Combinatorics Summary & Practice Worksheet

Ben Kettle

22 January 2020

1 Combinatorics Review

Last week, we covered a variety of topics we can use for counting the **size** of **sets**. For more intuitive understanding of these topics (which you can use in case you forget the formulas!), look back at the slides from the last two sessions.

Sum Rule
If $A_1, \dots A_n$ are disjoint (no elements in common) then $|A_1 \cup \dots \cup A_n| = |A_1| + \dots + |A_n|$

Product Rule
If $P_1, \dots, P_n$ are sets, then $|P_1 \times \dots \times P_n = |P_1| \cdot \dots \cdot |P_n|$

Subsets
For a set S of size n, there are 2^n possible subsets, including $\varnothing$ and S (each element must be in or out).

Overcounting
Think about whether it's possible to represent one item in multiple ways with the system you choose—a common mistake is counting one item multiple times.

Bijections
A one-to-one mapping between the elements of two sets. If there is a bijection from A to B, then $|A| = |B|$.

Division Rule
If there is a k-to-1 mapping between A and B, then $|B| = k \cdot |A|$.

Permutations
A permutation is an ordering of all the items in a set. There are $n!$ permutations of an n-element set.

Combinations
A combination is any selection of items from a set where order does not matter.

Binomial Coefficient
The number of combinations of k items from an n-element set:

$$\binom{n}{k}$$

For the following exercises, feel free to leave your answers in terms of factorials, binomial coefficients, multiplication, etc.

2 Poker Exercises

In Poker, certain hands—collections of 5 cards—are rated based on their rarity. Poker is played with a 52-card deck, consisting of 4 **suits** (Clubs, Spades, Diamonds, and Hearts) of 13 **ranks**: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. To practice counting, we'll find out how many ways there are to achieve each hand. As you do these problems, think about mapping from hands to sequences—what are the defining characteristics of each hand that fits the description?

1. How many total 5-card hands are possible from a 52-card deck?

2. A four-of-a-kind is a set of four cards with the same rank. For example, $\{8\spadesuit, 8\diamond, Q\heartsuit, 8\heartsuit, 8\clubsuit\}$ is one possible hand that contains a four-of-a-kind. How many possible four-of-a-kind hands are there?

3. A Full House is a hand with three cards of one rank and two cards of another rank. How many possible Full Houses exist?

4. A hand contains two pairs when it has two cards of one rank, two cards of a second rank, and another card of a third rank. How many of these hands exist?

3 Coin Flipping

Next, we're going to talk about probability. To get a sense for how probability works, we'll have everyone flip a coin 20 times. Keep track of your results in the chart below, with a "T" for tails and an "H" for heads. Then, add up your number of tails and number of heads.

trial	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
result																				

num. heads	num. tails

4 Let's Make a Deal: Don't do this yet!

Later on, we'll use this chart to keep track of more results:

trial	correct door	contestant picked	host revealed	stay/switch	contestant win?
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					