What is the difference between a point and a vector?

Author Name Wang Jun
Zhejiang Normal University
Advanced Game Engineering

1.Introduction

A "point" is a general term, with more than one mathematical formalization. A vector, in contrast, is a very specific term which is used in a precise mathematical context - that of vector spaces.

Overview

Technically, the classical "point" is from euclidean geometry, but it is common to informally speak of any element in some mathematical space as a "point", especially if we can visualize the space. For example you can talk about some point in a metric space, or some point on a manifold. It's a relatively loose concept.Vector spaces obey a very specific set of axioms: Vectors can be added, addition distributes over multiplication by a scalar, and so on and so on. Vector spaces arise in all sorts of places in mathematics. Some intuitive and easy to visualize, others not so much.

3.Summary

It so happens that the "canonical" vector spaces, the ones that naturally spring to mind, are euclidean spaces such as R3 , where all the elements are both vectors and can be easily conceived as "points" (indeed, they formally are called that in the context of geometry). But some vector spaces deal with very different concepts than the geometry that we are used to, and calling the elements in them 'points' is weird and runs counter to intuition -- in some vector spaces the elements are polynomials, would you call a polynomial a "point"? Conversely (and much more markedly), in many, many mathematical contexts where you can intuitively speak of "points", the points are not vectors and there are no vectors anywhere to be found at all, unless you explicitly construct them - e.g. in point-set topology.