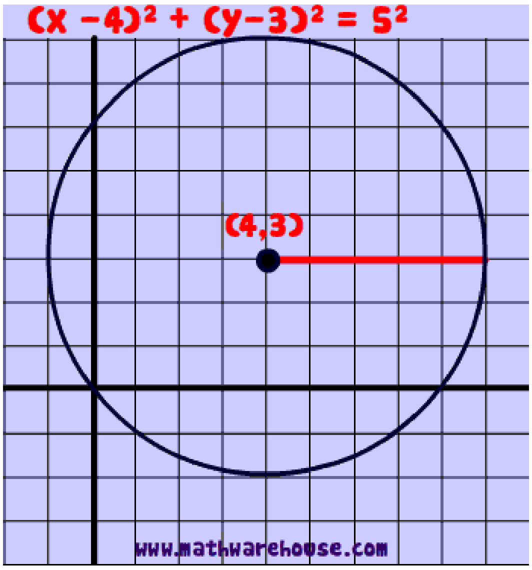
The standard form equation of a circle is a way to express the definition of a circle on the coordinate plane.

On the coordinate plane, the formula becomes (x−h)2+(y−k)2=r2

h and k are the x and y coordinates of the center of the circle

(x−9)2+(y−6)2=100 is a circle centered at (9,6) with a radius of 10





How do you convert an arbitrary vector to a unit vector?

In mathematics, a unit vector in a normed vector space is a vector (often a spatial vector) of length 1. A unit vector is often denoted by a lowercase letter with a "hat": (pronounced "i-hat"). The term direction vector is used to describe a unit vector being used to represent spatial direction, and such quantities are commonly denoted as d. Two 2D direction vectors, d1 and d2 are illustrated. 2D spatial directions represented this way are equivalent numerically to points on the unit circle. The same construct is used to specify spatial directions in 3D. As illustrated, each unique direction is equivalent numerically to a point on the unit sphere. The normalized vector or versor û of a non-zero vector u is the unit vector in the direction of u, i.e.,

$$\hat{\mathbf{u}} = \frac{\mathbf{u}}{\|\mathbf{u}\|}$$

where ||u|| is the norm (or length) of u. The term normalized vector is sometimes used as a synonym for unit vector. Unit vectors are often chosen to form the basis of a vector space. Every vector in the space may be written as a linear combination of unit vectors.

By definition, in a Euclidean space the dot product of two unit vectors is a scalar value amounting to the cosine of the smaller subtended angle. In three-dimensional Euclidean space, the cross product of two arbitrary unit vectors is a 3rd vector orthogonal to both of them having length equal to the sine of the smaller subtended angle. The normalized cross product corrects for this varying length, and yields the mutually orthogonal unit vector to the two inputs, applying the right-hand rule to resolve one of two possible directions.

How do you compute the cross product of two vectors? Informally, what does the cross product do?
Give an example where we need the cross product for Computer Graphics applications.

Example: Given $\mathbf{a} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, compute $\mathbf{a} \times \mathbf{b}$.

Solution: $\mathbf{a} \times \mathbf{b}$ is equal to

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 3 \\ -2 & 3 & 1 \end{vmatrix} = (-1-9, -(1+6), 3-2) = (-10, -7, 1).$$

What are implicit and parametric expressions for a plane?

When we have a curve in the plane,

whether given by a function or an implicit equation, we so far treated it as a shape. However, sometimes there is more information included in the situation, namely when the curve is actually a record of a movement, the path traced by, say, a bug.
The shape itself is not enough then, since two bugs can crawl along the same path in very much different ways (speed and even direction can vary, a bug can even turn back for a while and retrace the same path). This additional information has to be somehow stored and the most natural way is something called the parametric curve.
You obtain such a curve rather easily. At every time t that you are interested in you record the position in plane, that is, the x and y coordinate. Such a parametric curve is then described by two functions depending on time, x = x(t) and y = y(t) for t from some interval I. Here we will always assume that the two functions are continuous on I, so that the path is just one uninterrupted curve.
Such a parametric description carries complete information including instantaneous velocity at every time. Given the general way in which a bug can move, the curve described by a parametric equation can be very complicated and there is little hope of describing it also as a graph of a function. However, we can attempt to describe at least parts of it using functions so that we can use the arsenal we already have at our disposal, above all our skills in graph sketching.

Write an implicit equation for a plane given three points.

Thus, there are many ways to represent a plane **P**. Some methods work in any dimension, and some work only in 3D. In any dimension, one can always specify ***3 non-collinear points*** $V_0=(x_0,y_0,z_0)$, $V_1=(x_1,y_1,z_1)$, $V_2=(x_2,y_2,z_2)$ as the vertices of a triangle, the most primitive planar object. In 3D, this uniquely defines the plane of points $P=(x,y,z)$ satisfying the ***implicit equation***:

$$\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ x_1-x_0 & y_1-y_0 & z_1-z_0 \\ x_2-x_0 & y_2-y_0 & z_2-z_0 \end{vmatrix}=0$$

represented as a determinant.

Write a parametric equation for a plane given three points.

（x1,y1,z1），（x2,y2,z2），（x3,y3,z3）

| X- X1   Y-Y1   Z-Z1 |
|X1-X2   Y1-Y2   Z1-Z2 | =0
|X1-X3   Y1-Y3   Z1-Z3 |