Assignment 1
FIE450
Spring 2021

Before you start read the following points carefully!

1. For each of the four tasks use a separate `R`-script labeled `Assignment-1-1.R`, `Assignment-1-2.R`, etc.
2. Start each `R`-script with the line `rm(list=ls())`.
3. Assign the results of each subtask to the variable names given in brackets. Only these variables are considered for grading. If they are missing or misspelled then no points are given for this subtask.
4. Do not round results.
5. Do not use any packages.

Task 1 (8 points)

Use the price data on Equinor given in `Data-EQNR-OL.csv` to estimate annual expected returns.

1. Estimate the annual expected log return and a corresponding 99% confidence interval based on all available adjusted close prices. [`mu1`, `ci1`]
2. Estimate the annual expected log return and a corresponding 99% confidence interval based on the adjusted close prices starting with February 01, 2016. [`mu2`, `ci2`]
3. Which estimate in (a) and (b) is more accurate and why? [*Explain in one line as a comment in your code.*]
4. Quantify *exactly* by how much one estimate is more accurate than the other? [`f`]

Task 2 (14 points)

Simulate 1000 paths of stock prices 5 years into the future. Assume that the current stock price is 100 kr, the expected rate of return is 5% p.a. and the volatility is 10% p.a.

1. Simulate stock price paths using a daily sampling frequency. Estimate the annual expected percentage return for each scenario. Based on these estimates compute the standard error. [`se.a`]
2. Use the first path in (a) to compute the annualized standard error. [`se.b`]
3. Simulate stock price paths using a monthly sampling frequency. Estimate the annual expected percentage return for each scenario. Based on these estimates compute the standard error. [`se.c`]
4. Use the first path in (c) to compute the annualized standard error. [`se.d`]
5. Compare all four standard errors qualitatively. What do you find? [*Explain in one line as a comment in your code.*]
6. Which sampling frequency do you prefer and why? [*Explain in one line as a comment in your code.*]

Task 3 (4 points)

Estimate the implied volatility of a put option on the OBX index with Strike 880.00 and expiration date February 19, 2021. Use the market quotes obtained for Jan 26, 2021 and given in Figures 1 and 2. Assume the risk-free rate is 1% p.a.

1. Estimate the implied volatility using the bid price. [`sigma.bid`]
2. Estimate the implied volatility using the ask price. [`sigma.ask`]

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SETTL.	LAST	BID	ASK		STRIKE		BID	ASK	LAST	SETTL.
52.07	-	51.50	55.25	C	830.00	P	4.60	5.10	-	5.45
43.29	-	42.75	46.50	C	840.00	P	5.90	6.40	7.20	6.37
35.05	-	34.75	37.75	C	850.00	P	7.70	8.25	9.25	8.09
27.12	-	27.50	29.75	C	860.00	P	9.75	10.50	-	10.34
20.20	-	20.75	22.25	C	870.00	P	12.50	13.75	-	13.55
14.09	-	14.50	16.00	C	880.00	P	16.25	17.50	-	17.74
9.32	-	9.75	10.75	C	890.00	P	20.75	22.50	-	22.67
5.77	-	5.80	6.80	C	900.00	P	26.75	29.00	-	28.98
3.36	-	3.15	4.00	C	910.00	P	33.75	36.50	-	36.44
1.84	-	1.45	2.25	C	920.00	P	41.50	45.00	-	44.69
0.99	-	0.50	1.30	C	930.00	P	50.50	54.25	-	53.74

Updated 26/01/2021 15:08:54

Figure 1: Market data on put and call options on the OBX index on Feb 26, 2021.

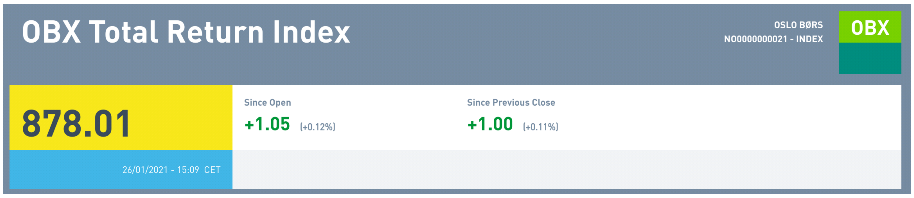

Figure 2: OBX index market data on Feb 26, 2021.

Task 4 (14 points)

Estimate the value of an at-the-money *down-and-in barrier call option* on OBX based on the market information for Jan 26, 2021 and shown in Figure 2. Assume the stock index volatility to be 18% p.a. The barrier shall be $b = 850$. The payoff function at maturity in 1.25 years is given by

$$I\{\tau(b) \leq T\}(S(T) - K)^+, \tag{1}$$

where

$$\tau(b) = \inf\{t_i : S(t_i) < b\} \tag{2}$$

is the first time $t_i \in \{0.25, 0.5, 0.75, 1, 1.25\}$ the price of the underlying asset S drops below b and $I\{\}$ denotes the indicator of the event in braces, i.e. I is one if the expression within braces is true and zero otherwise. Thus, a down-and-in call gets "knocked in" only when the underlying asset crosses the barrier b from above.
Incorporate the term structure of risk-free interest rates into your Monte-Carlo simulation framework using

$$S(t_{i+1}) = S(t_i)\frac{Z_i(t_0)}{Z_{i+1}(t_0)}\exp\left(-\frac{1}{2}\sigma^2(t_{i+1}-t_i) + \sqrt{t_{i+1}-t_i}\sigma W_{i+1}\right), \tag{3}$$

where, $Z_i(t_0)$ denotes the price of a *risk-free* zero coupon bond at t_0 with maturity t_i and W is a standard normal distributed random variable. Simulate $n = 10000$ price paths in total.

Assume that the term structure of risk-free interest rates is given by the prices of *risk-free* zero coupon bonds, $Z_i(t_0)$, at time t_0 and with maturity $t_i \in \{0.25, 0.5, 0.75, 1, 1.25\}$ years. That is, the price of 1 kr received at t_i is worth Z_i kr at t_0. The following zero coupon bond prices are given

$$Z_1(T_0) = 0.99 \quad Z_2(T_0) = 0.98 \quad Z_3(T_0) = 0.97 \quad Z_4(T_0) = 0.96 \quad Z_5(T_0) = 0.95$$

1. Estimate the option price. [`V`]
2. Compute a 99.9% confidence interval for the Monte Carlo estimator in (a). [`ci`]
3. Estimate the option price using Antithetic variates. [`V.as`]
4. Compute a 99.9% confidence interval for the Monte Carlo estimator in (c). [`ci.as`]
5. Does antithetic sampling improve the accuracy of the estimator? [`rho`]