Combinatorics Summary & Practice Worksheet

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Combinatorics Review 1

Last week, we covered a variety of topics we can use for counting the size of sets. For more intuitive understanding of these topics (which you can use in case you forget the formulas!), look back at the slides from the last two sessions.

Product Rule

Sum Rule

If $A_1, \ldots A_n$ are disjoint (no elements in common) then $|A_1 \cup \cdots \cup A_n| = |A_1| + \cdots + |A_n|$

Overcounting

Think about whether it's

possible to represent one item

in multiple ways with the

system you choose—a common

mistake is counting one item multiple times.

Permutations

A permutation is an ordering

of all the items in a set. There

are n! permutations of an

n-element set.

If P_1, \ldots, P_n are sets, then $|P_1 \times \cdots \times P_n = |P_1| \cdot \cdots \cdot |P_n|$

A one-to-one mapping between the elements of two sets. If

Bijections

there is a bijection from A to B, then |A| = |B|.

Combinations

A combination is any selection of items from a set where order does not matter.

Subsets

For a set S of size n, there are 2ⁿ possible subsets, including \emptyset and S (each element must be in or out).

Division Rule

If there is a k-to-1 mapping between A and B, then $|B| = k \cdot |A|$.

Binomial Coefficient

The number of combinations of k items from an n-element set:

For the following exercises, feel free to leave your answers in terms of factorials, binomial coefficients, multiplication, etc.

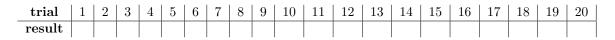
2 Poker Exercises

In Poker, certain hands—collections of 5 cards—are rated based on their rarity. Poker is played with a 52-card deck, consisting of 4 suits (Clubs, Spades, Diamonds, and Hearts) of 13 ranks: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. To practice counting, we'll find out how many ways there are to achieve each hand. As you do these problems, think about mapping from hands to sequences—what are the defining characteristics of each hand that fits the description?

e de	fining characteristics of each hand that fits the description?
1. 1	How many total 5-card hands are possible from a 52-card deck?
j	A four-of-a-kind is a set of four cards with the same rank. For example, $\{8\spadesuit, 8\diamondsuit, Q\heartsuit, 8\heartsuit, 8\spadesuit\}$ is one possible hand that contains a four-of-a-kind. How many possible four-of-a-kind hands are there?
	A Full House is a hand with three cards of one rank and two cards of another rank. How many possible Full Houses exist?
	A hand contains two pairs when it has two cards of one rank, two cards of a second rank, and another card of a third rank. How many of these hands exist?

3 Coin Flipping

Next, we're going to talk about probability. To get a sense for how probability works, we'll have everyone flip a coin 20 times. Keep track of your results in the chart below, with a "T" for tails and an "H" for heads. Then, add up your number of tails and number of heads.



num. heads	num. tails

4 Let's Make a Deal: Don't do this yet!

Later on, we'll use this chart to keep track of more results:

trial	correct door	contestant picked	host revealed	stay/switch	contestant win?
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					