Induction Exercises

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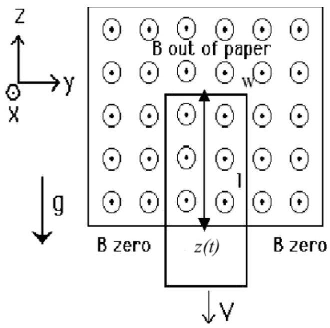
22 January 2020

1 Falling Loop

Suppose a rectangular loop with mass m, width w, vertical length l, and resistance R is falling out of a magnetic field under the influence of gravity. The magnetic field is uniform and out of the page ($\vec{B} = B\hat{x}$) within the area shown and zero outside of that area. The loop is exiting the magnetic field with $\vec{V}(t) = V(t)\hat{z}$, where $V(t) < 0$ (the loop is moving downward). Define the distance from the top of the loop to the bottom of the magnetic field at time t as $z(t)$.



(a) What is the relationship between the distance $z(t)$ and the speed $V(t)$? Remember that $z(t)$ is positive and decreasing in time, and $V(t)$ is negative.

(b) If we define the area vector $\vec{A}$ to be out of the page, what is the magnetic flux Φ_B through the circuit at time t (in terms of $z(t)$, not $V(t)$.).

(d) Find $d\Phi_B/dt$. Is it positive or negative at time t? Your answer should include $V(t)$.

(e) What is the direction of the induced current in the loop?

(f) What is the direction of the magnetic field created by this current?

(g) What is the magnitude of the current flowing in the circuit at the time t?

(h) What other force (besides gravity) is acting on the loop in the $\pm\hat{z}$ direction? Give the magnitude and direction of this force in terms of the quantities given.

(i) Assume the loop has reached terminal velocity—it is no longer accelerating. What is the magnitude of that terminal velocity in terms of the given quantities?

2 Power Transmission

You've started a new job at the electric company, and want to determine the best way to send power across Ancona. The stretch of power lines you are working with has a total resistance of $.40\Omega$, and the solar panels you are using as a power source generate $240V$ power. On average, $120kW$ of power is sent through the lines. You are considering using a transformer, and want to minimize power loss during transmission.

(a) Calculate the power loss over the length of the transmission lines without the transformer.

(b) Calculate the power loss through the transmission lines with a transformer added between the solar panels and the transmission lines. The transformer has 10 primary (input) turns and 1,000 secondary (output) turns.

(c) Should you use the transformer in this case? How could you decrease power loss further?

3 Circuit Concepts

A circuit consists of two resistors R_1 and R_2, a capacitor, and a switch. The switch has been open for a very long time and is closed at $t = 0$.

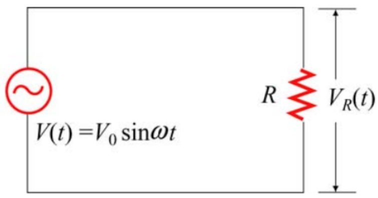
1. What is the current through the capacitor at $t = 0^+$, just after the switch is closed?

2. What are the currents through R_1 and R_2 at $t = 0^+$?

3. What is the current through the capacitor at $t = \infty$?

4. What are the currents through R_1 and R_2 at $t = \infty$?

4 Purely Resistive Circuit



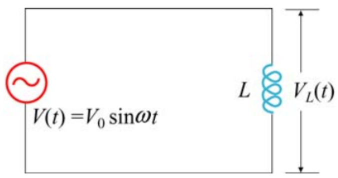
Recall that for AC Circuits, we define $V(t) = V_0 sin(\omega t)$ and $I(t) = I_0 sin(\omega t - \phi)$. The above circuit consists of an AC voltage source with voltage defined by $V(t) = V_0 sin(\omega t)$ and a resistor with resistance R.

(a) Use Kirchoff's loop rule to find the circuit equation.

(b) Find the values of I_0 and ϕ for this circuit.

(c) Find the room-mean-squared power P_{rms} disappated by this circuit. Recall that $\frac{1}{T}\int_0^T sin^2(t) = \frac{1}{2}$.

5 Purely Inductive Circuit

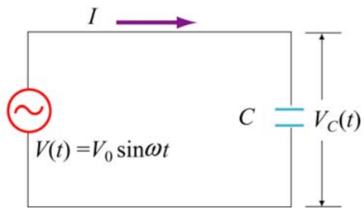


Now, the same voltage source with $V(t) = V_0 sin(\omega t)$ is connected to an inductor with inductance L.

(a) Use Kirchoff's loop rule to find the circuit equation.

(b) For current in the form $I(t) = I_0 sin(\omega t - \phi)$, find the values of I_0 and ϕ for this circuit.

6 Purely Capacative Circuit



Finally, the same voltage source is connected to only a capacitor. Once again, voltage is given by $V(t) = V_0 sin(\omega t)$.

(a) Use Kirchoff's loop rule to find the circuit equation.

(b) For current in the form $I(t) = I_0 sin(\omega t - \phi)$, find the values of I_0 and ϕ for this circuit.