

Production of photons (and not only) by high-energy particles (and not only)



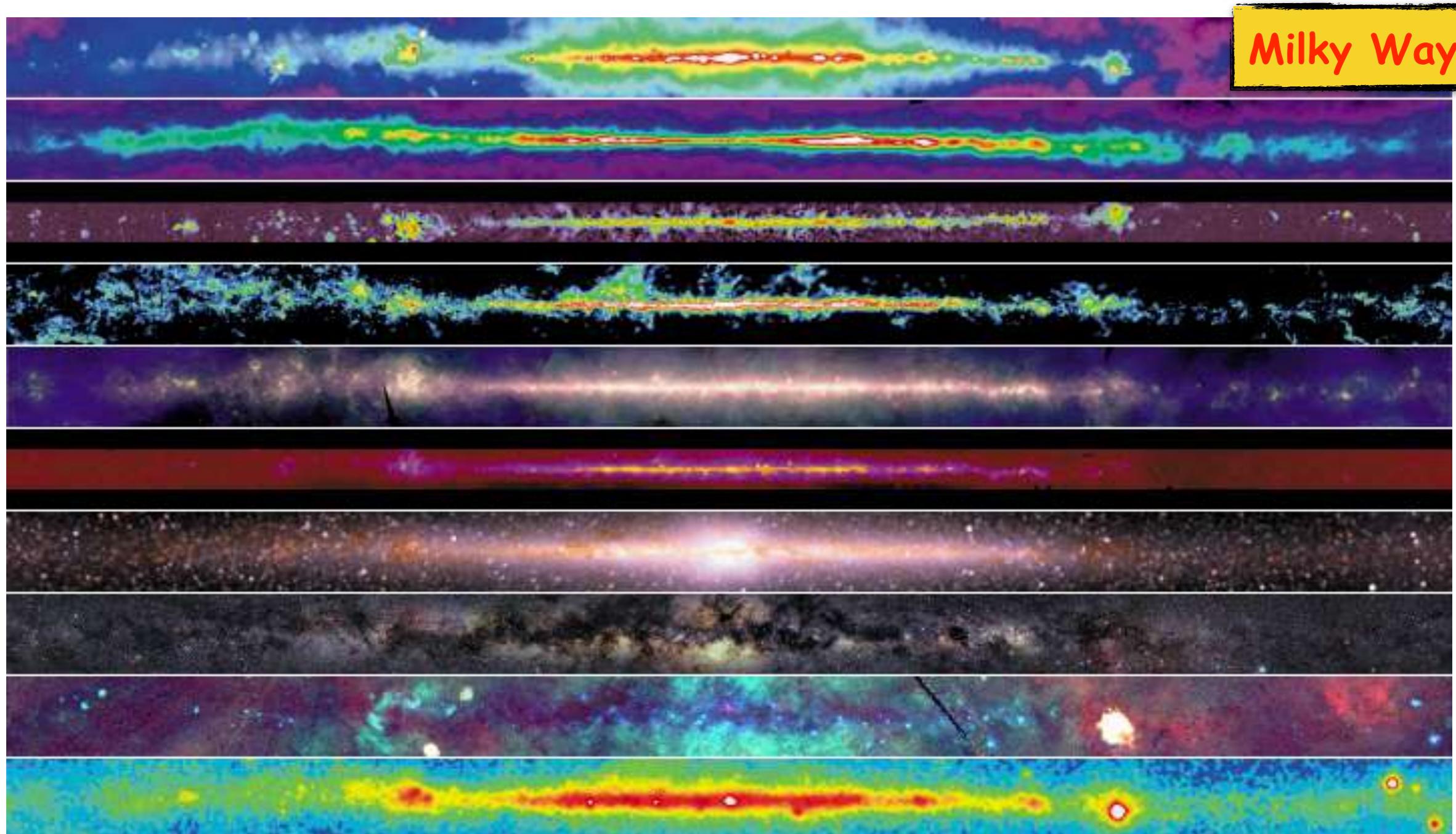
Stefano Gabici
APC, Paris



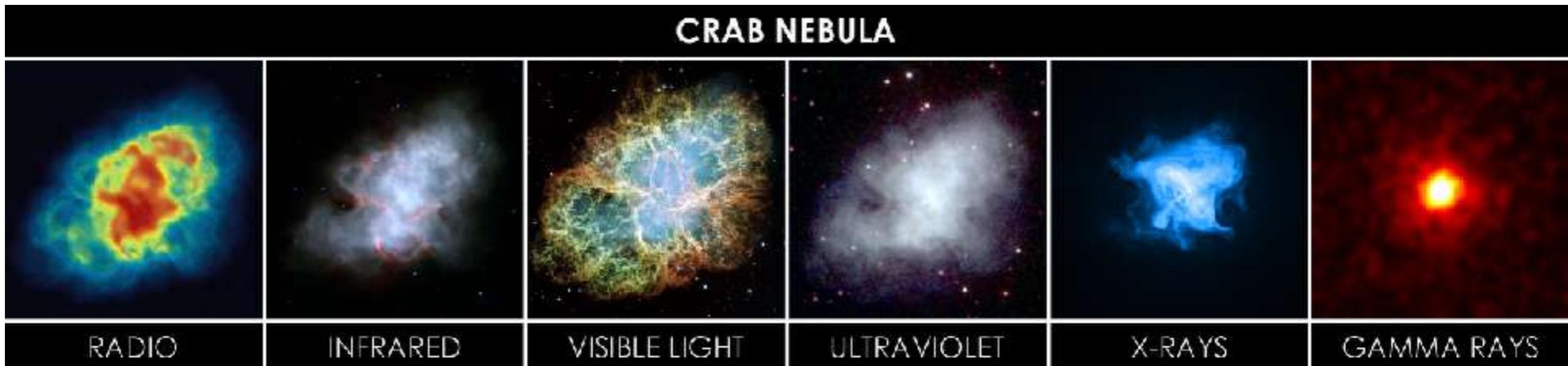
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Milky Way

radio waves — gamma-rays



CRAB NEBULA

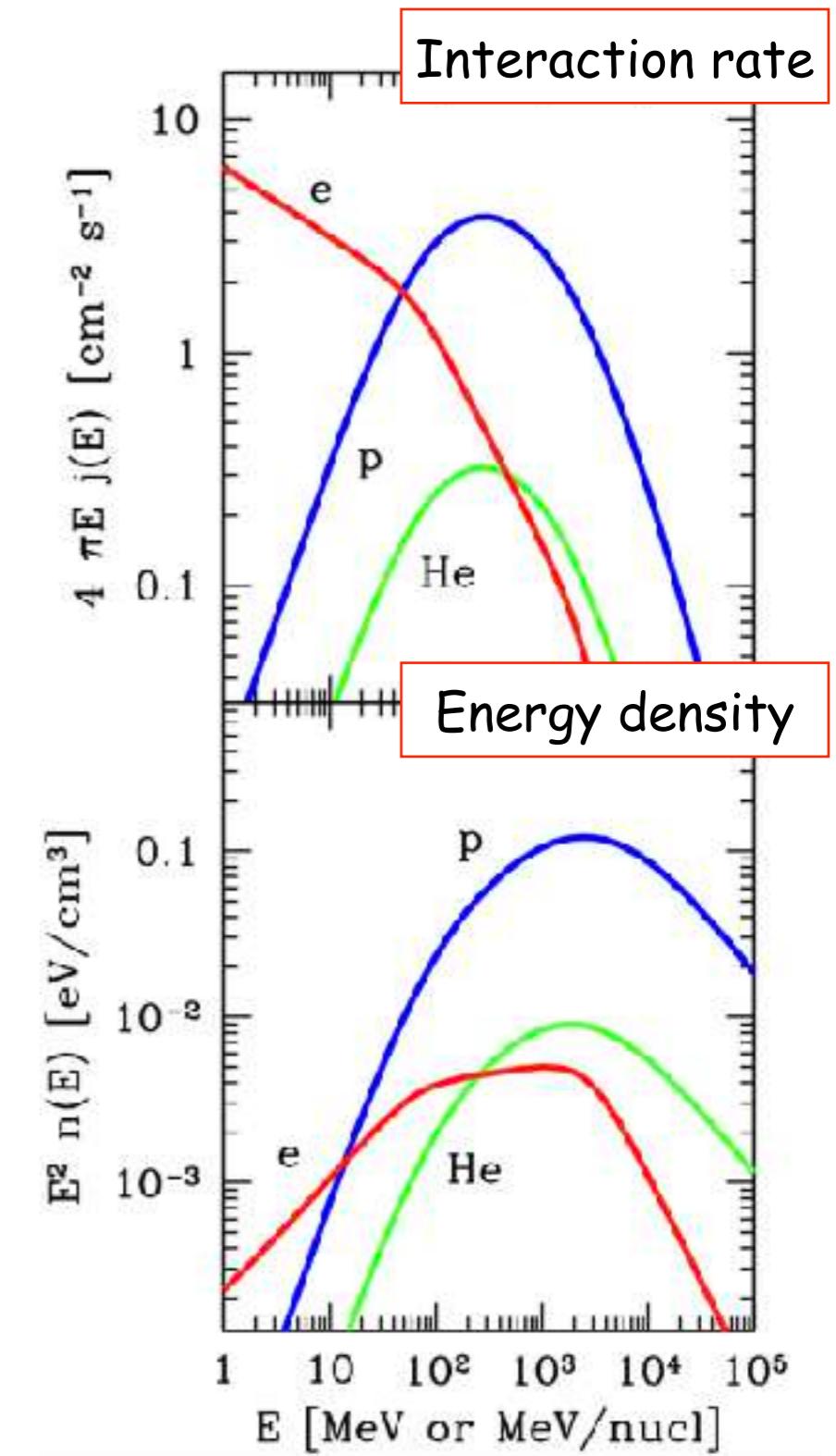
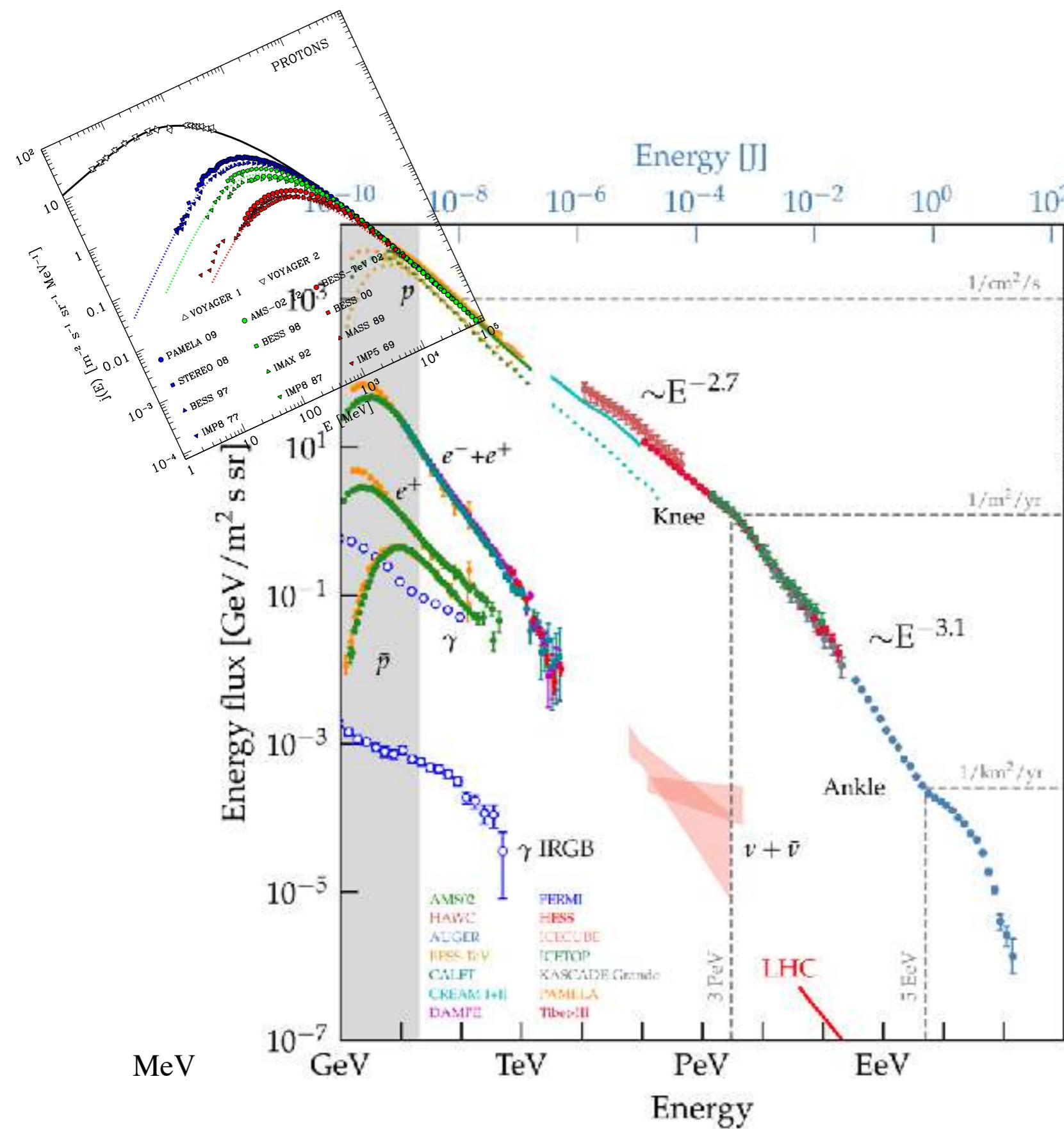


it is impossible to review in a decent way “all” radiative processes in 1.5 h*

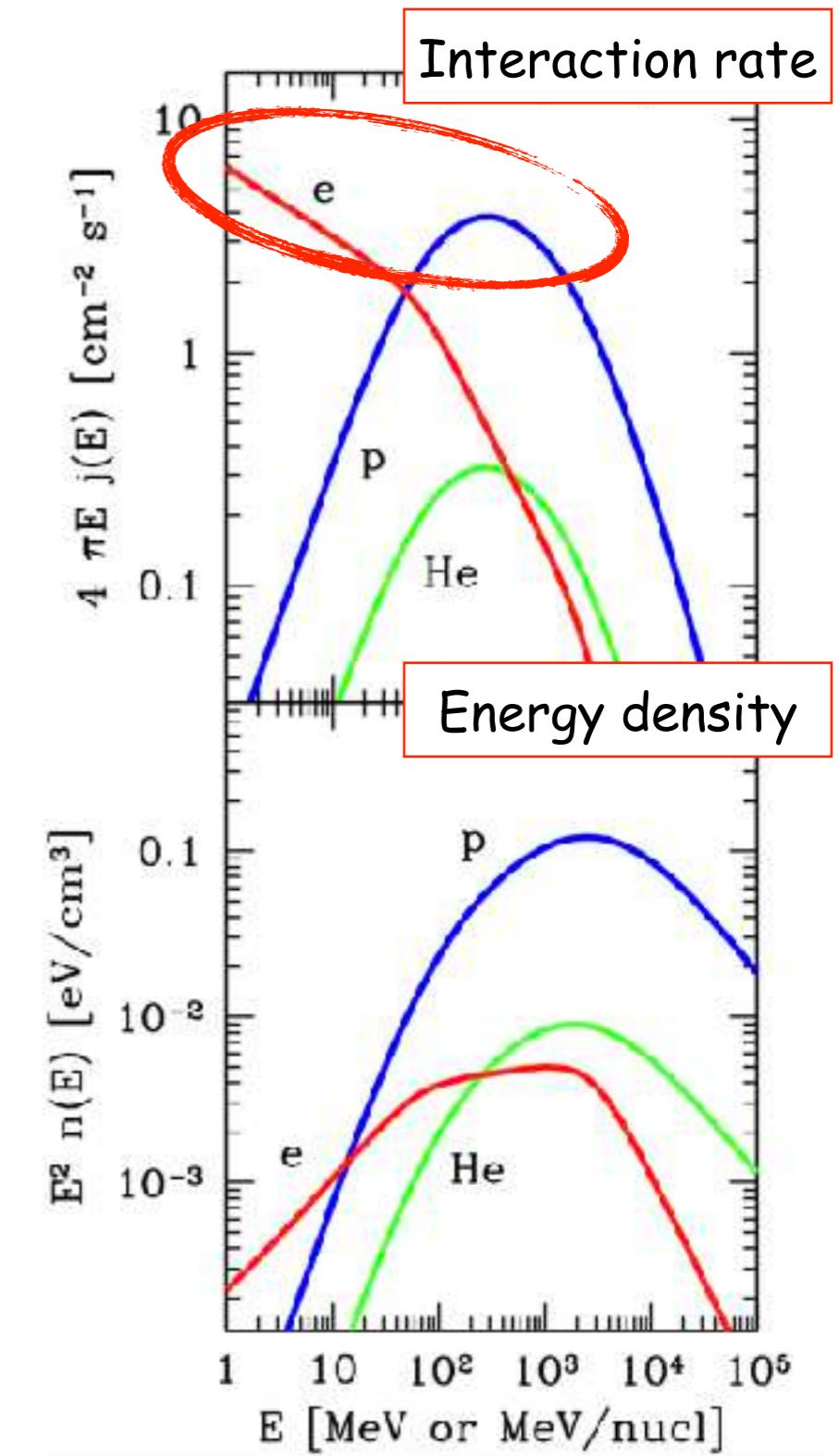
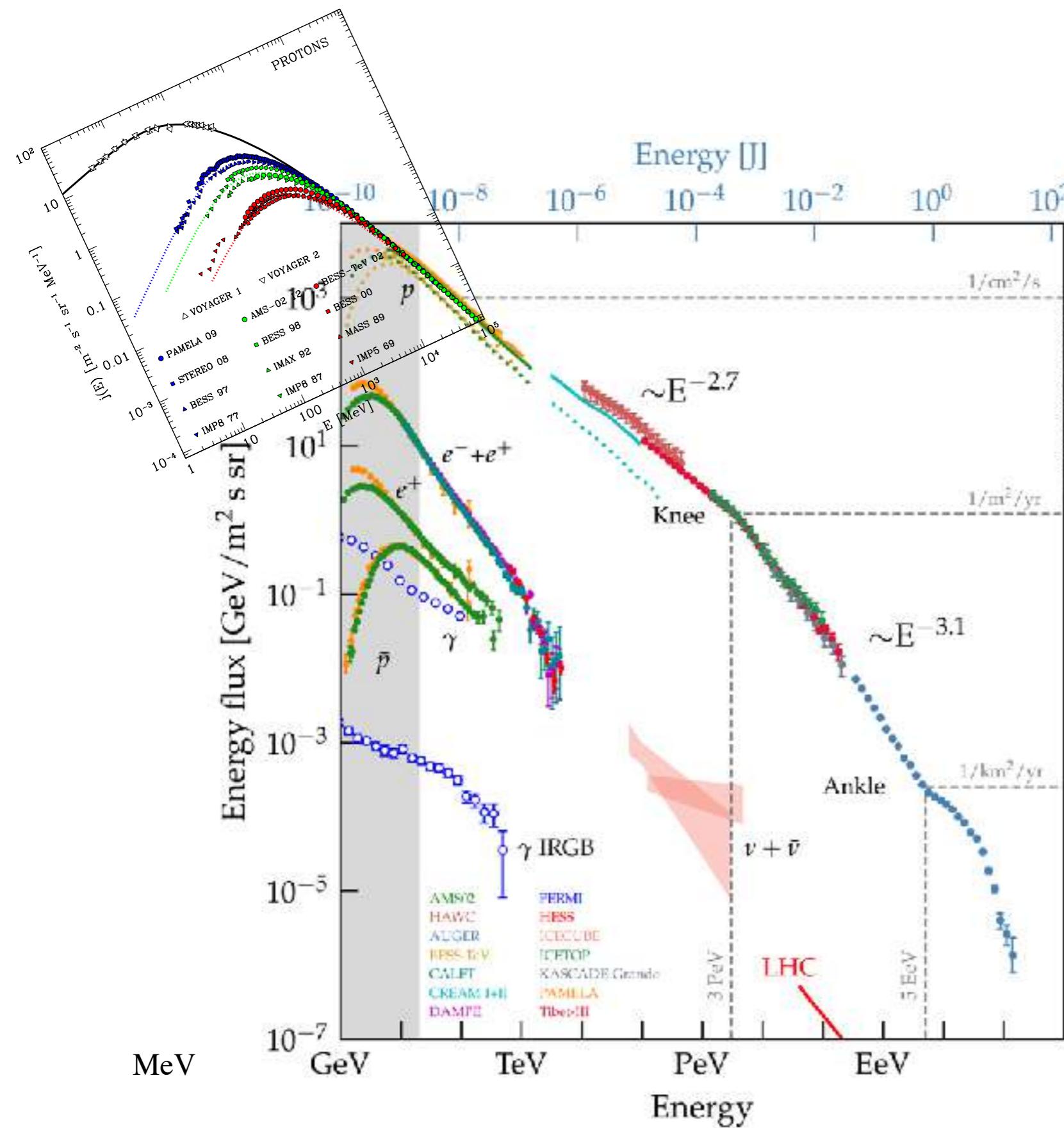


* see also Matteo's classes

One question every 3 (energy) decades...

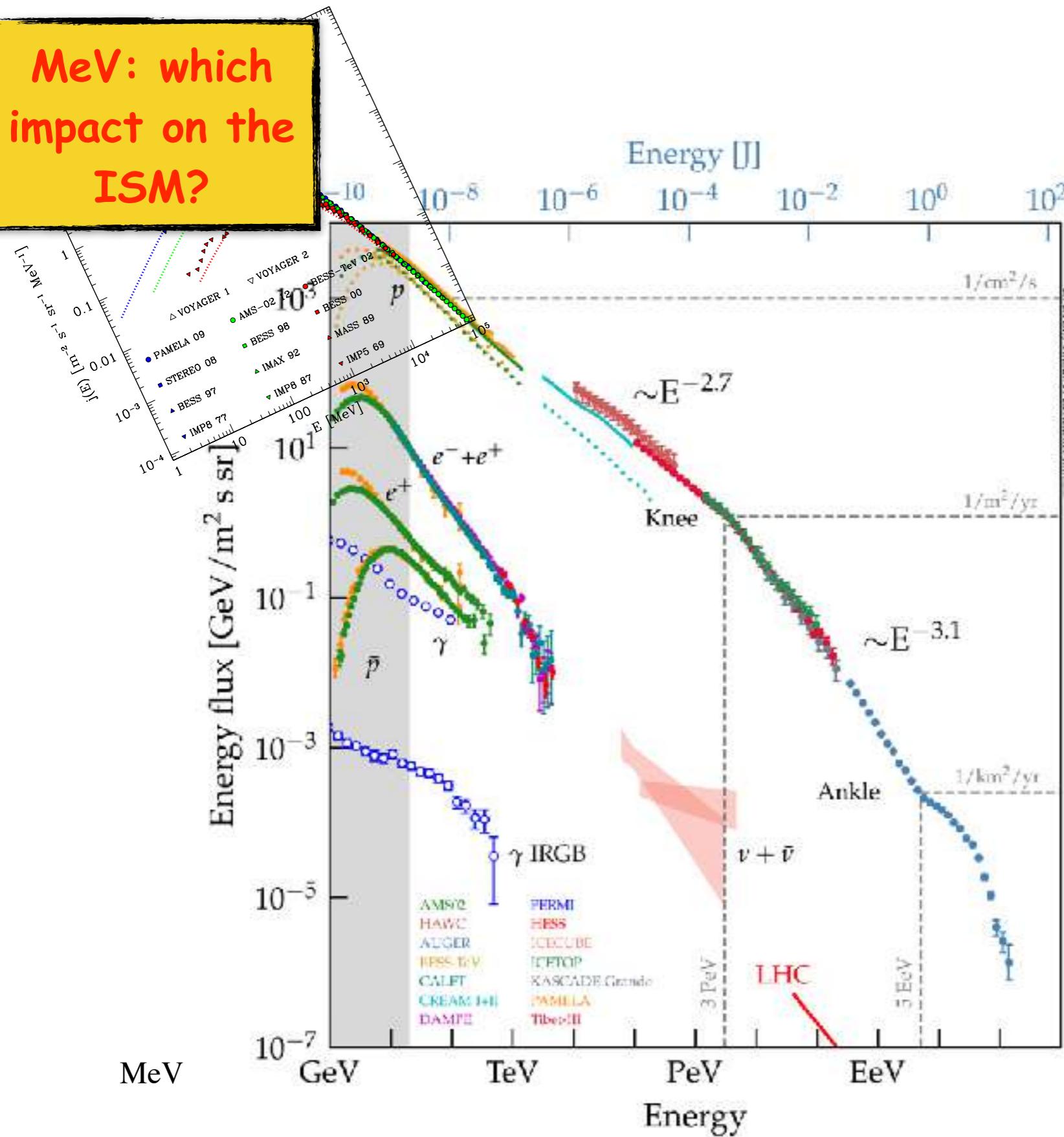


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MeV: which impact on the ISM?



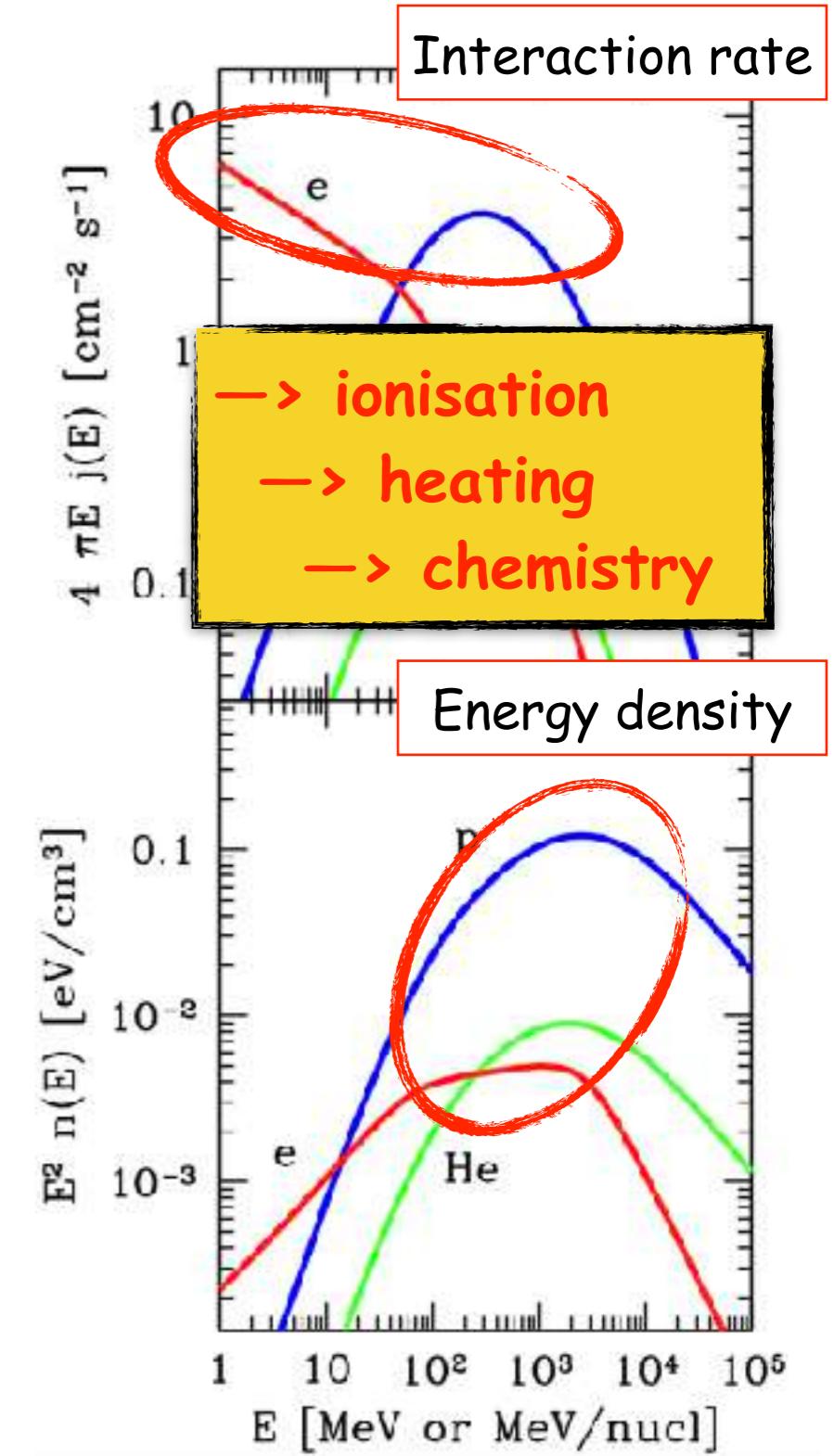
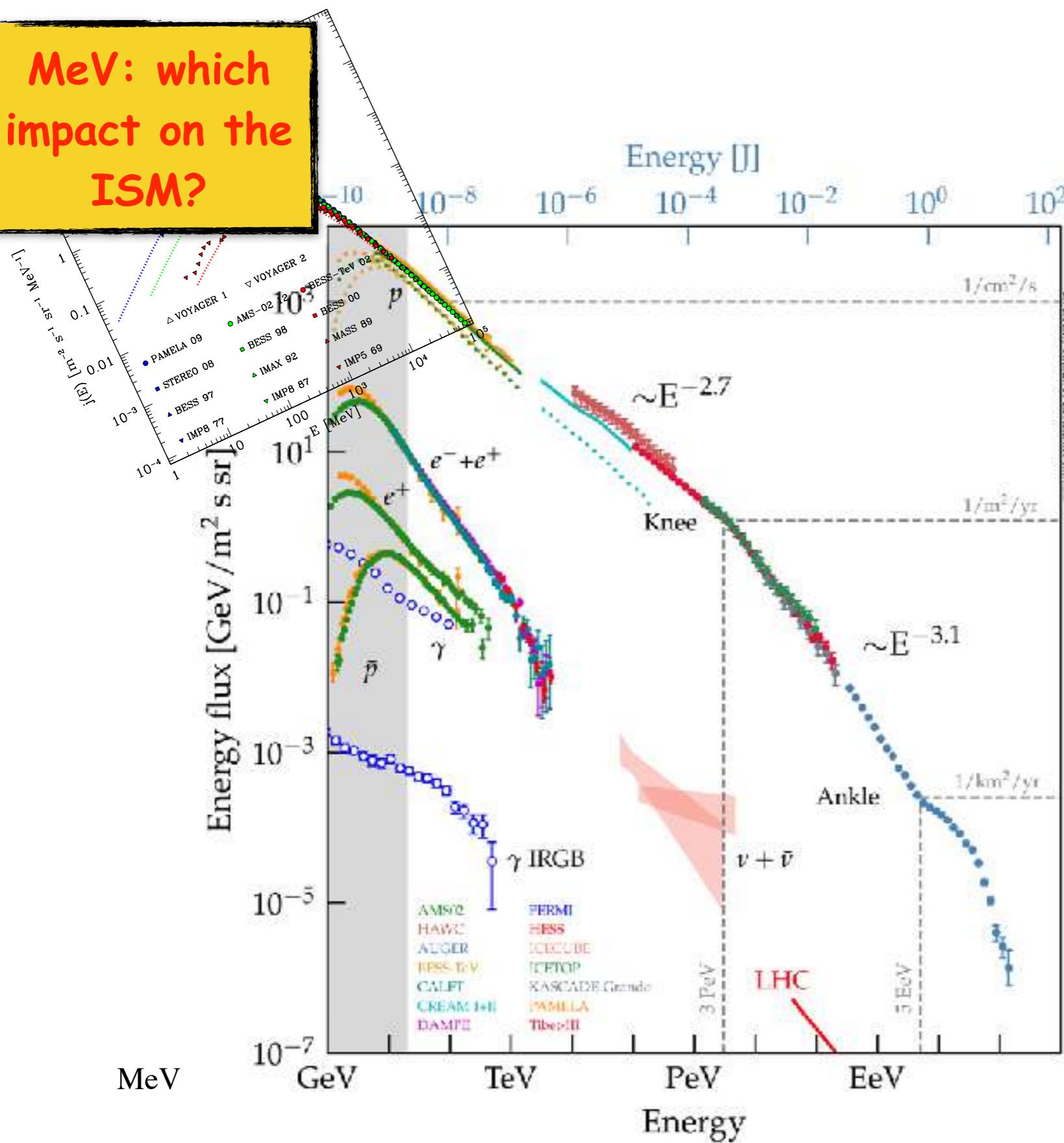
Interaction rate

- ionisation
- heating
- chemistry

Energy density

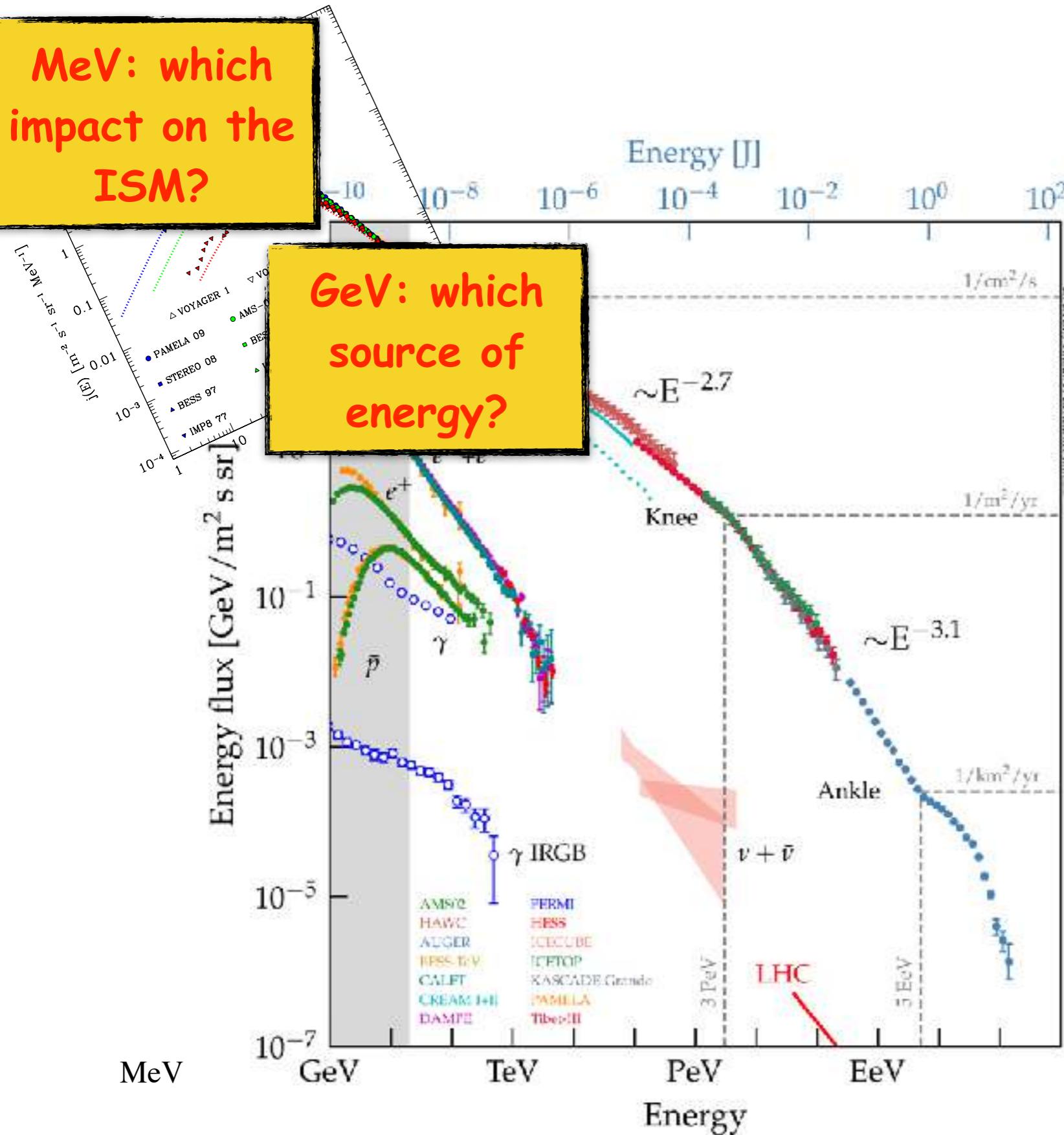
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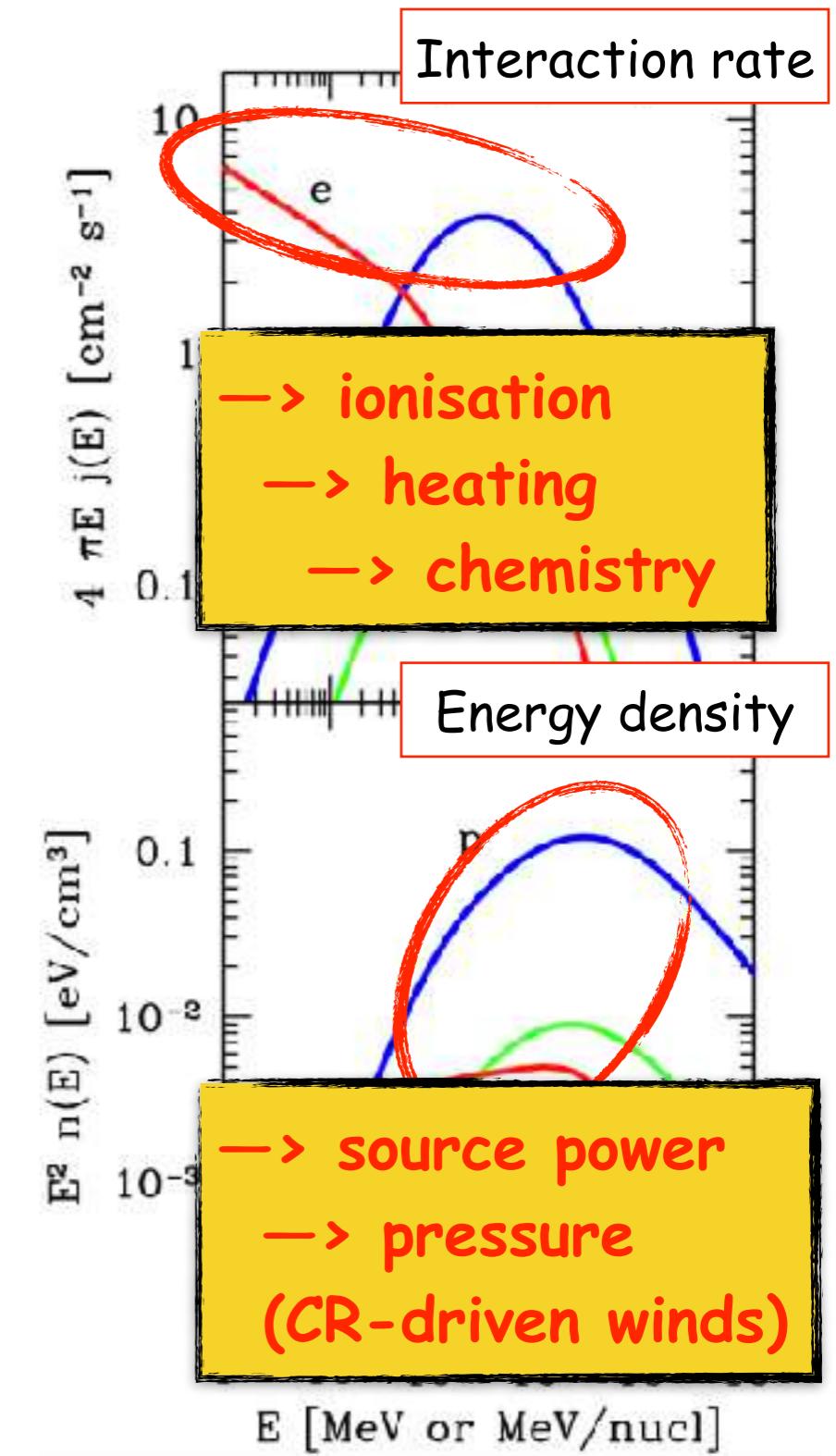


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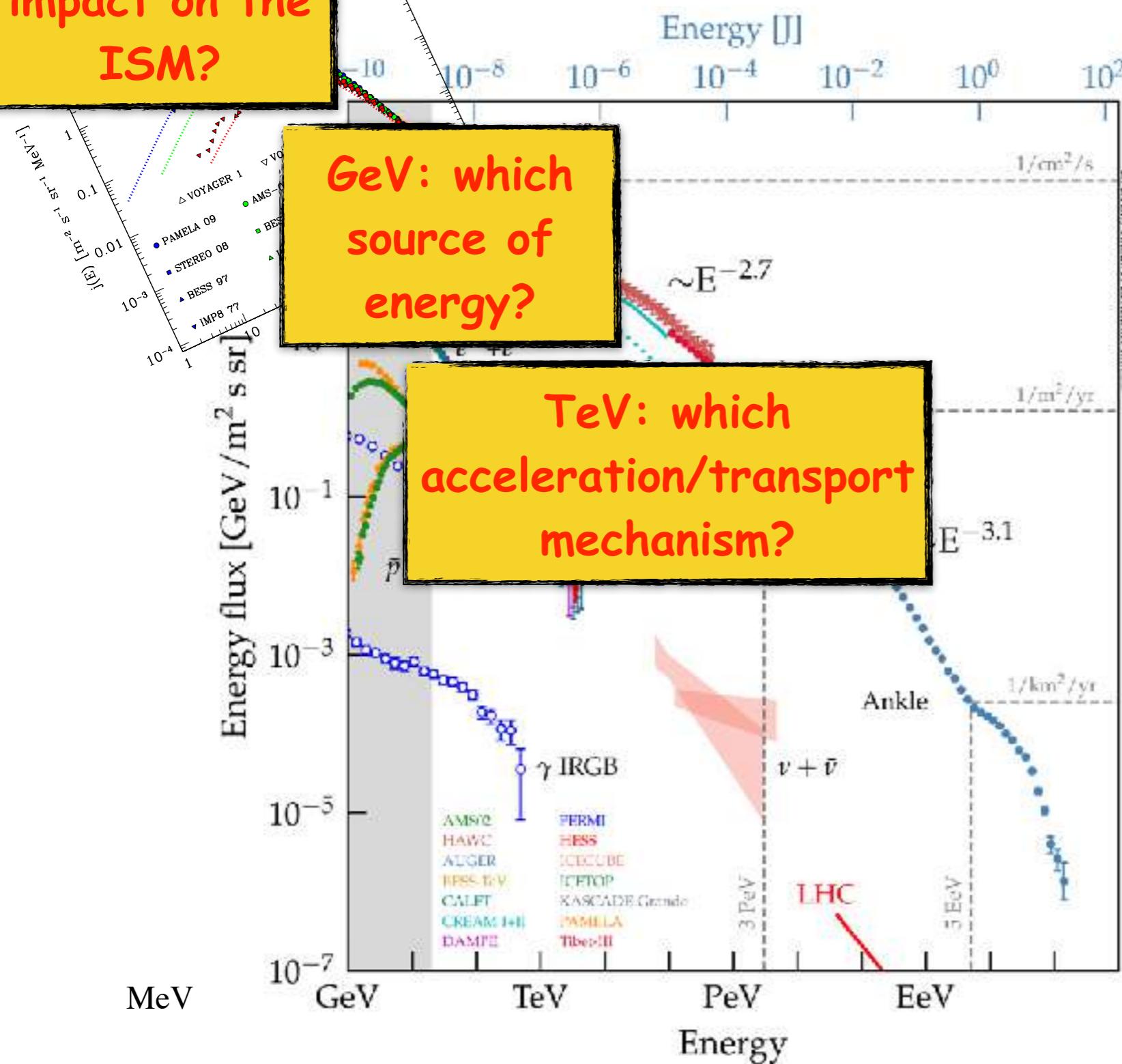


GeV: which source of energy?



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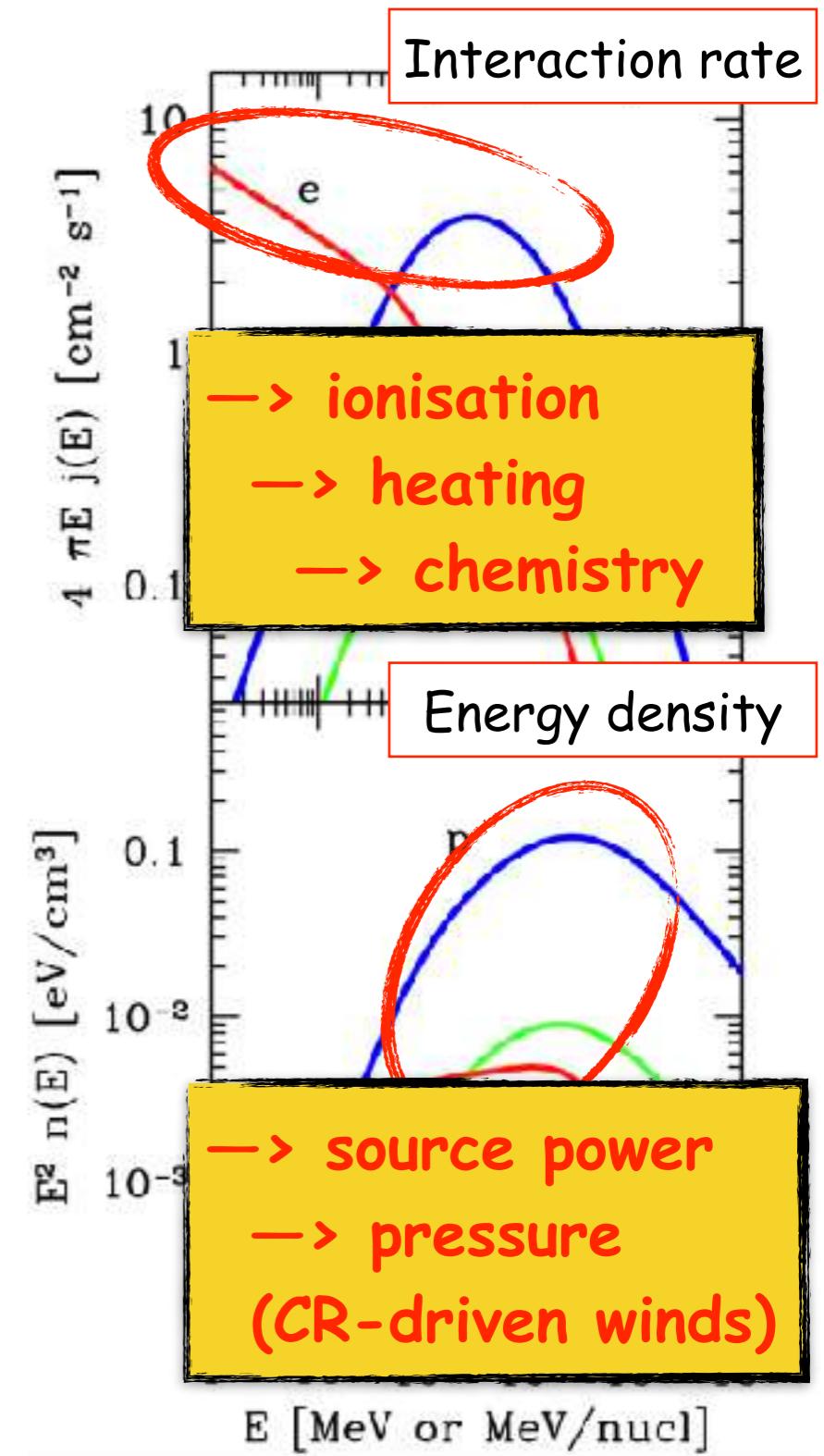
TeV: which acceleration/transport mechanism?

Interaction rate

→ ionisation
→ heating
→ chemistry

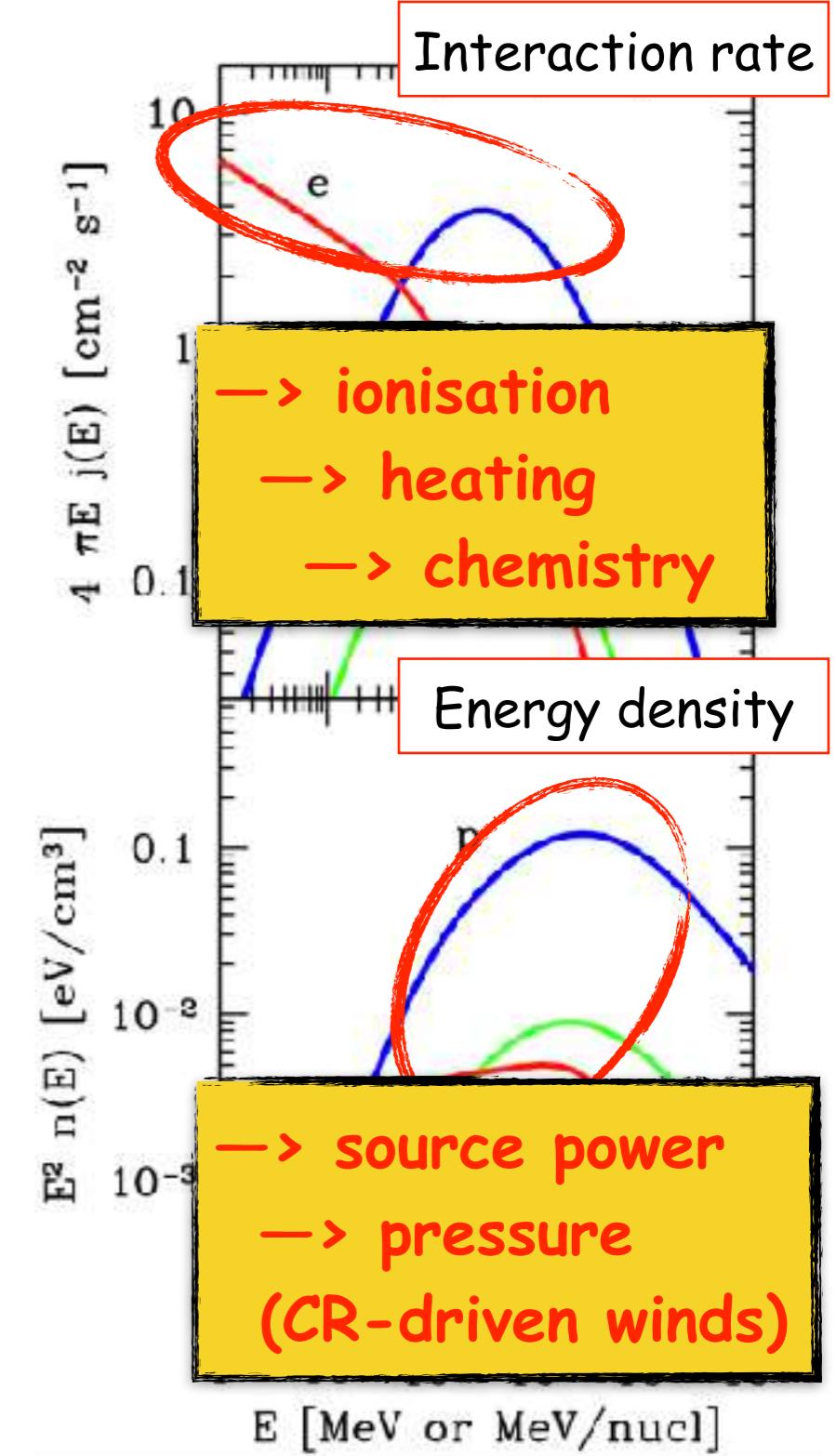
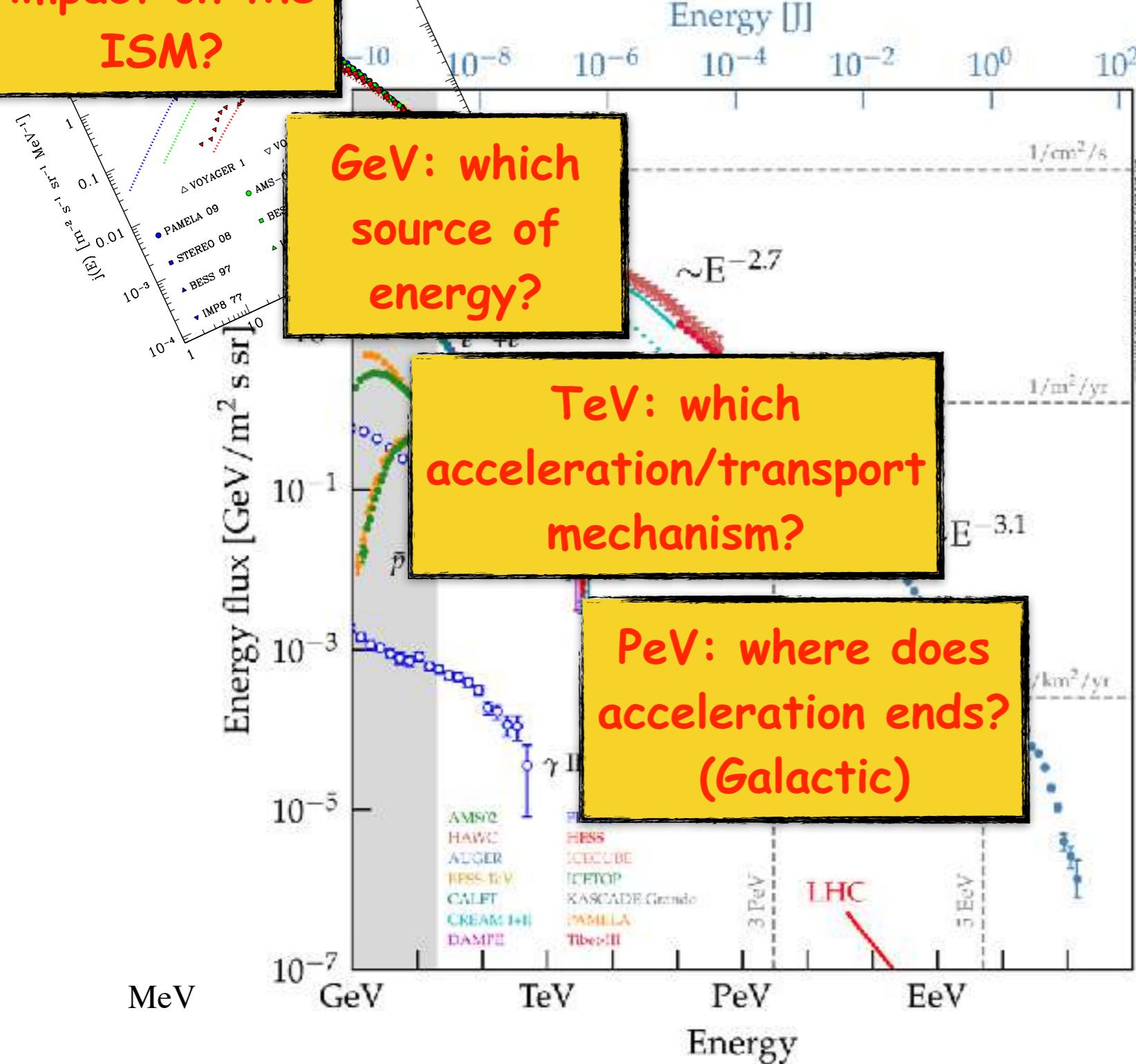
Energy density

→ source power
→ pressure
(CR-driven winds)



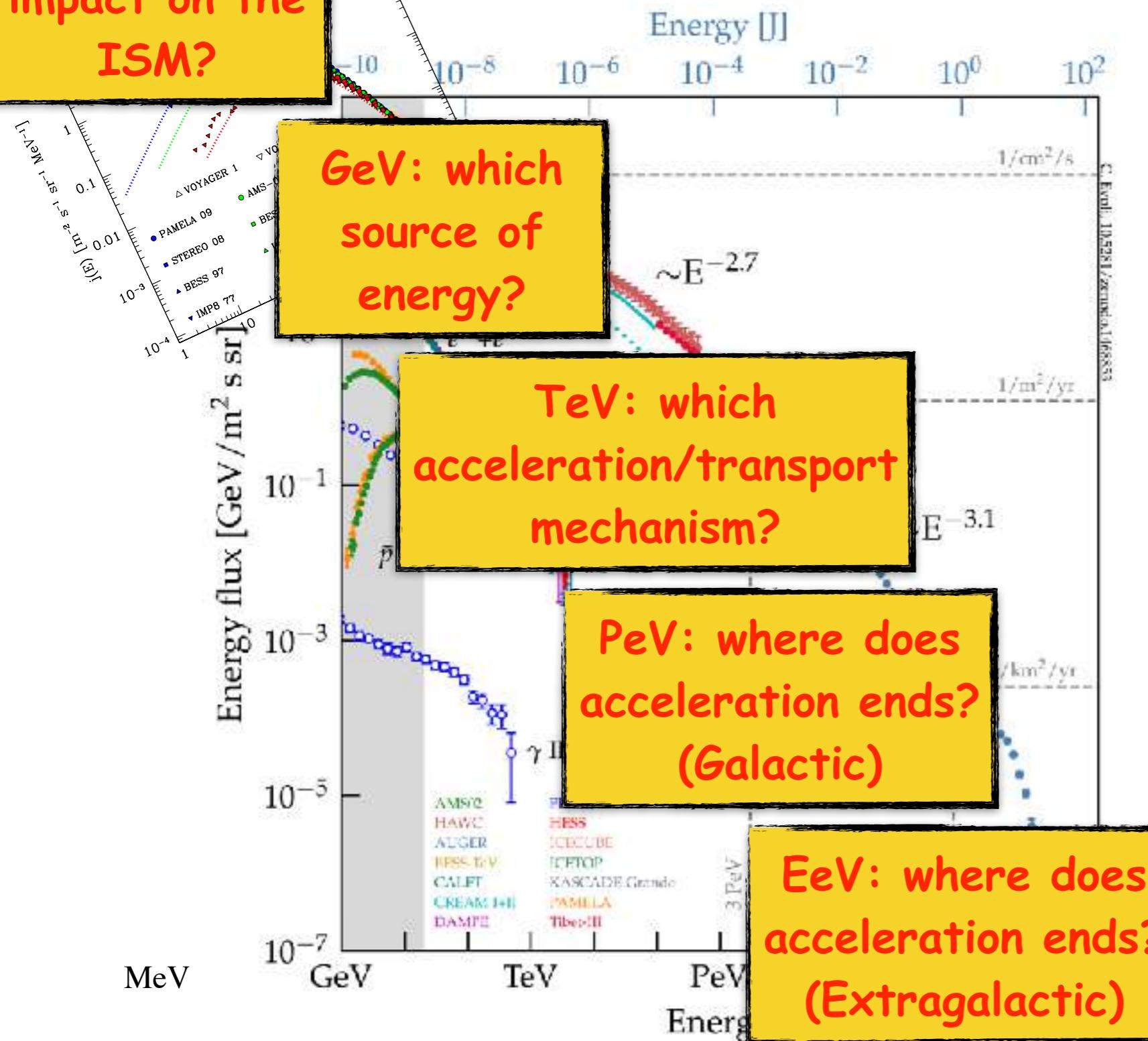
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Astrophysical environments

we will mostly discuss radiation produced
in interactions between energetic particles and matter or radiation/B-fields

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interstellar medium (ISM)

filling factor
↓

Phase	n_{tot} (cm $^{-3}$)	T (K)	M (10 $^9 M_{\odot}$)	f
Molecular	>300	10	2.0	0.01
Cold neutral	50	80	3.0	0.04
Warm Neutral	0.5	8,000	4.0	0.3
Warm ionized	0.3	8,000	1.0	0.15
Hot ionized	0.003	500,000	—	0.5

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interstellar
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$$B \sim 3 \mu\text{G}$$

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a remarkable
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just a coincidence)

$$\frac{B^2}{8\pi} \sim \omega_{CMB} \sim 0.25 \text{ eV/cm}^3$$

Astrophysical environments

we will mostly discuss
in int. but not only → high energy radiation without non-thermal particles
molecules and matter or radiation/B-fields

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Thermal emission

let's begin with estimating the emission of a thermal plasma of temperature T

Simplifying assumptions: fully ionised hydrogen plasma ($n_i = n_e$), protons and electrons in thermal equilibrium (same temperature T)

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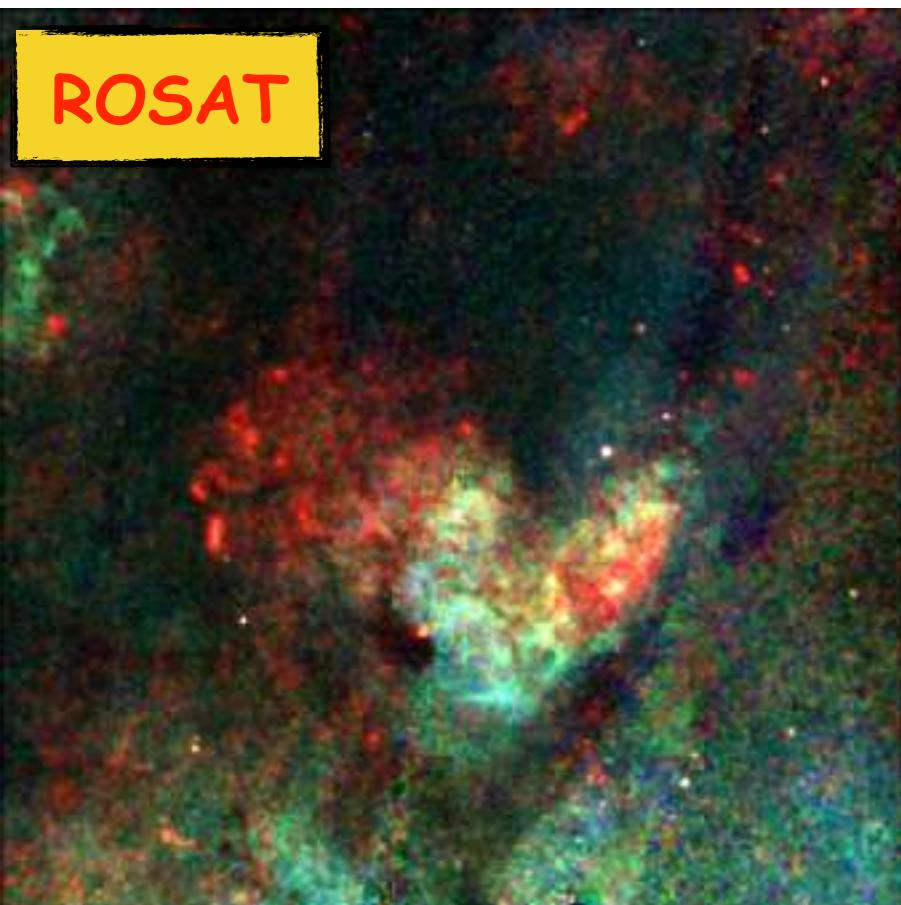
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ROSAT



→ the hot phase consists of cavities inflated by stellar winds or generated by supernova remnants and emits thermal X-rays

X-ray telescopes onboard of satellites: XMM, Chandra, NuSTAR, eROSITA, ...

Emission from hot plasmas: thermal Bremsstrahlung

Radiation from a thermal electron-proton plasma

Emission from hot plasmas: thermal Bremsstrahlung

Radiation from a thermal electron-proton plasma

$$v = \sqrt{\frac{3kT}{m}} \rightarrow v_e = \left(\frac{m_p}{m_e}\right)^{1/2} v_p \gg v_p$$

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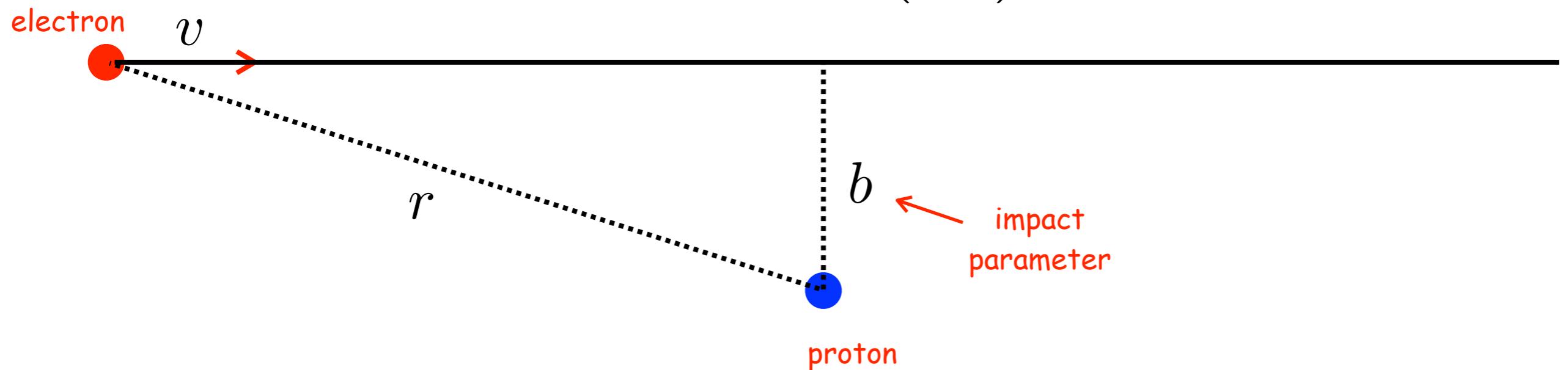
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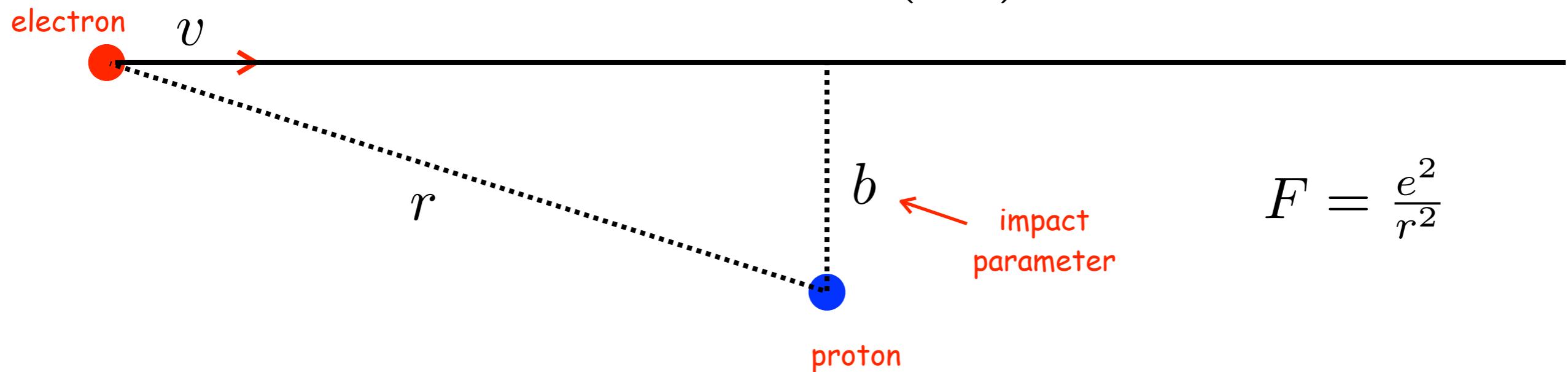
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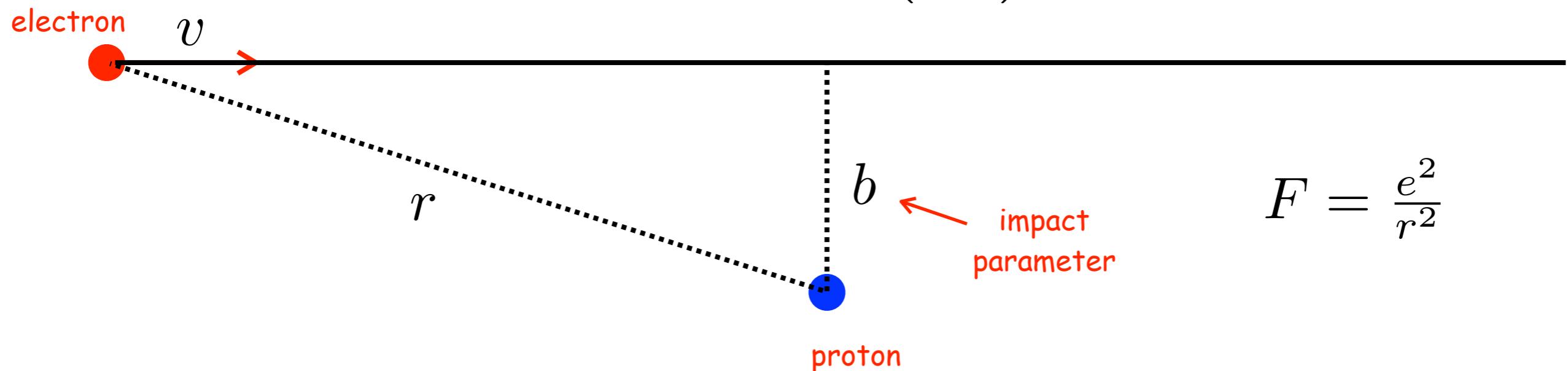
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$$F = \frac{e^2}{r^2}$$

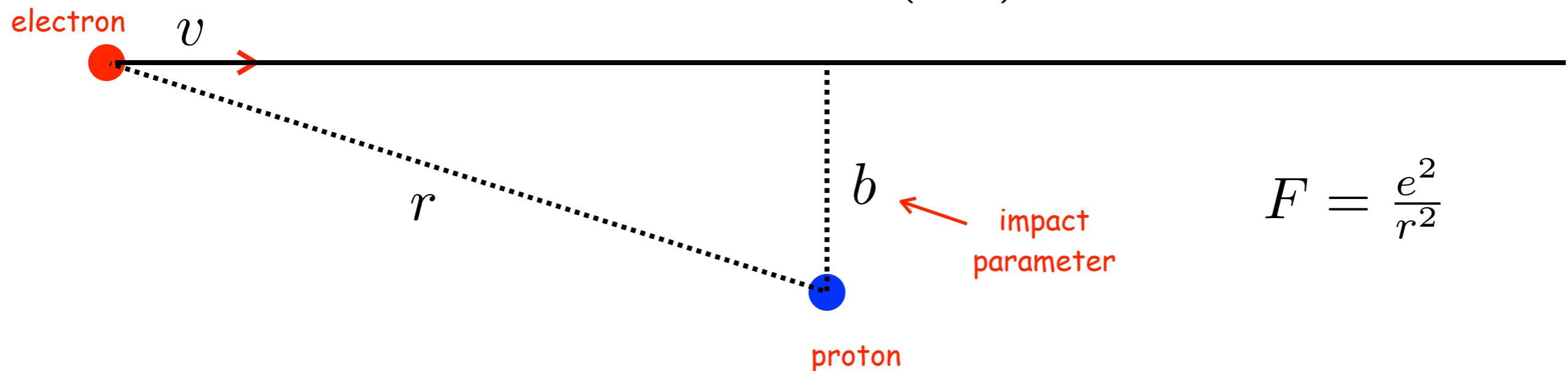
power emitted by an
accelerated charge

$$P = \frac{2e^2}{3c^3} a^2$$

Emission from hot plasmas: thermal Bremsstrahlung

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power emitted by an
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$$\begin{cases} r \approx b \rightarrow a \approx \frac{e^2}{m_e b^2} \\ r \gg b \rightarrow a \approx 0 \end{cases}$$

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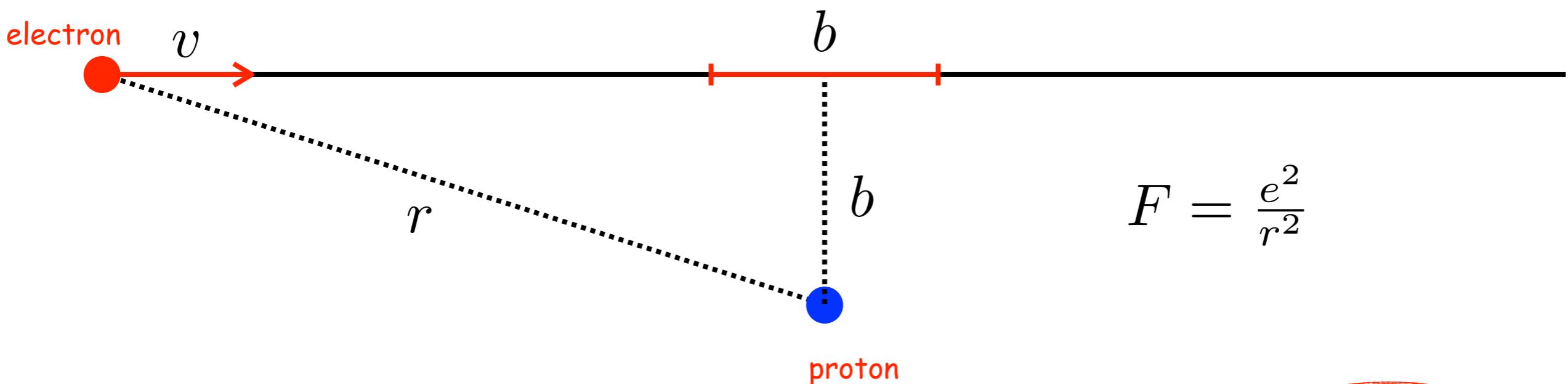
Thermal Bremsstrahlung

very brutal approximations...

characteristic time
for the interaction

$$\tau \approx \frac{b}{v} \longrightarrow \omega \approx \frac{1}{\tau} = \frac{v}{b}$$

characteristic
frequency of the
emitted radiation



power emitted by an
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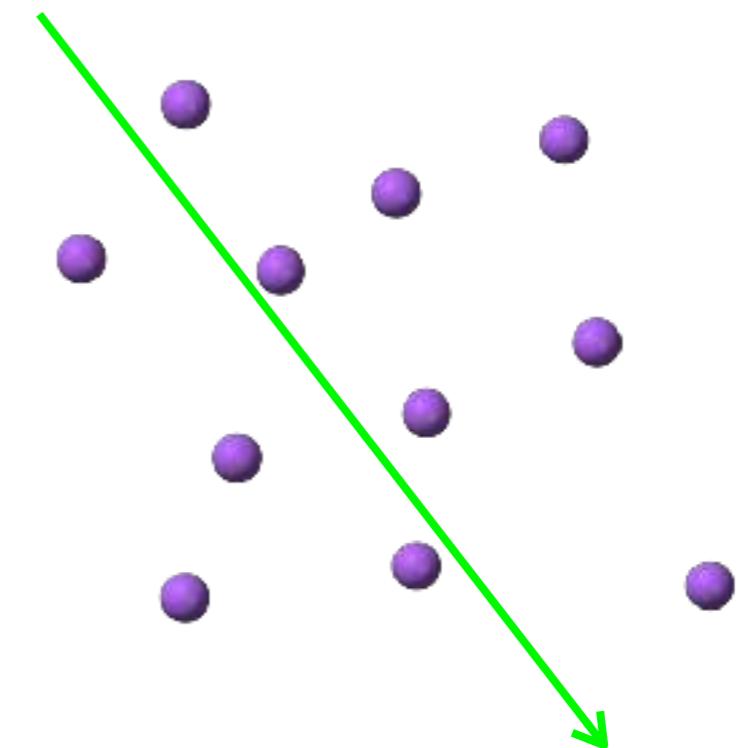
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Thermal Bremsstrahlung

rough estimate of the
impact parameter b

plasma proton density -> n_p

mean distance between protons -> $l_p \sim n_p^{-1/3} \approx b$

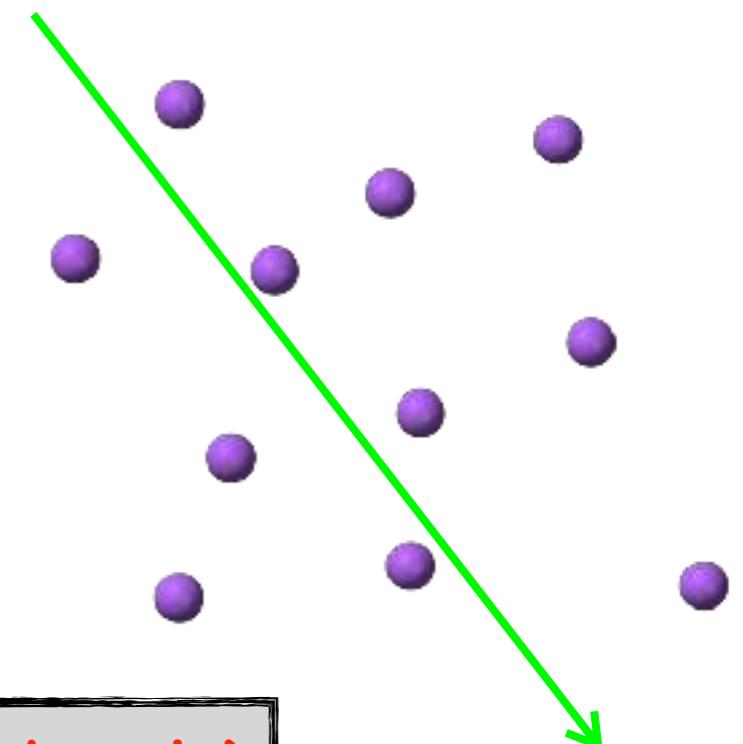


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emissivity (power per unit frequency, volume, and solid angle)

$$\omega \rightarrow v = \omega/2\pi$$

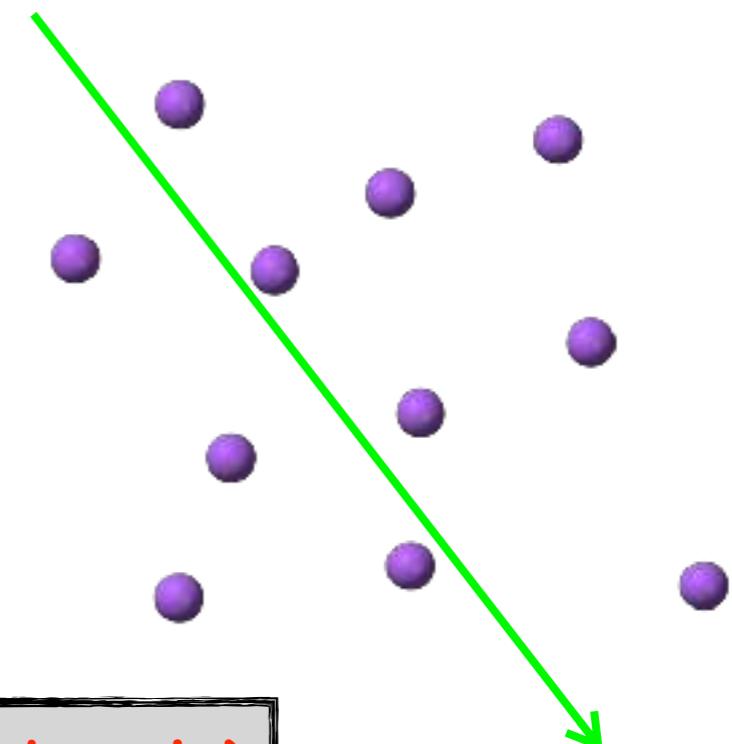
$$j(\nu) \approx \frac{n_e P}{(4\pi)} \frac{2\pi}{\omega} = \frac{n_e n_p e^6}{3c^3 m_e^2} \left(\frac{m_e}{kT}\right)^{1/2}$$

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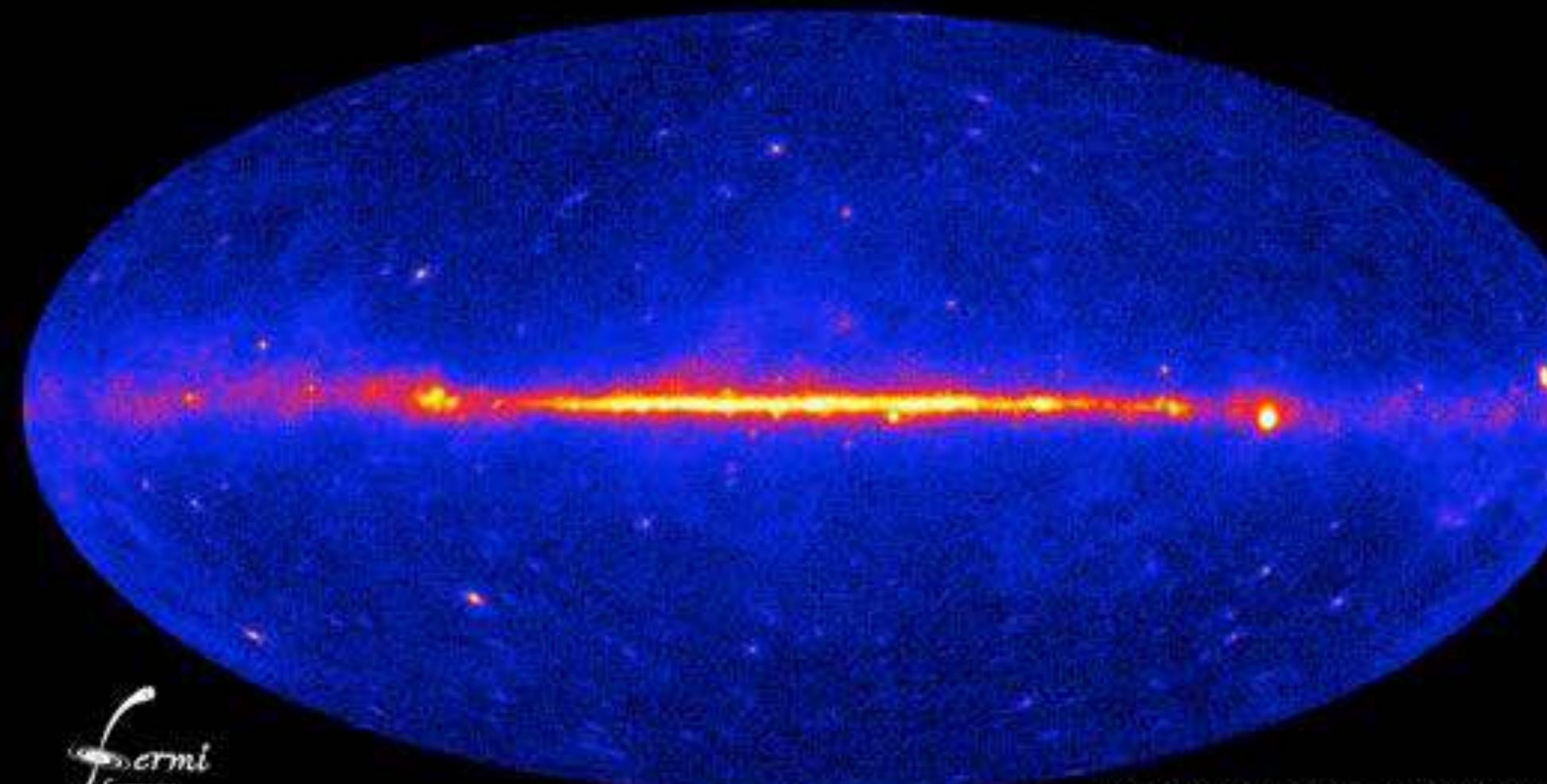
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exponential suppression
 \rightarrow no electrons with energy $\gg kT$



Cosmic ray interactions in the sky

NASA's Fermi telescope reveals best-ever view of the gamma-ray sky

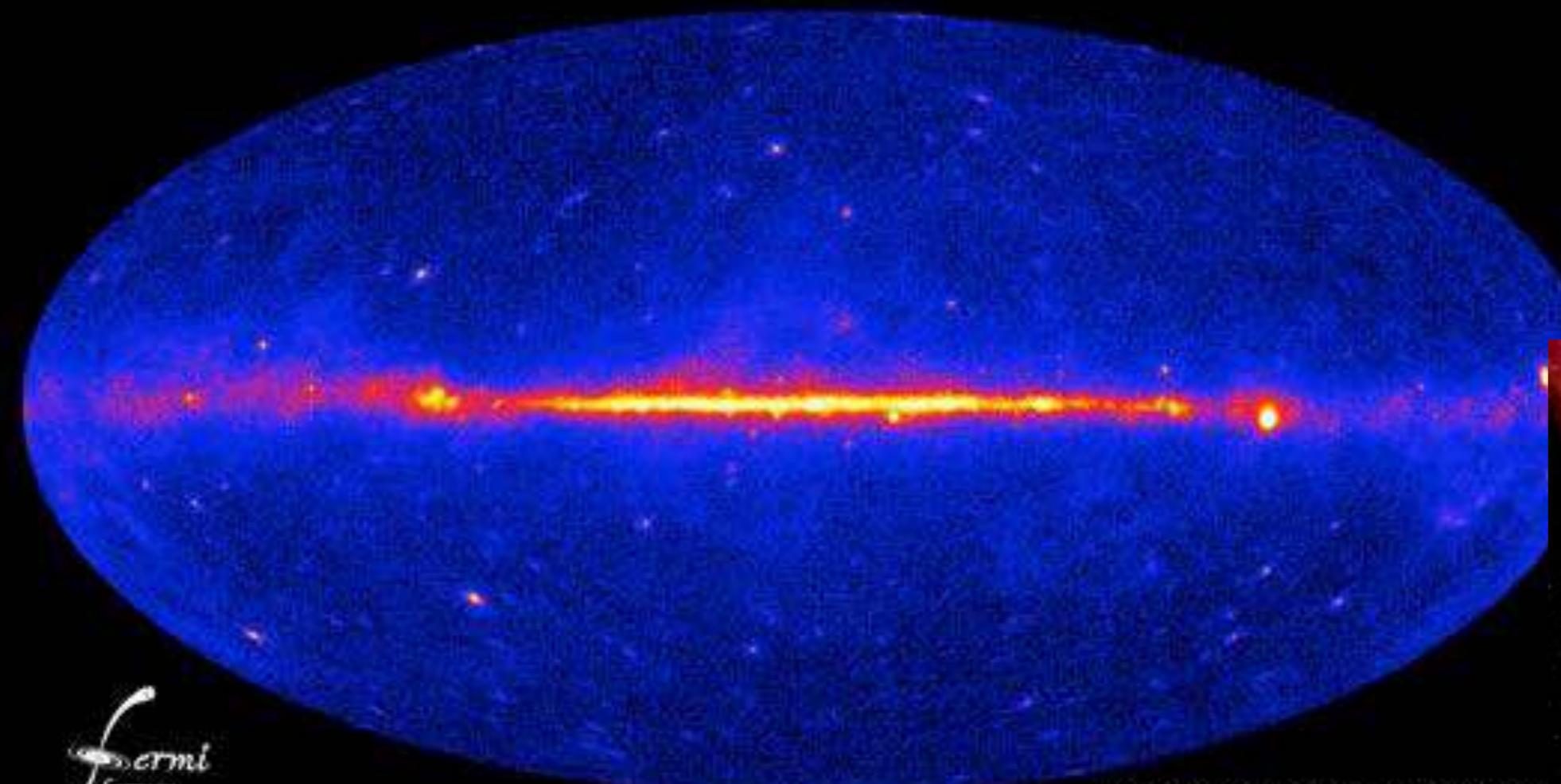


Credit: NASA/DOE/Fermi LAT Collaboration

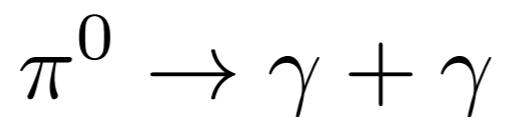
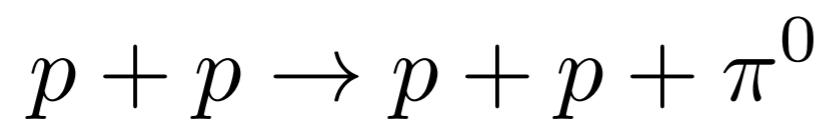


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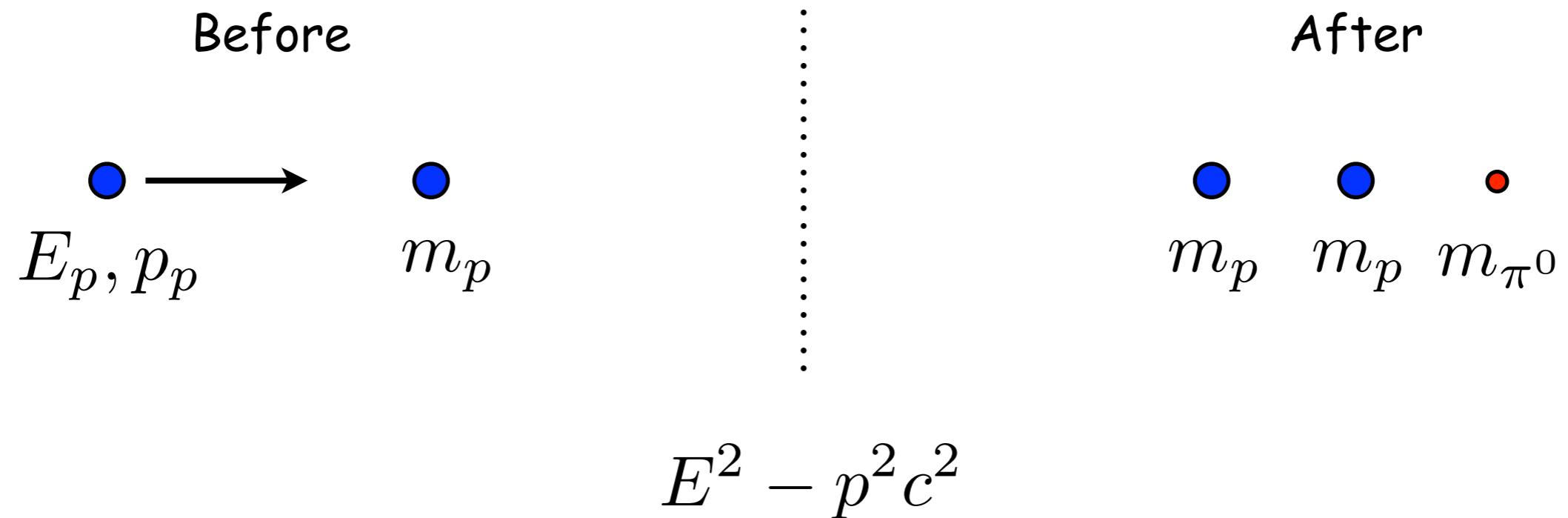
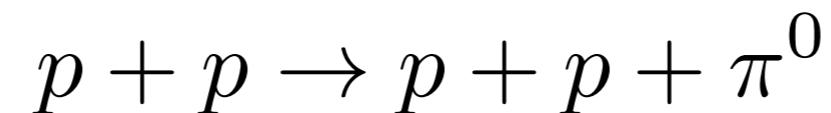


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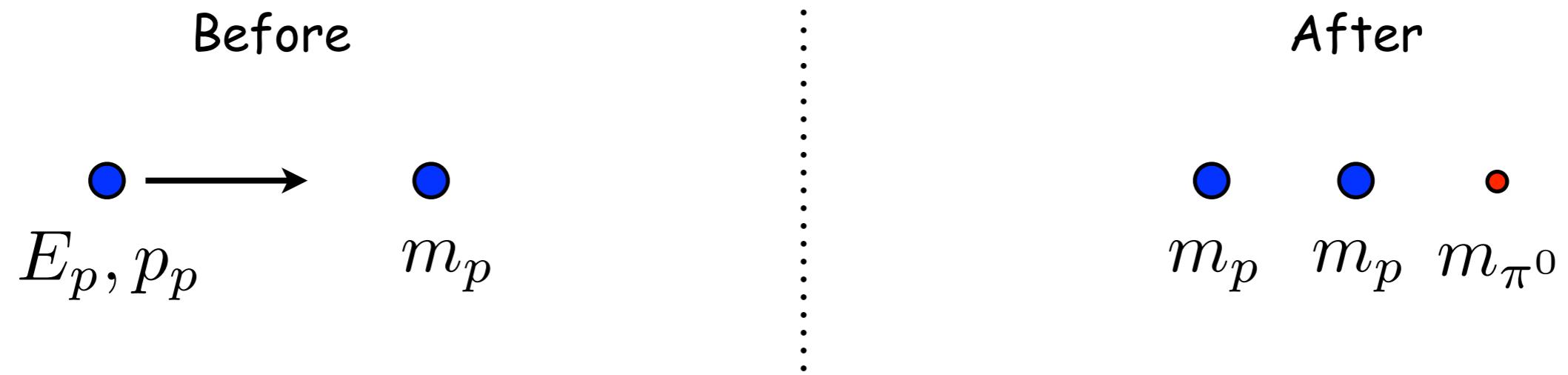
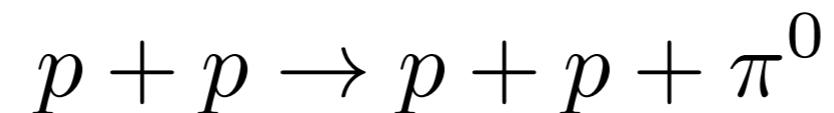
Gamma-Ray Astronomy: p-p interactions

Energy threshold for neutral pion production:



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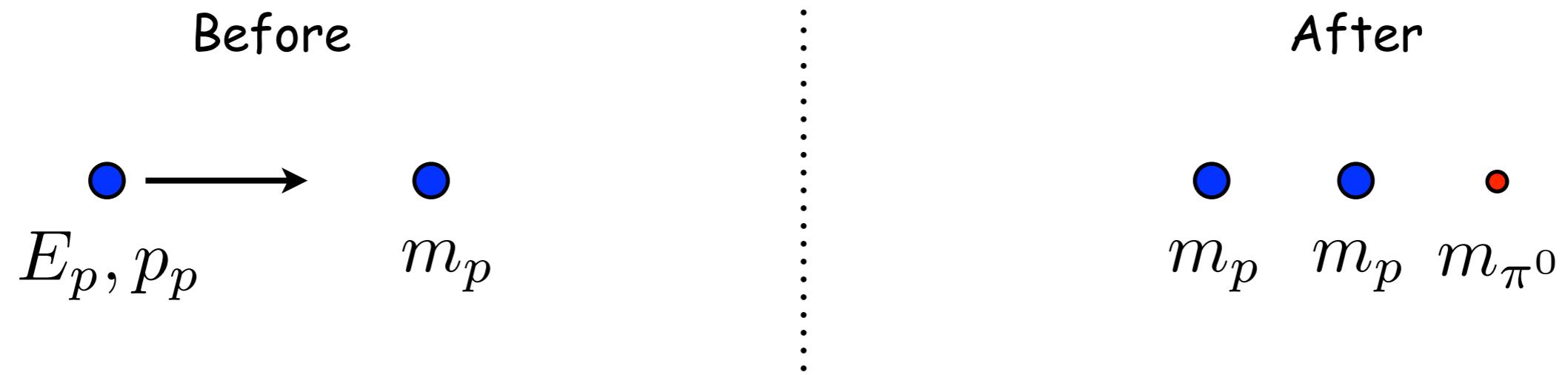
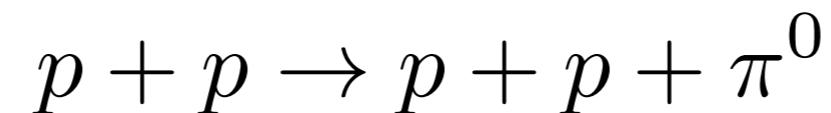
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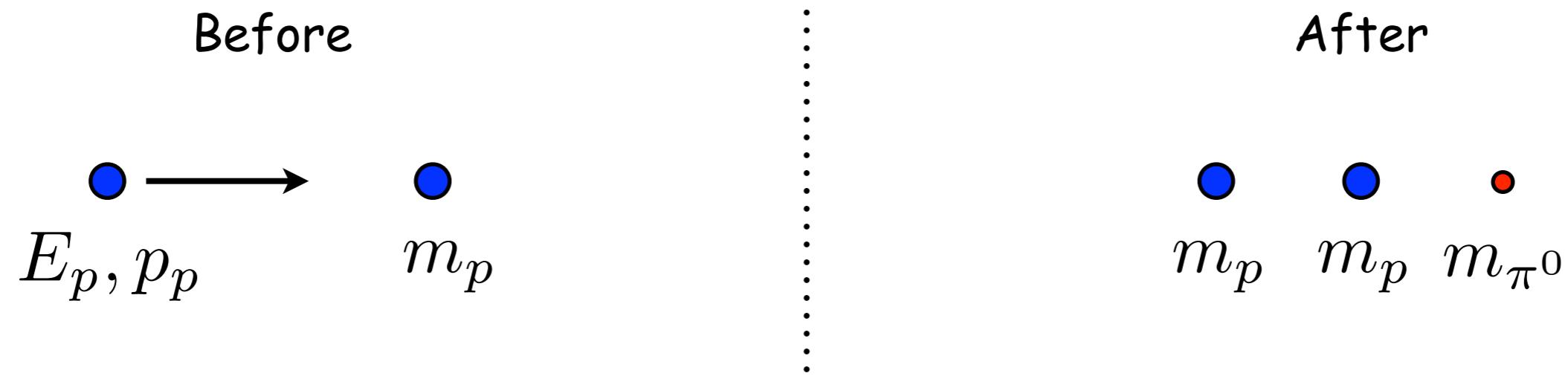
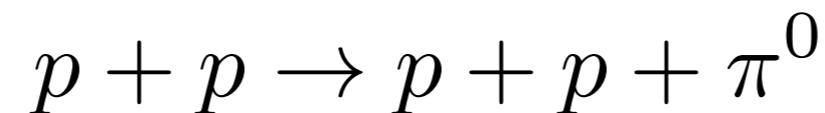
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$$E_p - m_p c^2 > 2m_{\pi^0} c^2 + \left(\frac{m_{\pi^0}}{2m_p} \right) m_{\pi^0} c^2 \approx 280 \text{ MeV}$$

energy threshold

Gamma-Ray Astronomy: p-p interactions

Let's calculate the spectrum of neutral pions:

We assume a power law spectrum for CRs: $N_p(E_p) \propto E_p^{-\delta}$

Fraction of proton kinetic energy transferred to pion (from data): $f_{\pi^0} \approx 0.17$

.....
production
rate

total cross
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$$q_{\pi^0} = \int dE_p \ N_p(E_p) \ \delta(E_{\pi^0} - f_{\pi^0} E_{p,kin}) \ \sigma_{pp}(E_p) \ n_{gas} \ c$$

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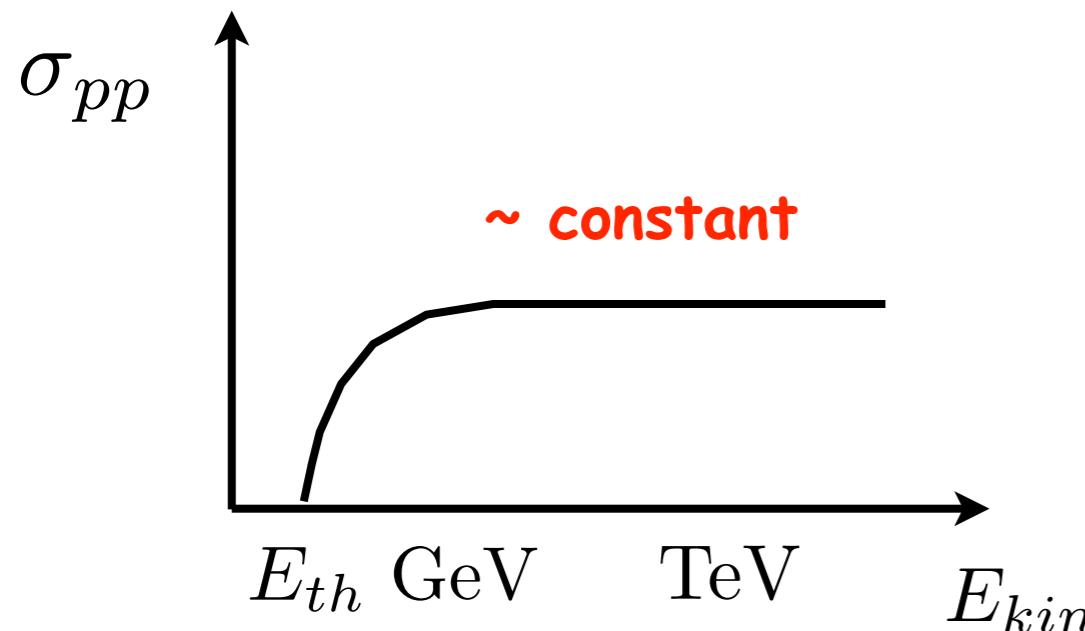
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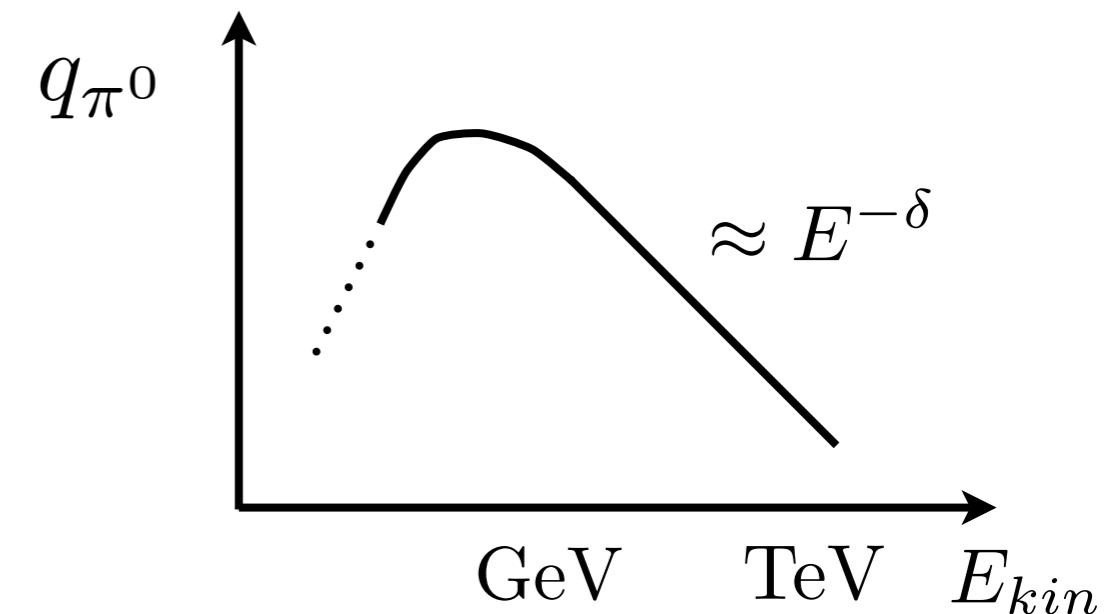
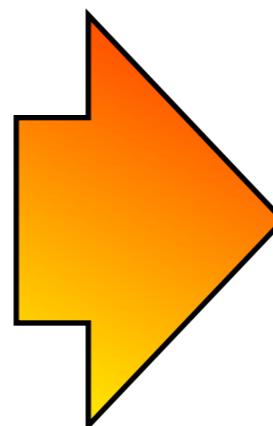
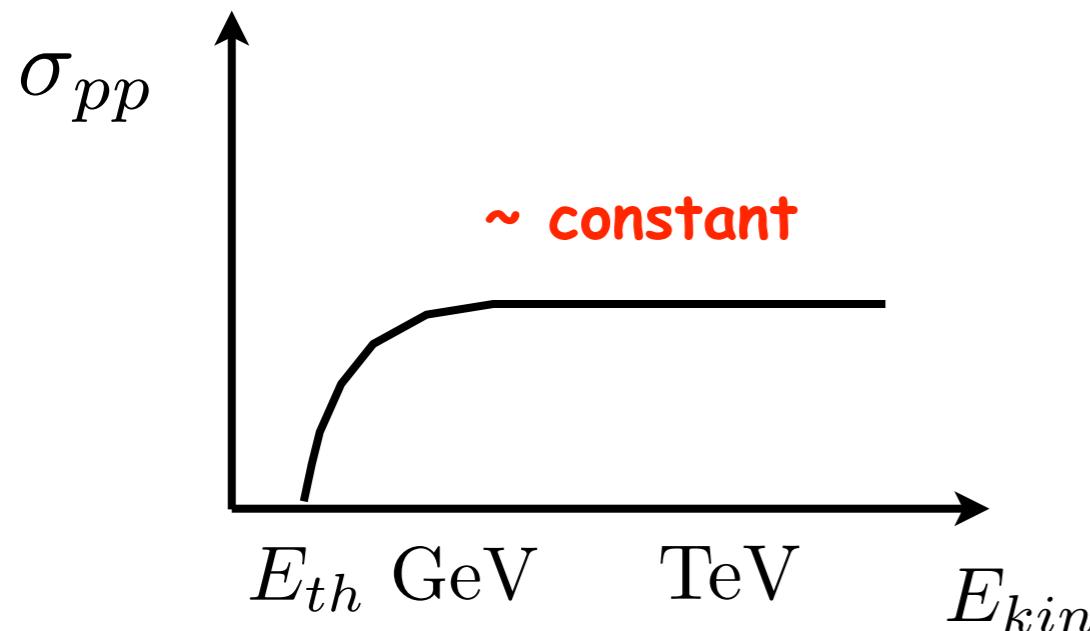
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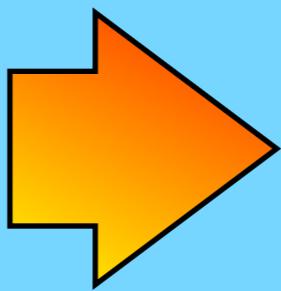
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Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - I

The photon spectrum is
the result of a "one-body-
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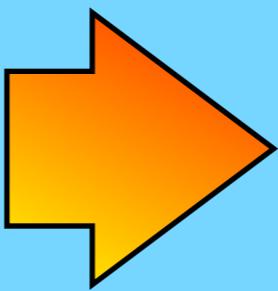


The photon spectrum MUST
exhibit a feature at an energy
related to the pion mass

Gamma-Ray Astronomy: p-p interactions

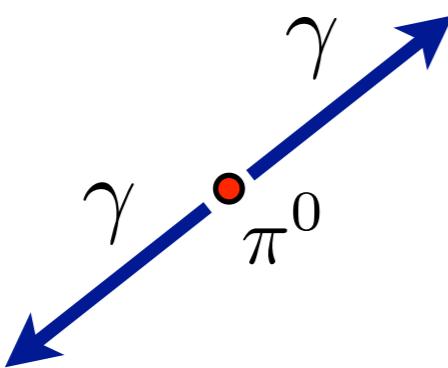
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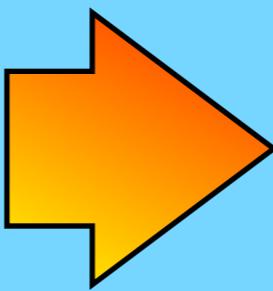


$$E_{\gamma}^{*} = \frac{m_{\pi^0}}{2}$$

Gamma-Ray Astronomy: p-p interactions

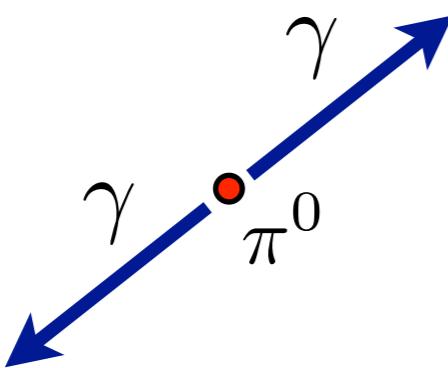
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Pion rest frame:



$$E_{\gamma}^{*} = \frac{m_{\pi^0}}{2}$$

Lab frame:

$$E_{\gamma} = \gamma (E_{\gamma}^{*} + vp_{\gamma}^{*} \cos \theta^{*})$$

max and min energies $\rightarrow \cos \theta^{*} = \pm 1$

$$\frac{m_{\pi^0}}{2} \sqrt{\frac{1-\beta}{1+\beta}} \leq E_{\gamma} \leq \frac{m_{\pi^0}}{2} \sqrt{\frac{1+\beta}{1-\beta}}$$

Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - II

$$E_{\gamma}^{min} = \frac{m_{\pi^0}}{2} \sqrt{\frac{1-\beta}{1+\beta}} \leq E_{\gamma} \leq \frac{m_{\pi^0}}{2} \sqrt{\frac{1+\beta}{1-\beta}} = E_{\gamma}^{max}$$

(1)
$$\frac{\log E_{\gamma}^{max} + \log E_{\gamma}^{min}}{2} = \log \left(\frac{m_{\pi^0}}{2} \right)$$

in log-scale, the centre
of the interval is half
the pion mass

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$$E_{\gamma}^{min} = \frac{m_{\pi^0}}{2} \sqrt{\frac{1-\beta}{1+\beta}} \leq E_{\gamma} \leq \frac{m_{\pi^0}}{2} \sqrt{\frac{1+\beta}{1-\beta}} = E_{\gamma}^{max}$$

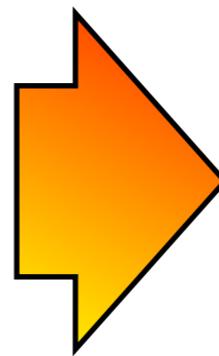
(1) $\frac{\log E_{\gamma}^{max} + \log E_{\gamma}^{min}}{2} = \log \left(\frac{m_{\pi^0}}{2} \right)$

in log-scale, the centre
of the interval is half
the pion mass

(2) in the pion rest frame the photon distribution is isotropic $\frac{dn_{\gamma}}{d\Omega^*} = \frac{1}{4\pi}$

$$d\Omega^* \propto d(\cos \theta^*)$$

$$E_{\gamma} = \gamma (E_{\gamma}^* + vp_{\gamma}^* \cos \theta^*) \rightarrow dE_{\gamma} \propto d(\cos \theta^*)$$

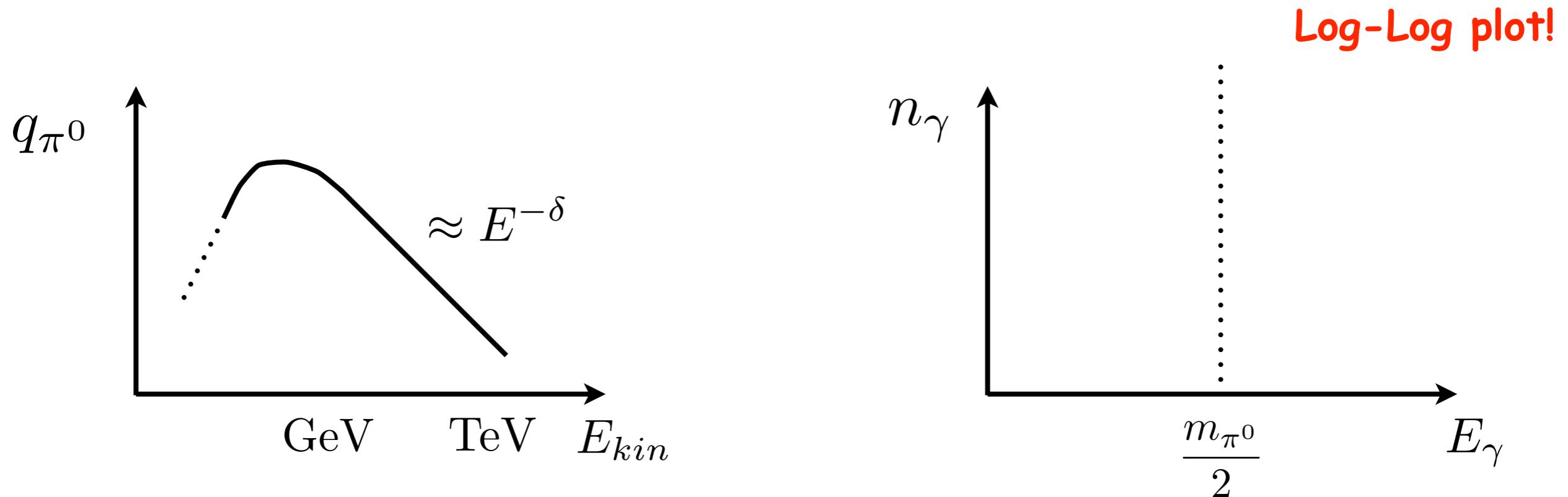


$$\frac{dn_{\gamma}}{dE_{\gamma}} = const$$

The spectrum is flat!

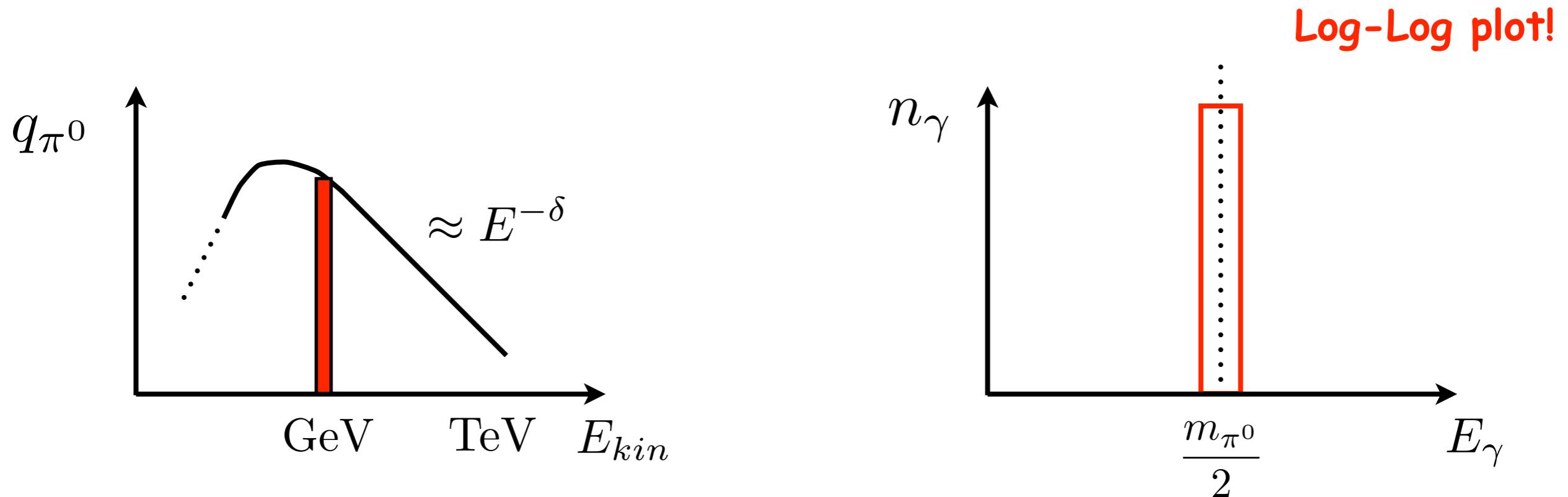
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



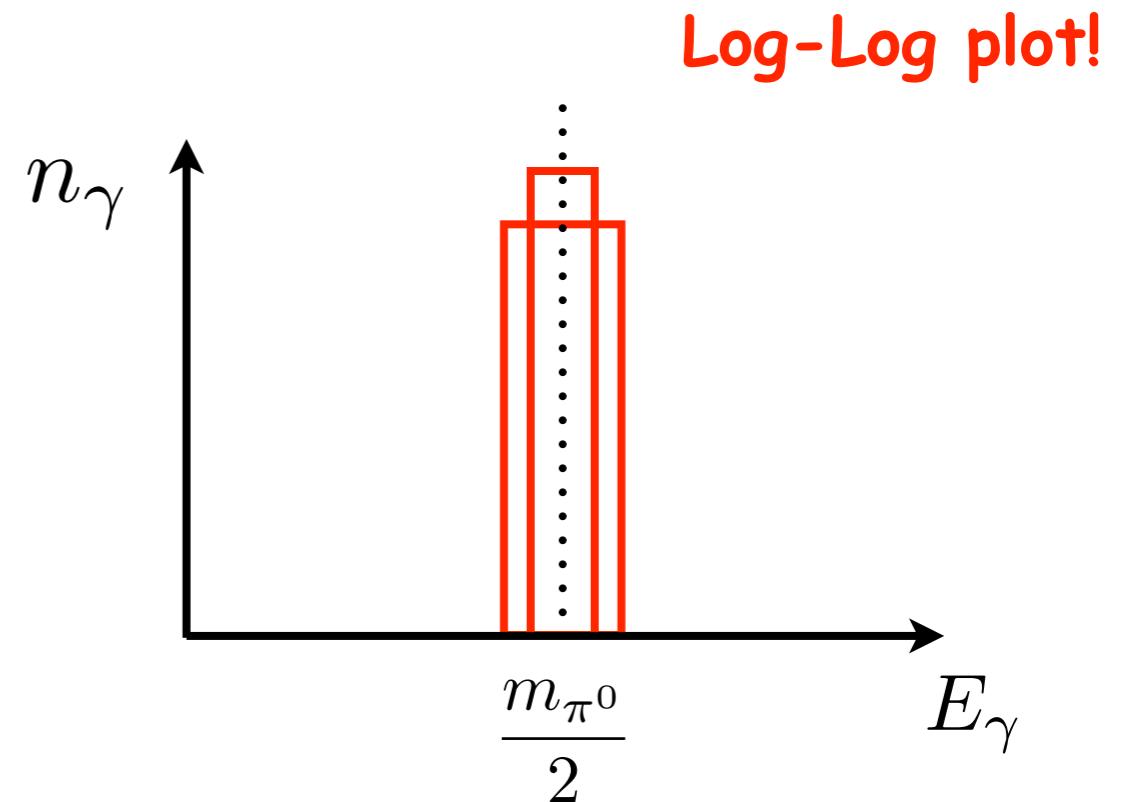
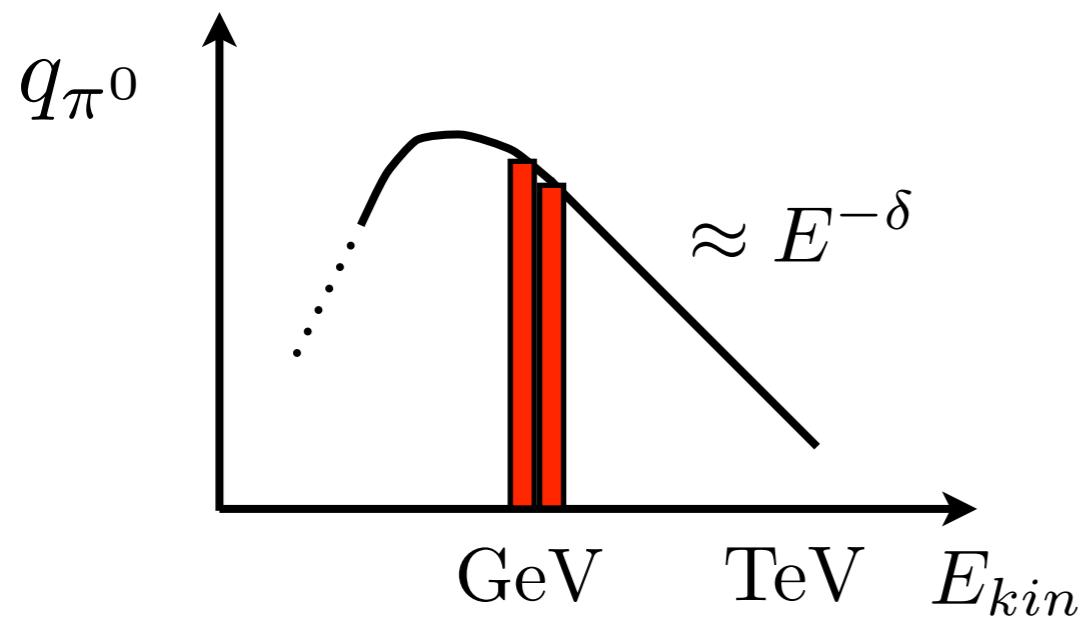
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



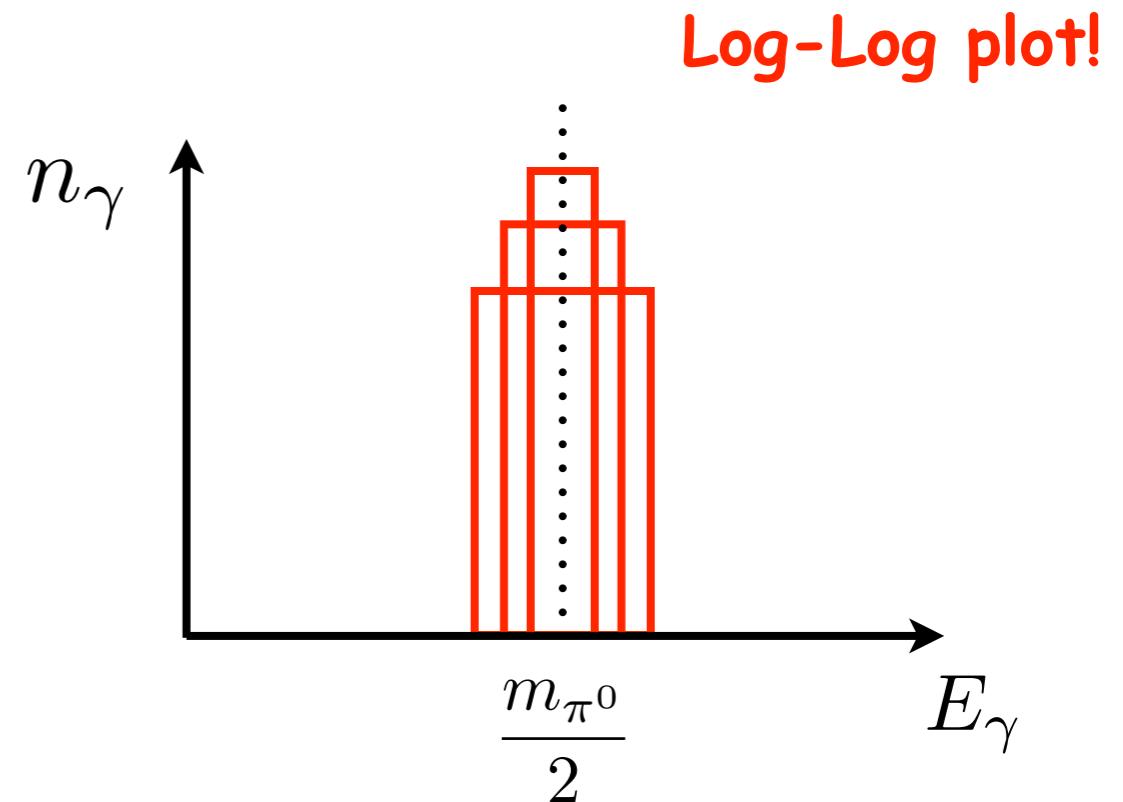
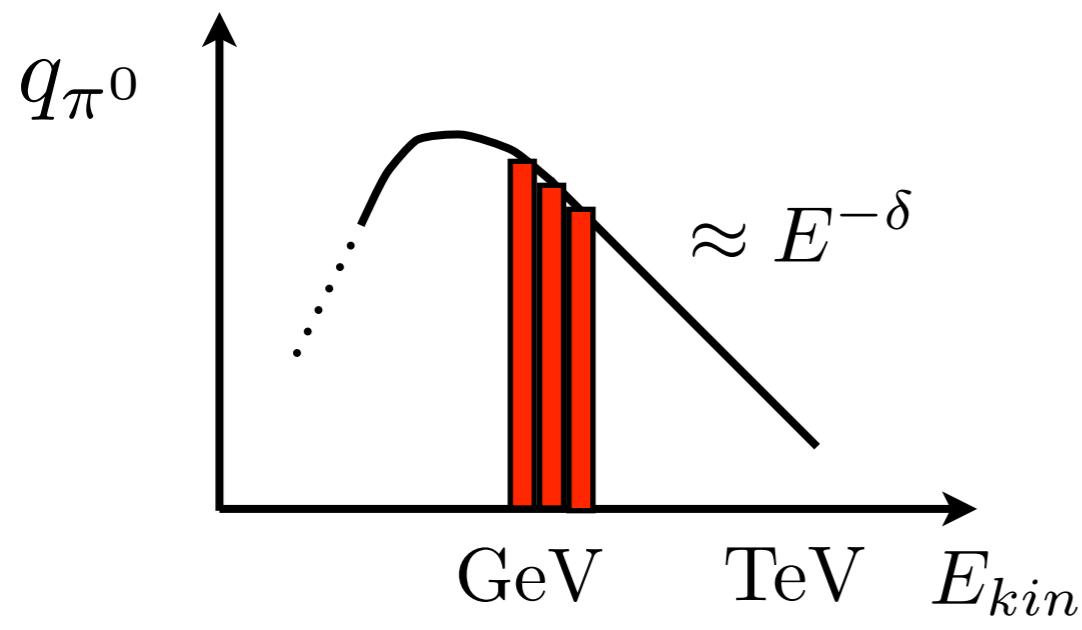
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



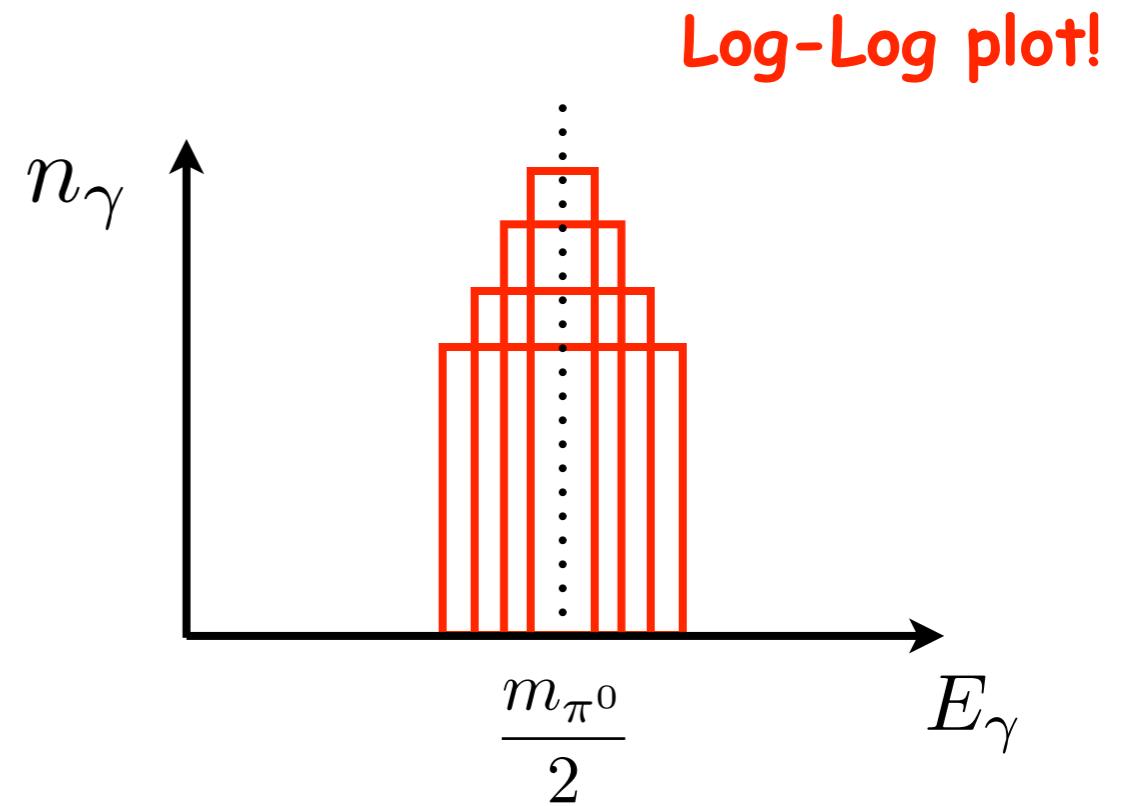
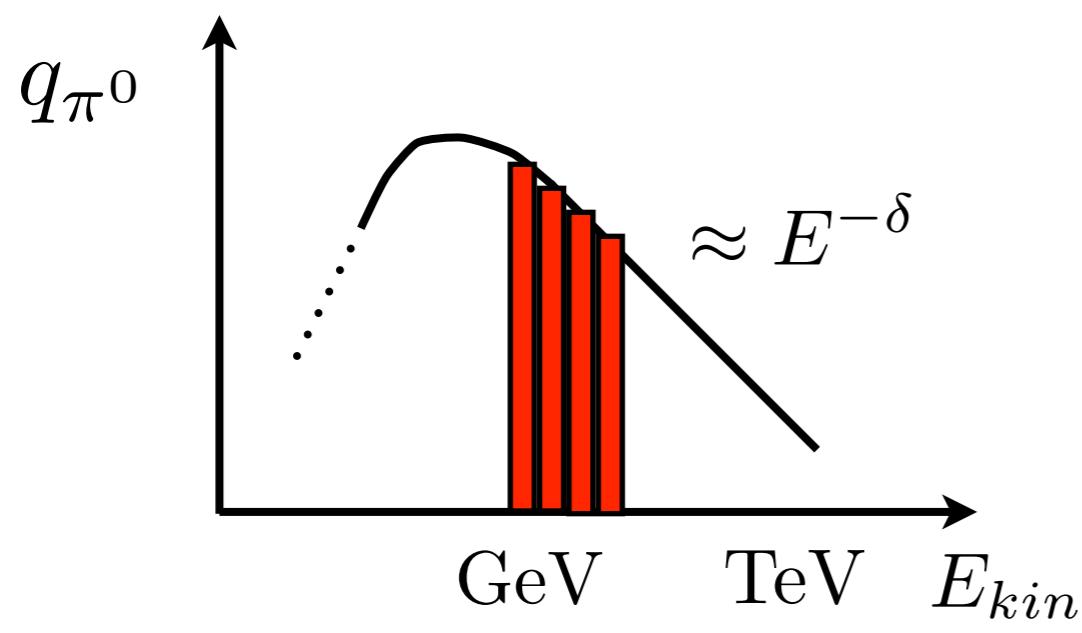
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



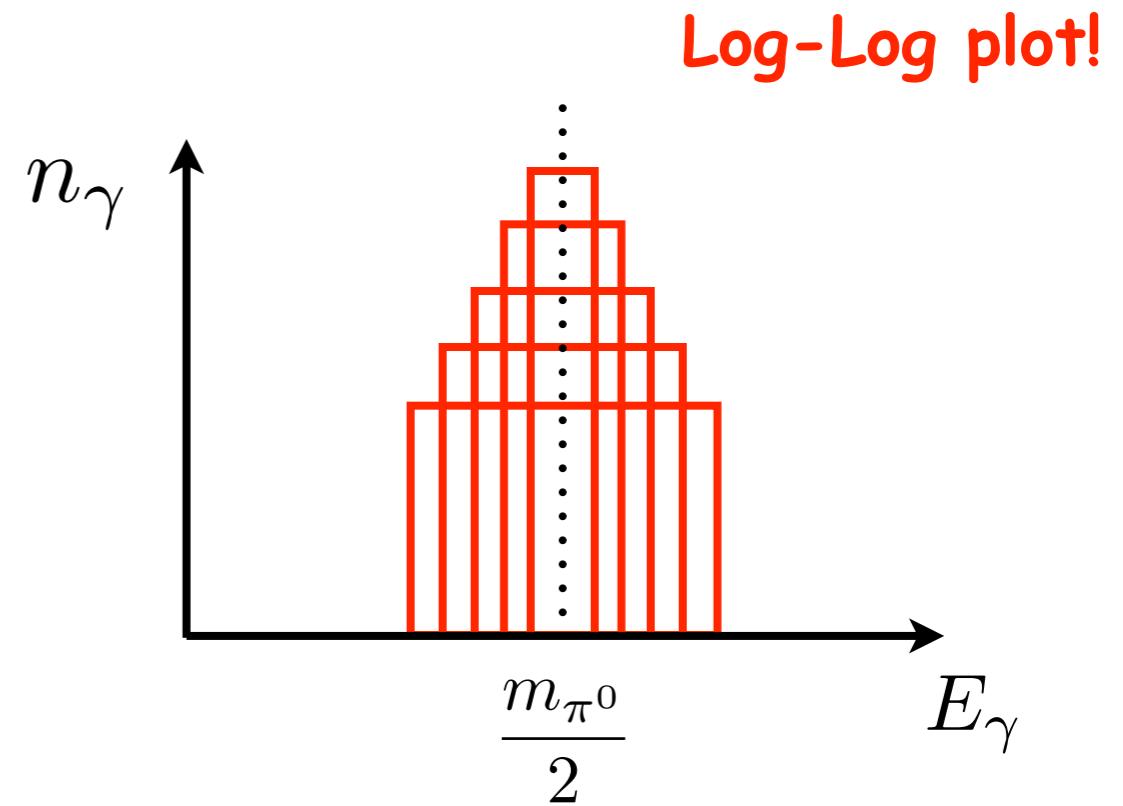
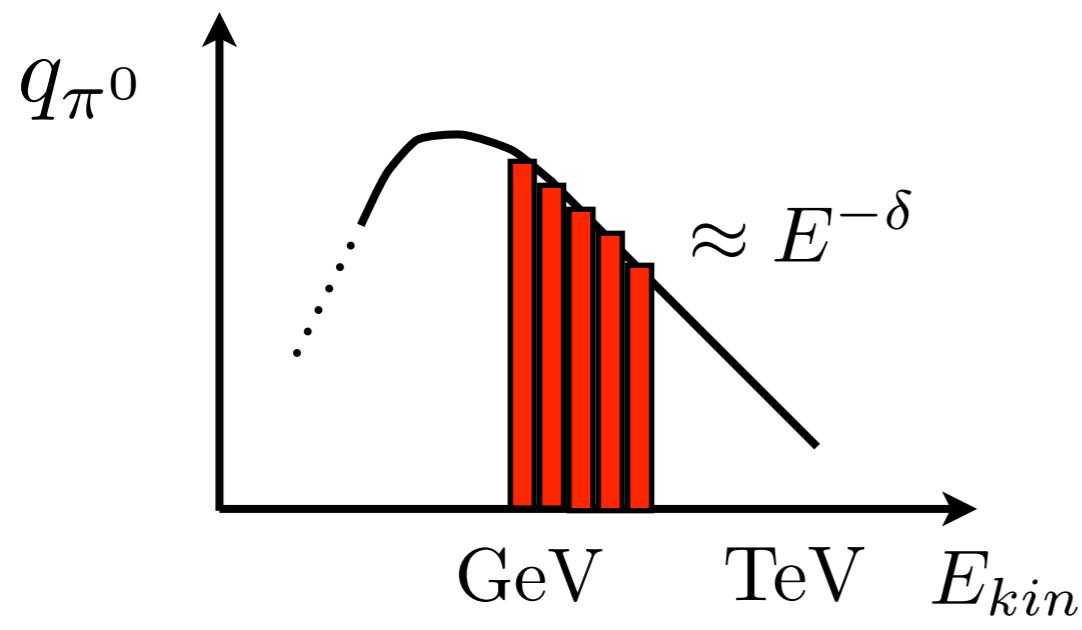
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



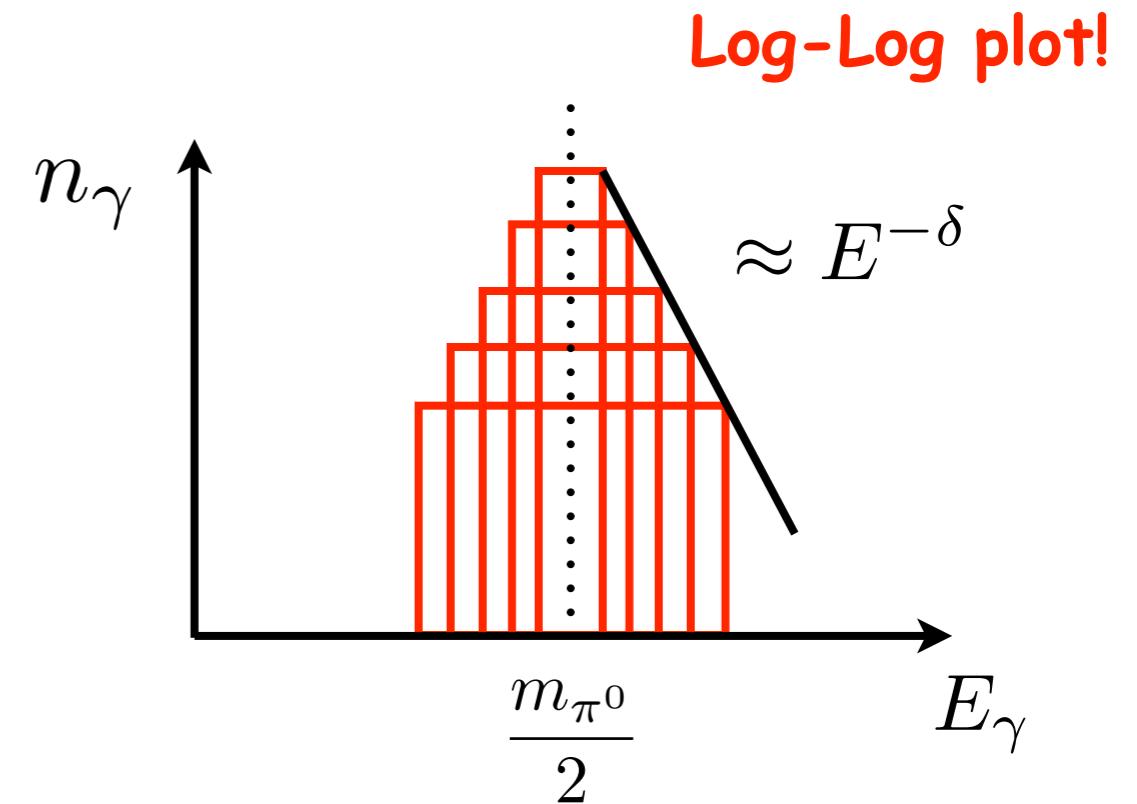
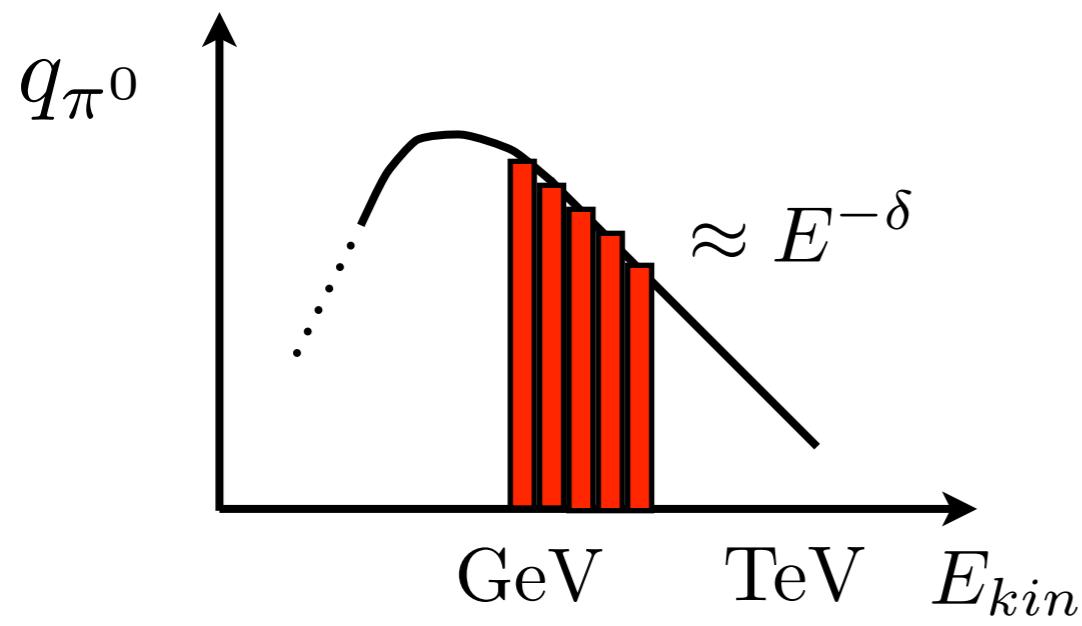
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



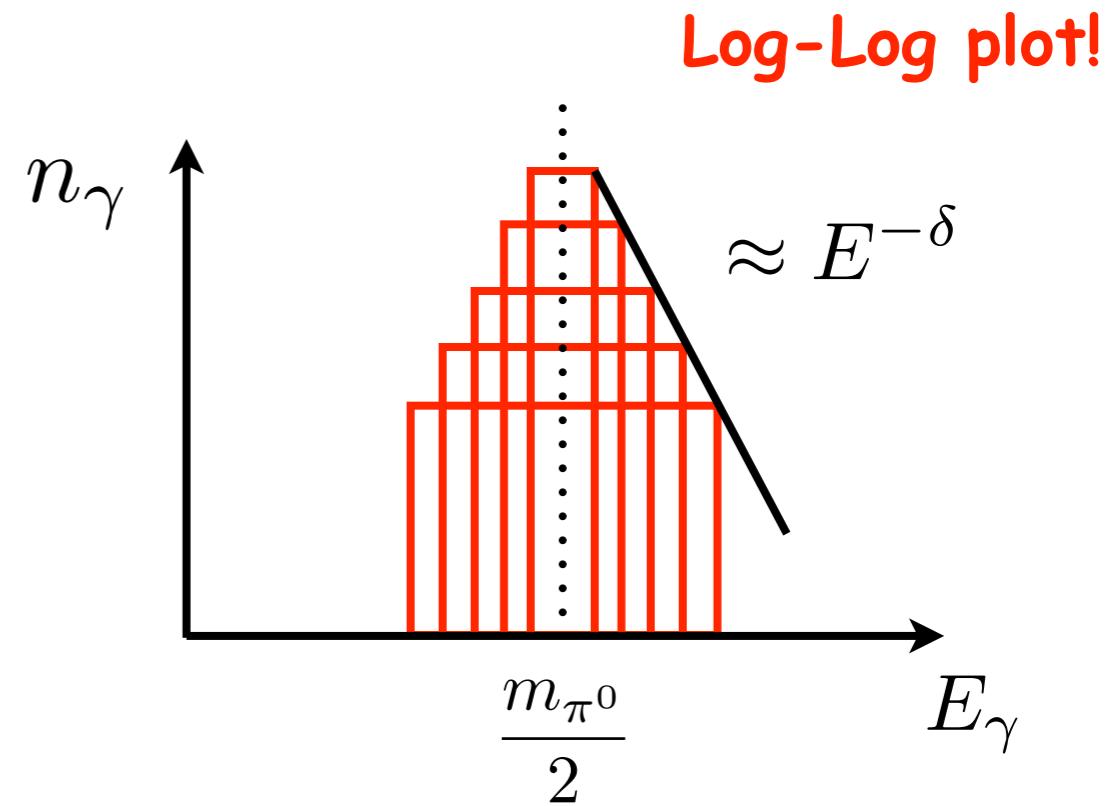
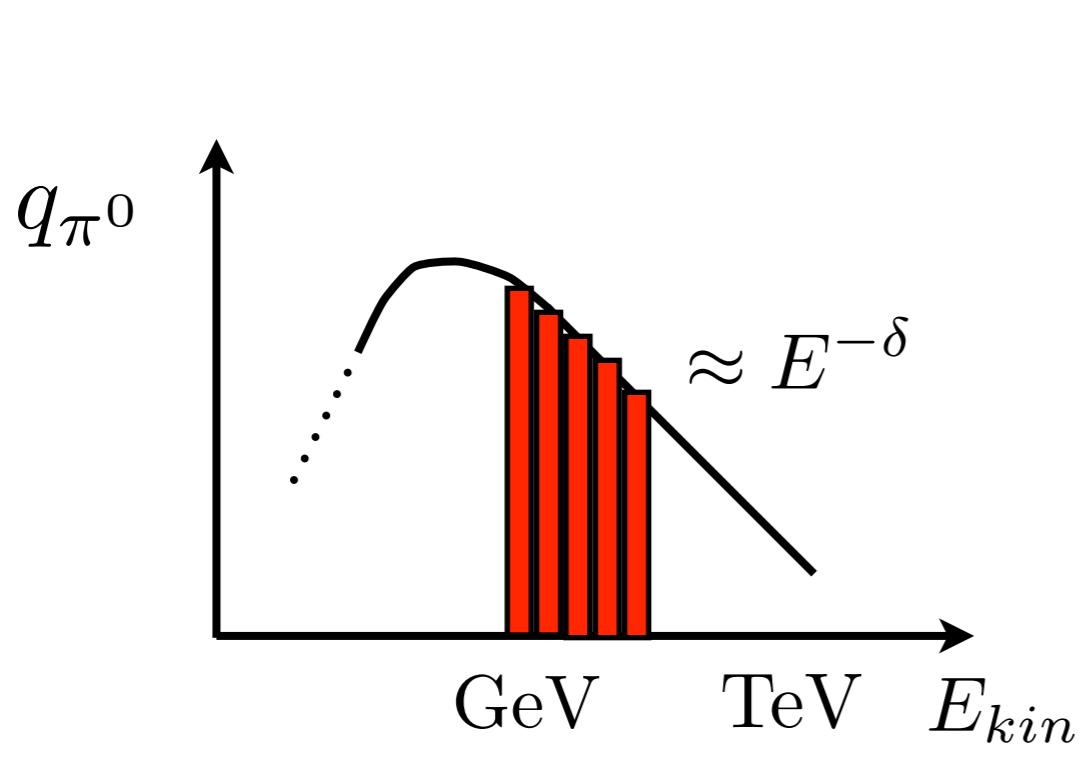
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III



Gamma-Ray Astronomy: p-p interactions

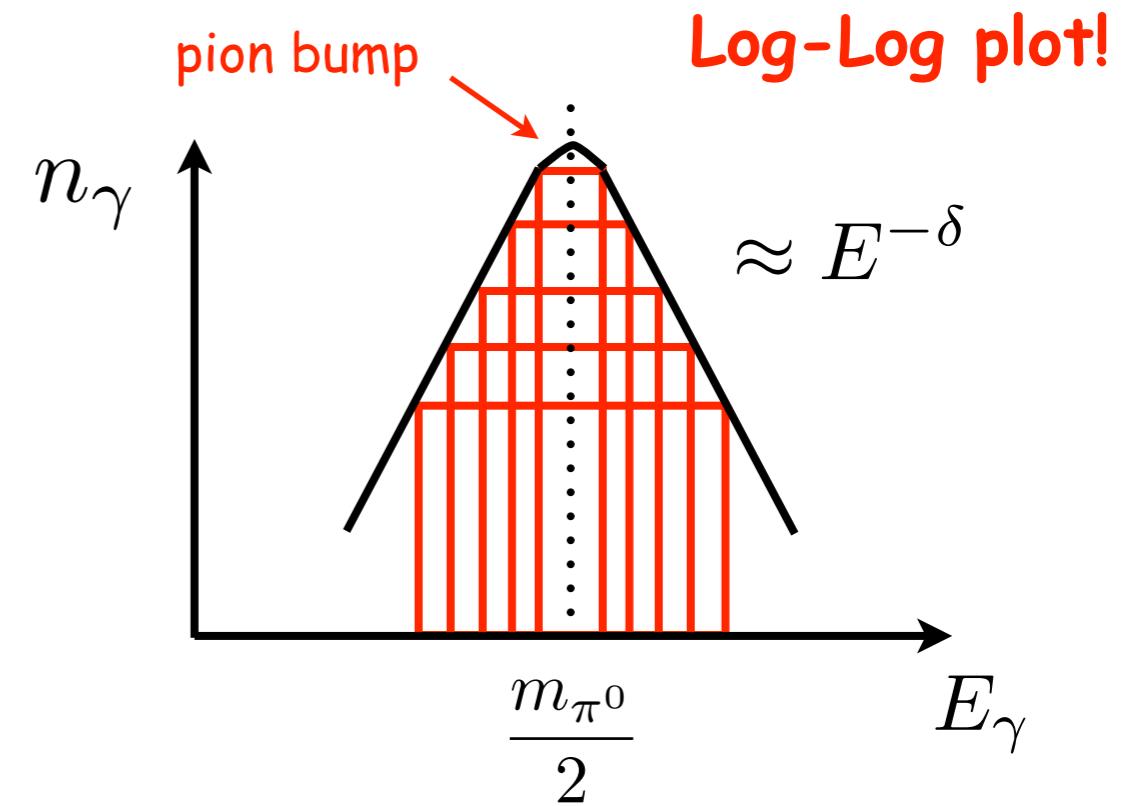
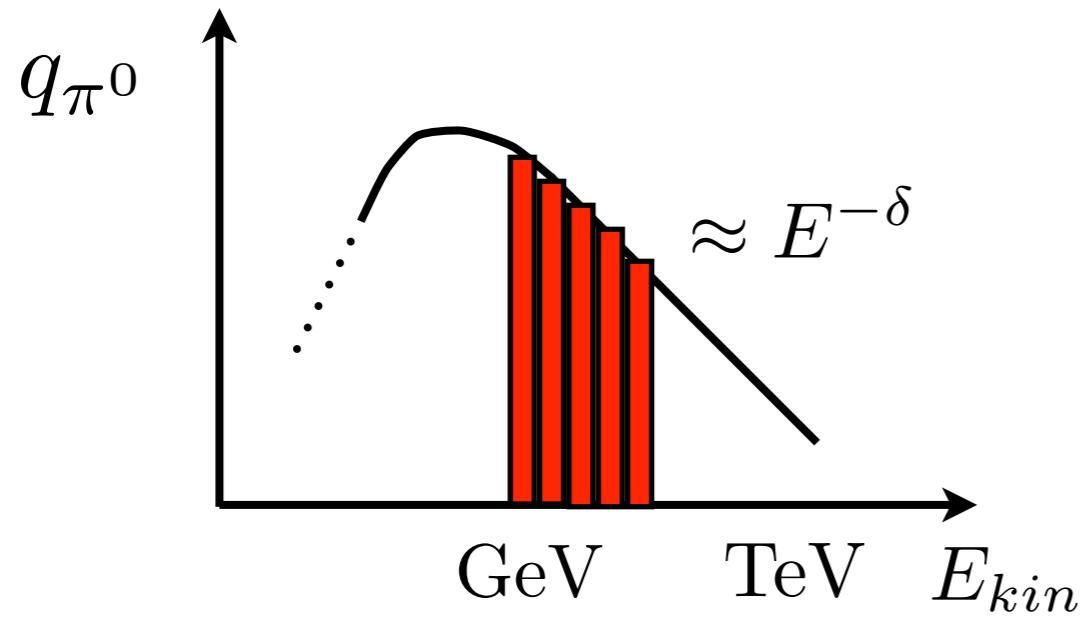
Let's now calculate the spectrum of photons from pion decay - III



- the gamma ray spectrum is symmetric (in log-log) with respect to: $\frac{m_{\pi^0}}{2} \sim 70 \text{ MeV}$
- at high energy the spectrum mimics the CR spectrum, with (roughly): $E_{\gamma} \approx \frac{E_{CR}}{10}$

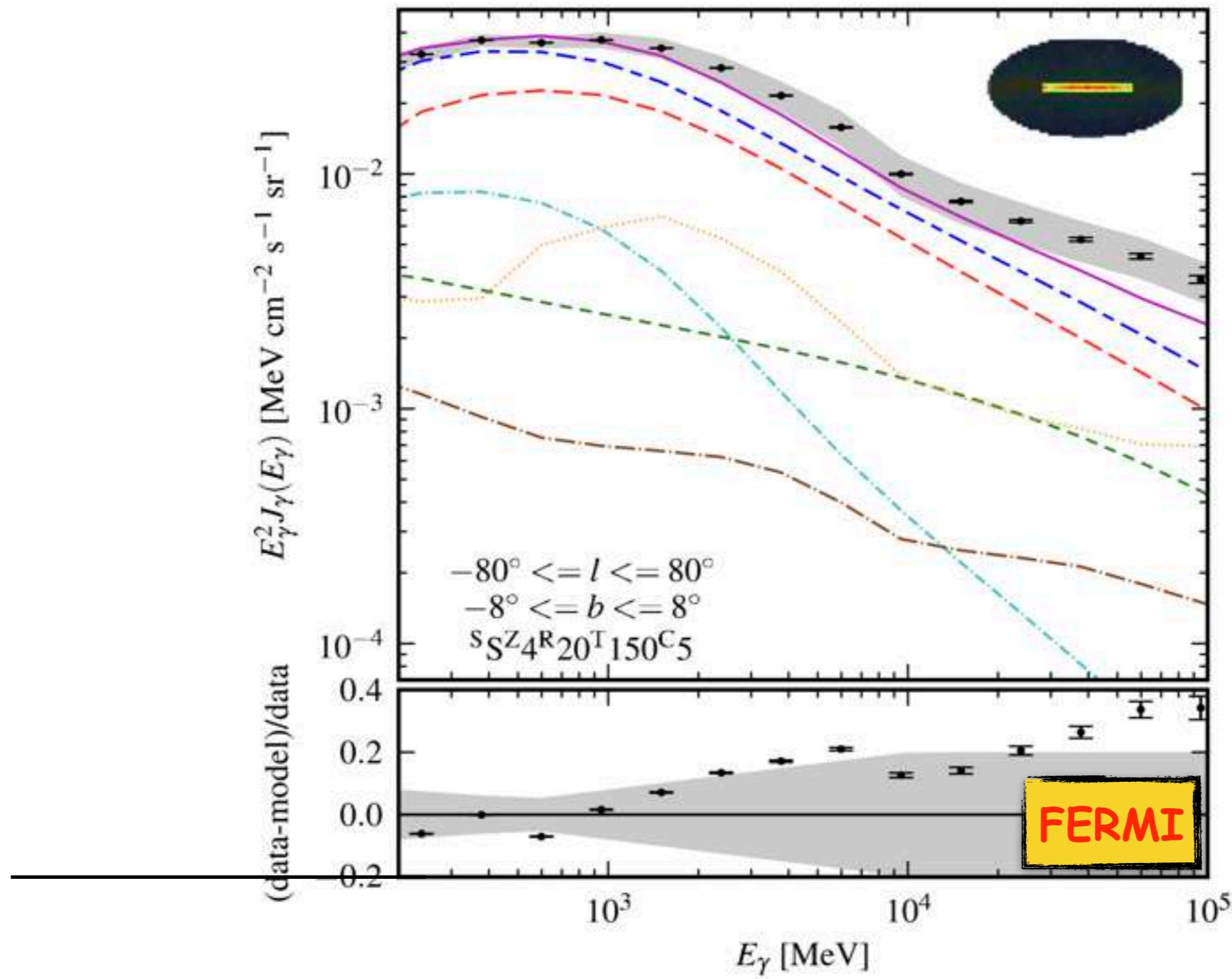
Gamma-Ray Astronomy: p-p interactions

Let's now calculate the spectrum of photons from pion decay - III

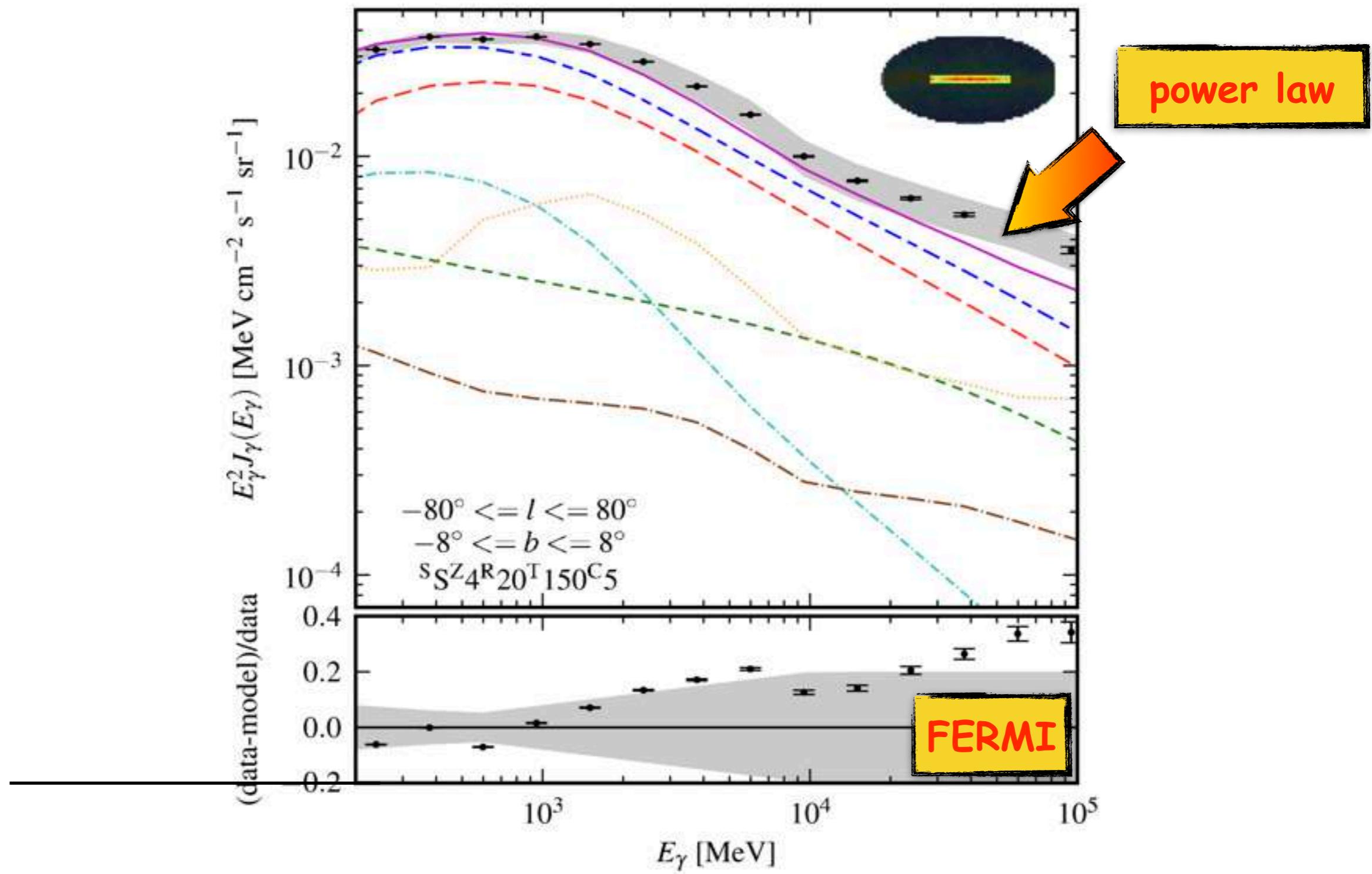


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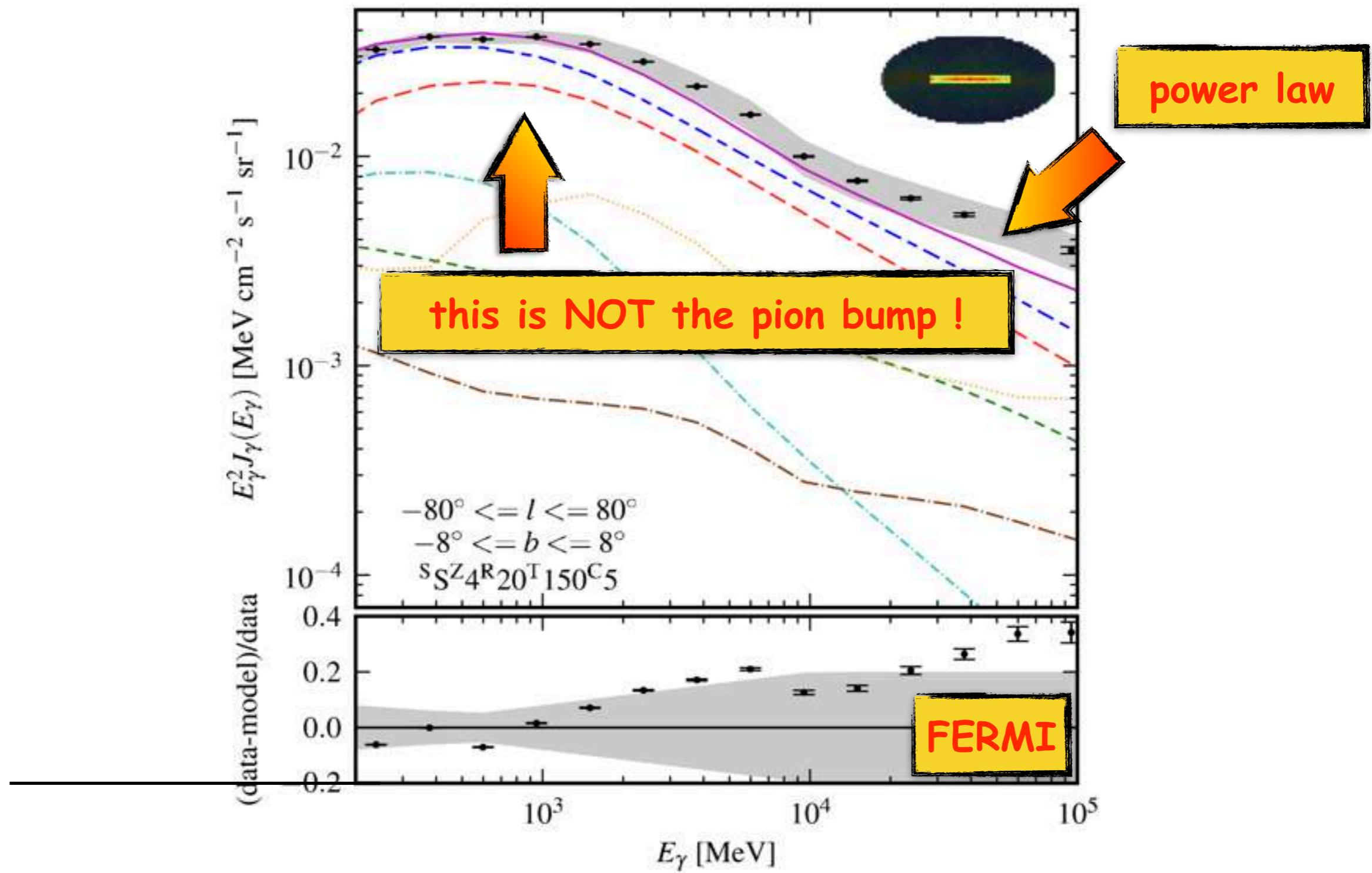
Diffuse emission from the inner Galaxy



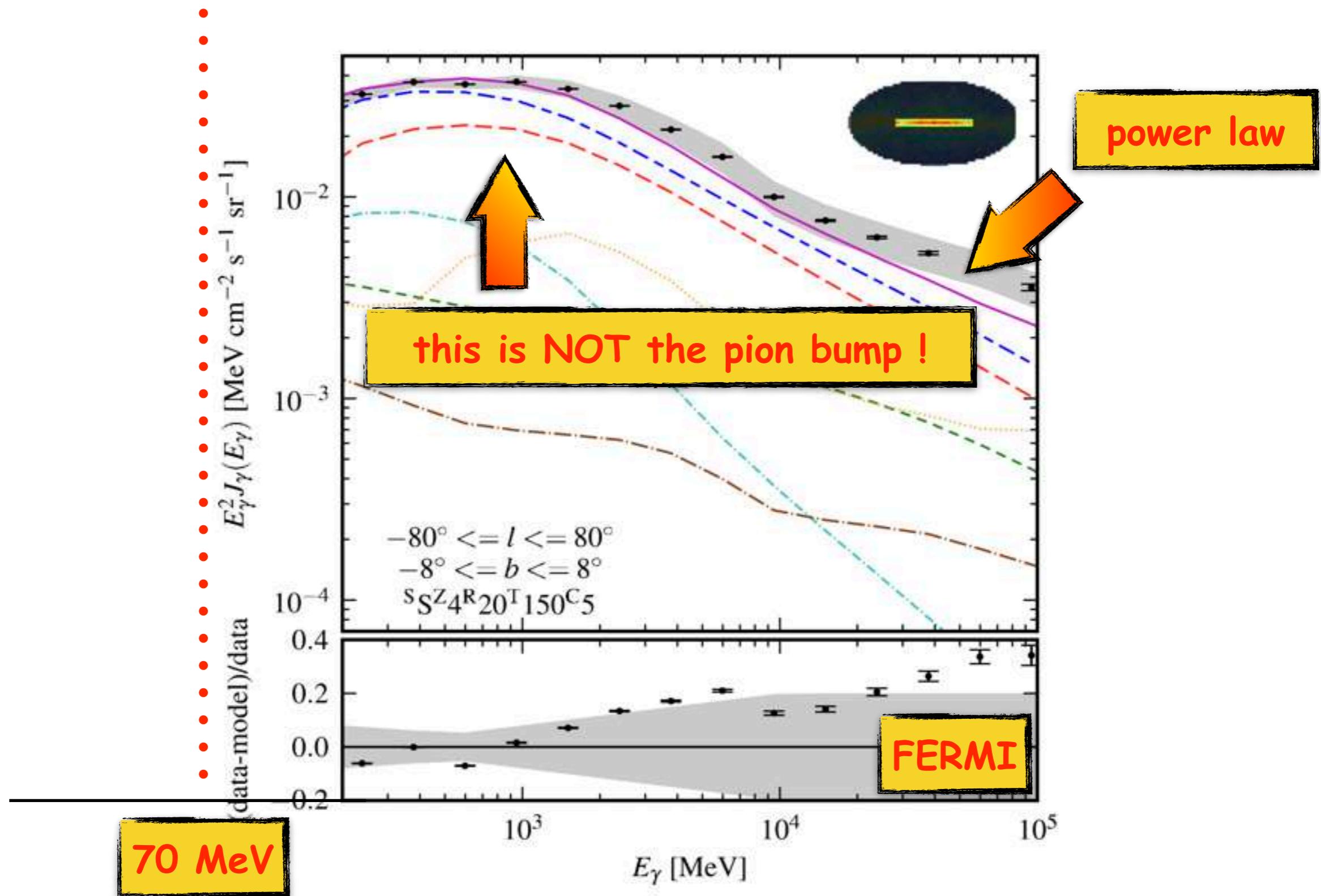
Diffuse emission from the inner Galaxy



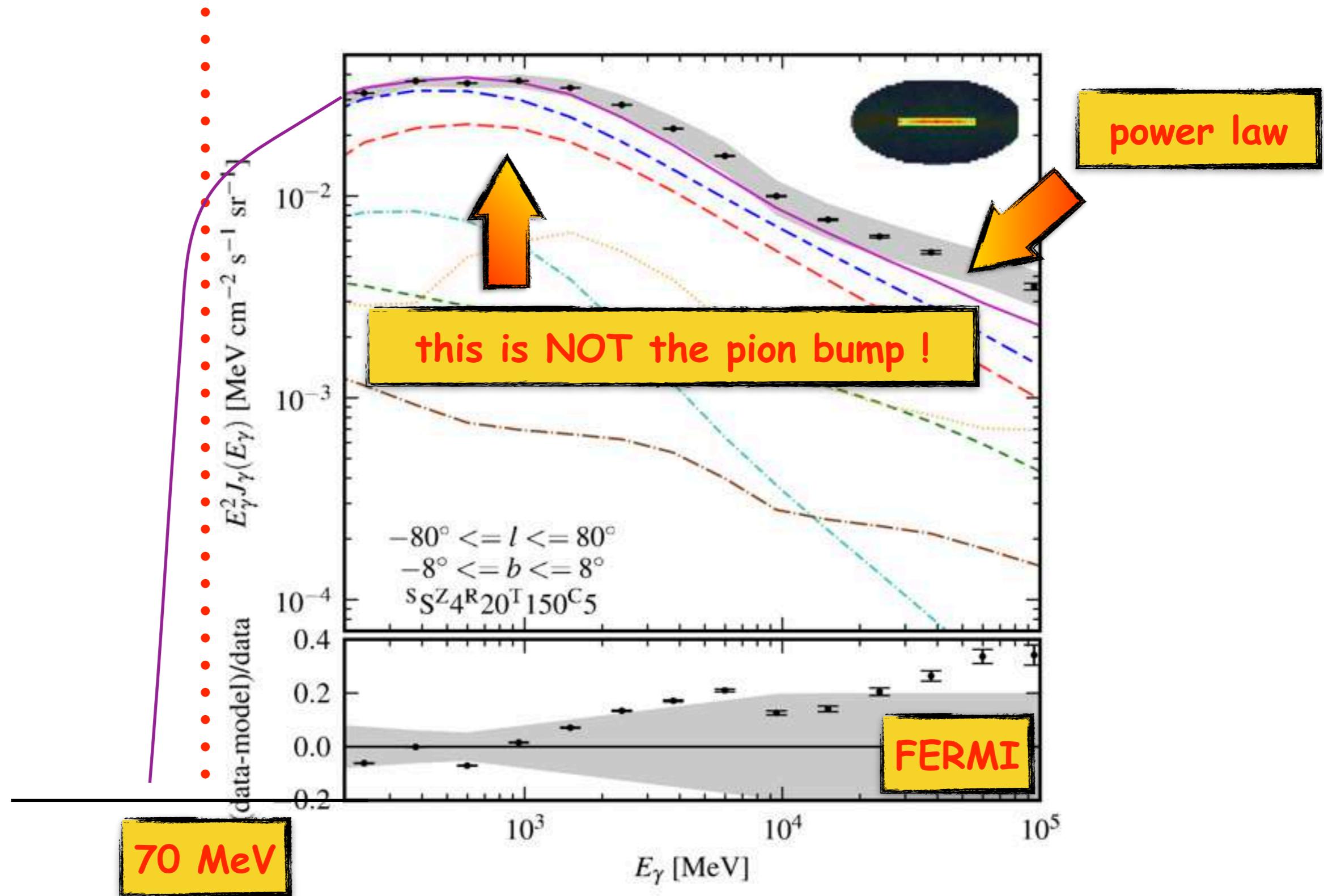
Diffuse emission from the inner Galaxy



Diffuse emission from the inner Galaxy

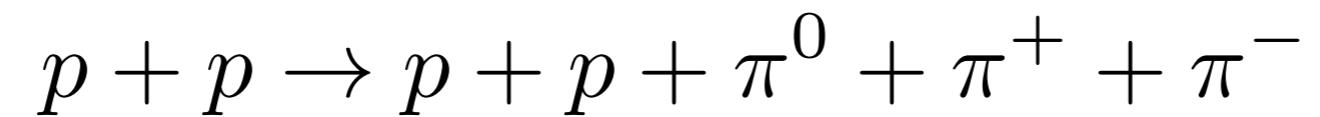


Diffuse emission from the inner Galaxy



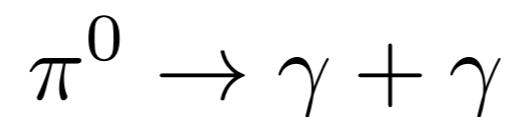
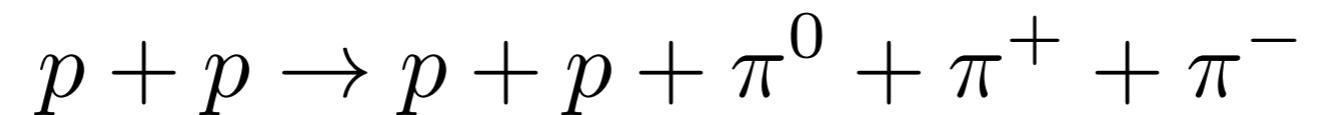
Not only gammas: neutrinos & electrons

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions



Not only gammas: neutrinos & electrons

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions



Not only gammas: neutrinos & electrons

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions

$$p + p \rightarrow p + p + \pi^0 + \pi^+ + \pi^-$$

$$\pi^0 \rightarrow \gamma + \gamma$$

$$\left\{ \begin{array}{l} \pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \\ \mu^\pm \rightarrow e^\pm + \bar{\nu}_\mu (\nu_\mu) + \nu_e (\bar{\nu}_e) \end{array} \right.$$

Not only gammas: neutrinos & electrons

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions

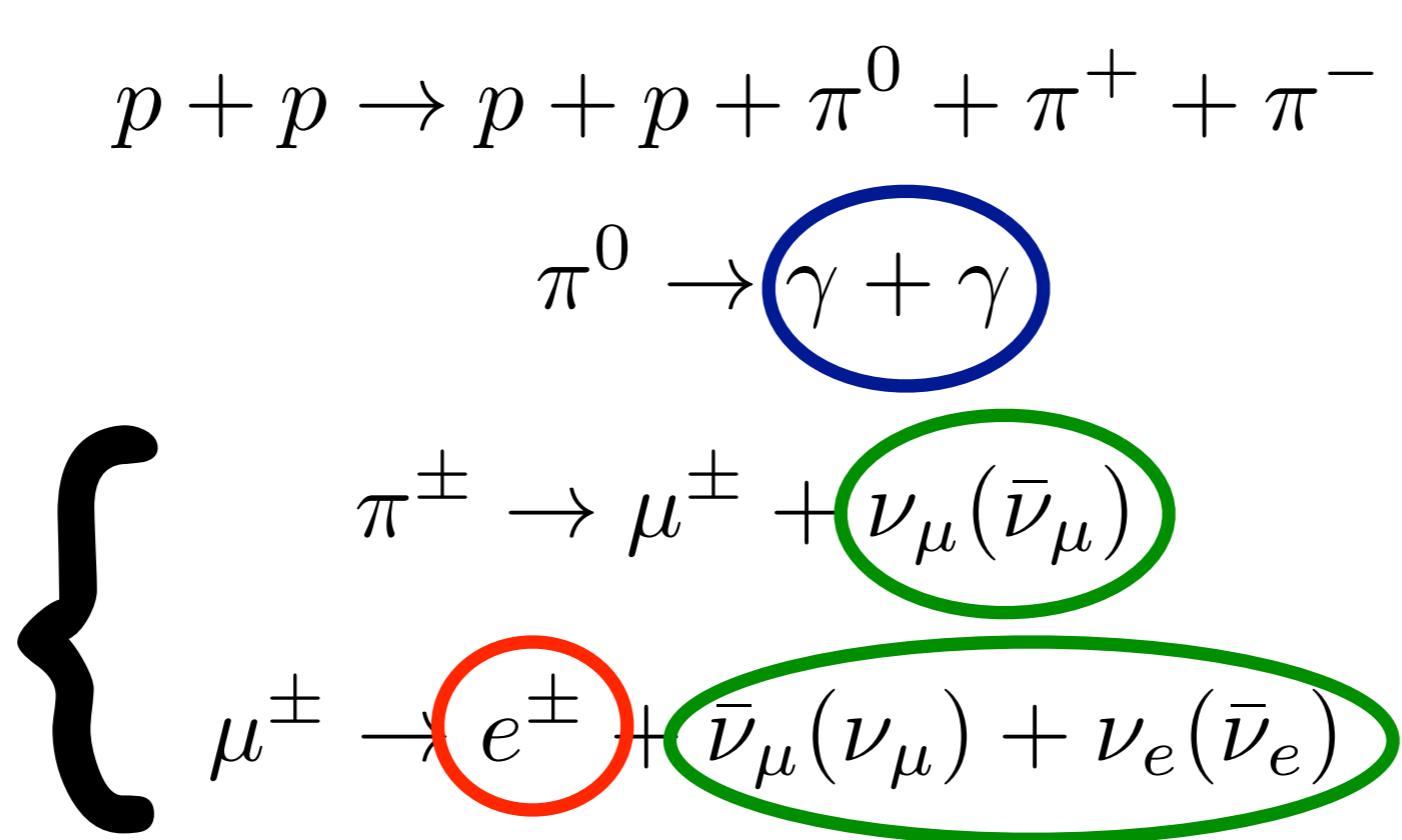
$$p + p \rightarrow p + p + \pi^0 + \pi^+ + \pi^-$$
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Final products of proton-proton interactions are not only **gamma ray photons** but also **neutrinos, anti-neutrinos, electrons and positrons**

$$E_e \approx E_\nu \approx \frac{E_p}{20}$$

Not only gammas: neutrinos & electrons

Neutrinos/antineutrinos & electrons/positrons are also produced in pp interactions



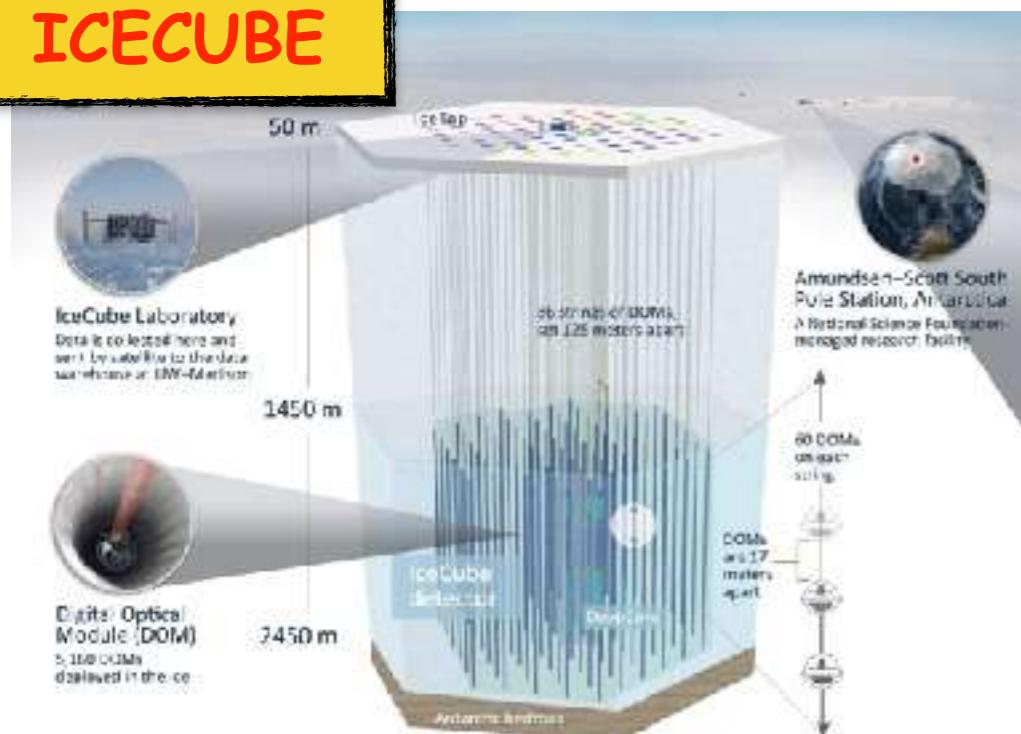
neutral and charged
pions produced with the
same probability
(1/3,1/3,1/3)

Final products of proton-proton interactions are not only **gamma ray photons** but
also **neutrinos, anti-neutrinos, electrons and positrons**

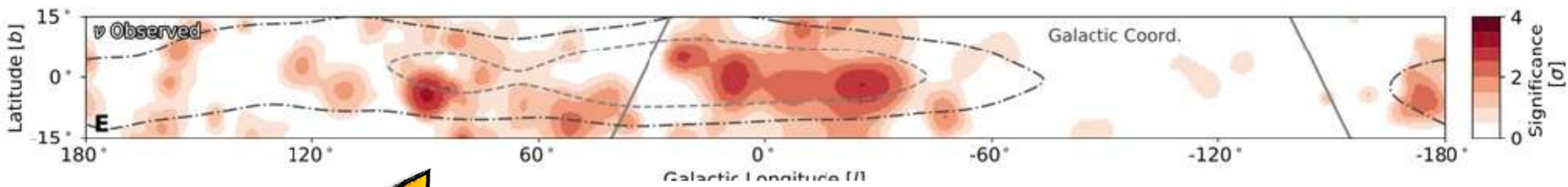
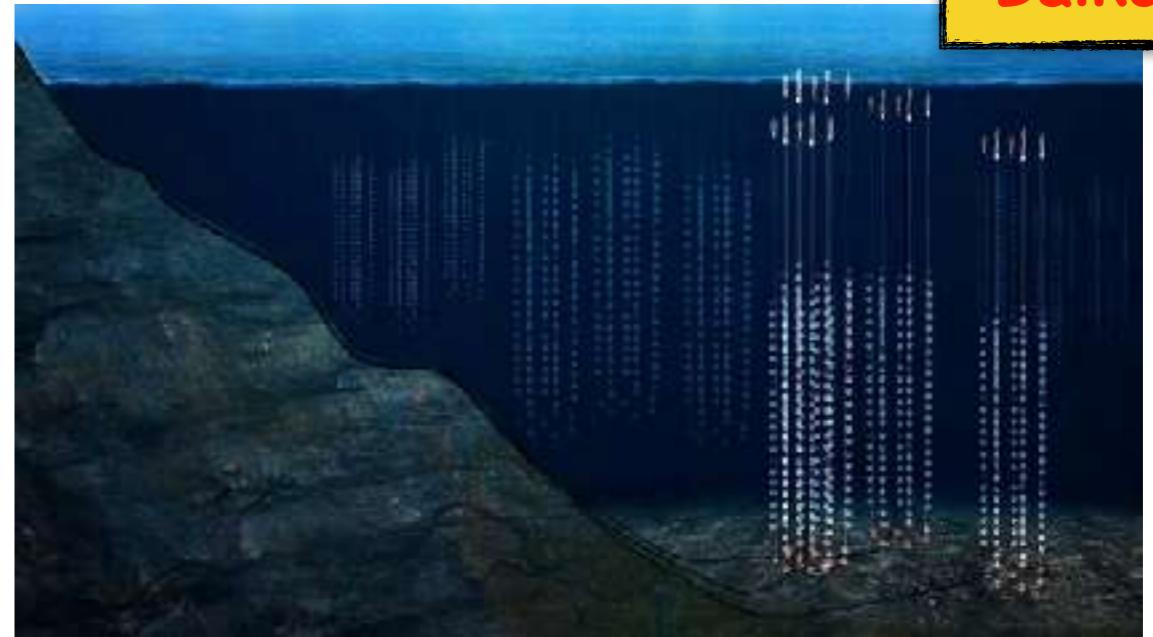
$$E_e \approx E_\nu \approx \frac{E_p}{20}$$

Neutrino telescopes

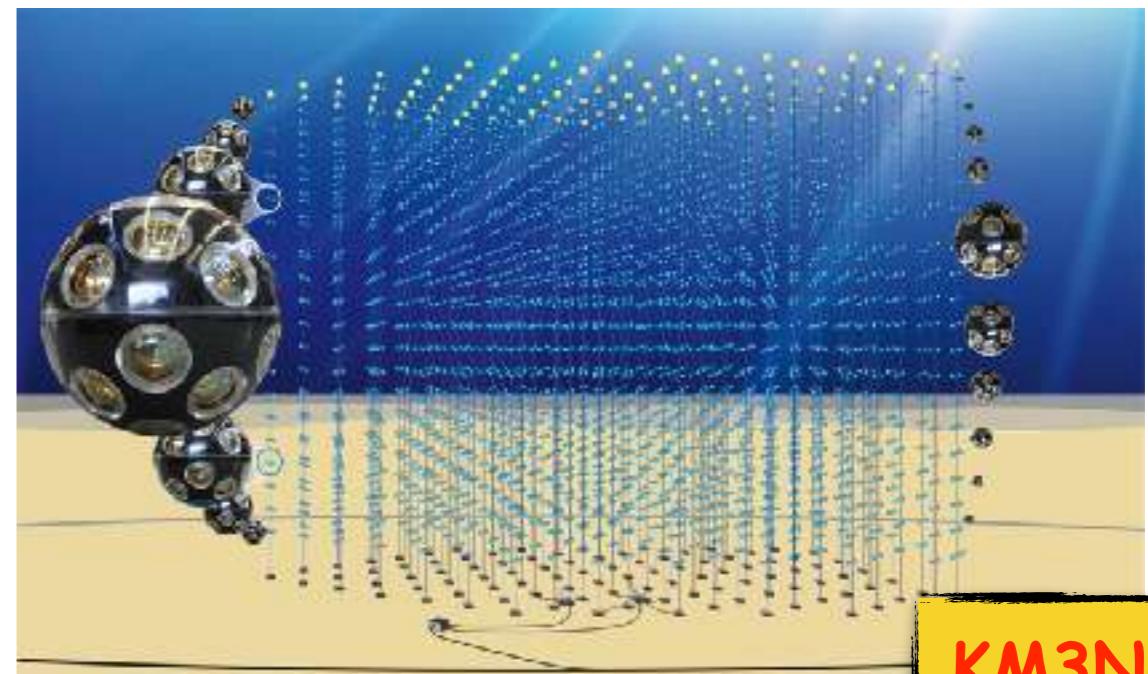
ICECUBE



Baikal



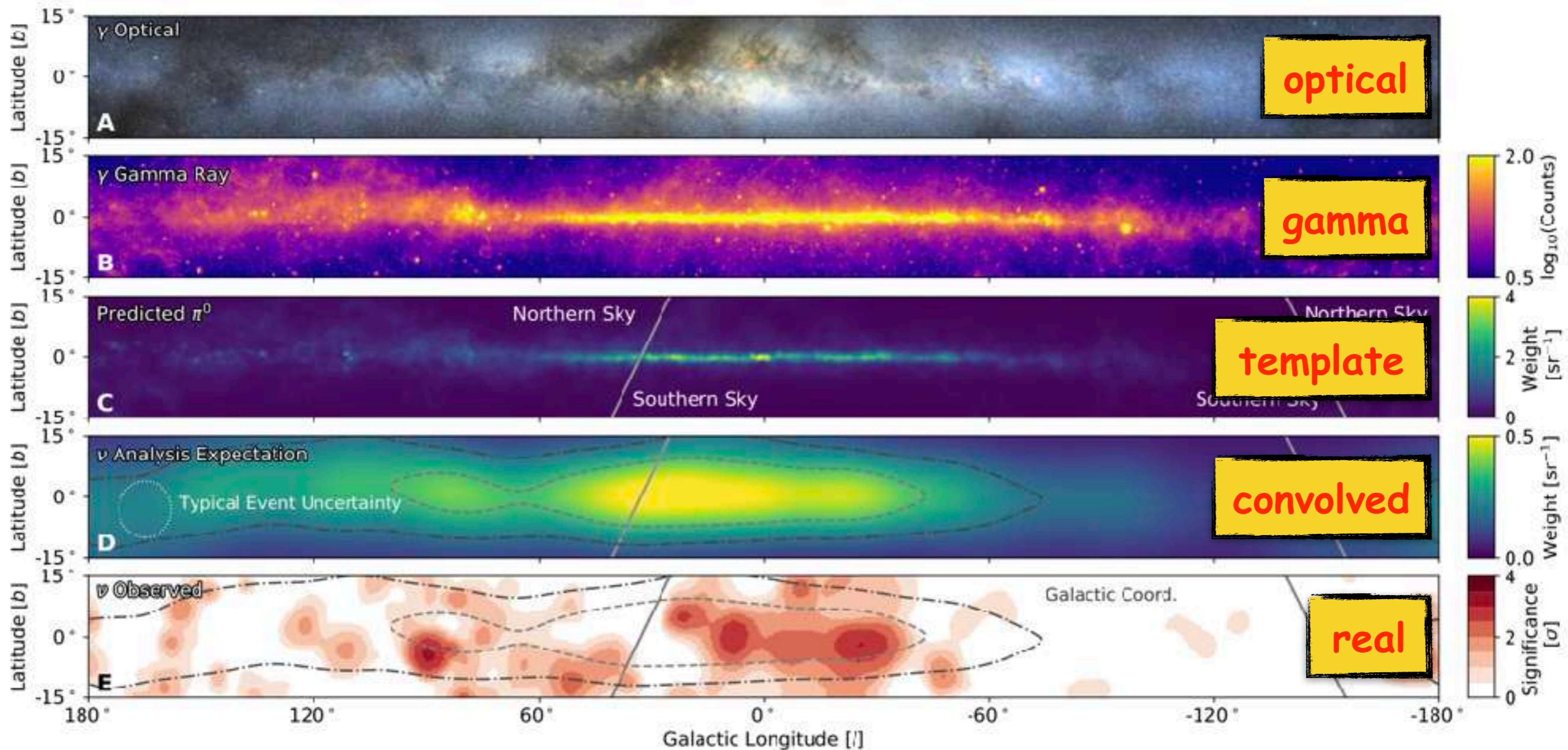
diffuse emission
from Galactic plane!



KM3NeT

The gamma-neutrino connection

a neutrino signal from the disk is unavoidable !



Simple order-of-magnitude calculations

gamma rays

valid at large energies

number of protons of
energy E_p produced per s

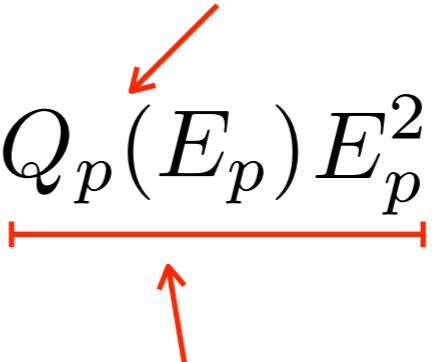
$$Q_p(E_p)$$


Simple order-of-magnitude calculations

gamma rays

valid at large energies

number of protons of
energy E_p produced per s

$$Q_p(E_p) E_p^2$$


power (energy
per unit time)

Simple order-of-magnitude calculations

gamma rays

valid at large energies

fraction of the
energy converted
into pions...

number of protons of
energy E_p produced per s

$$\eta_\pi Q_p(E_p) E_p^2$$

power (energy
per unit time)

Simple order-of-magnitude calculations

gamma rays

valid at large energies

fraction of the
energy converted
into pions...

number of protons of
energy E_p produced per s

$$\eta_\pi Q_p(E_p) E_p^2$$

power (energy
per unit time)

$$\eta_\pi = 1 - e^{-\left(\frac{\tau_{res}}{\tau_{pp}}\right)} \rightarrow \frac{\tau_{res}}{\tau_{pp}} \quad \tau_{pp} \gg \tau_{res}$$
$$\rightarrow 1 \quad \tau_{pp} \ll \tau_{res}$$

Simple order-of-magnitude calculations

gamma rays

valid at large energies

$$f_\gamma \eta_\pi Q_p(E_p) E_p^2$$

fraction of the energy converted into pions...

...and into gamma rays ($1/3 \rightarrow \pi^0$)

number of protons of energy E_p produced per s

power (energy per unit time)

A diagram illustrating the components of the formula. A bracket is placed under the term $Q_p(E_p) E_p^2$, which is labeled "number of protons of energy E_p produced per s". Red arrows point from the text "fraction of the energy converted into pions..." and "...and into gamma rays ($1/3 \rightarrow \pi^0$)" to the term $f_\gamma \eta_\pi$. Another red arrow points from the text "power (energy per unit time)" to the bracket under $Q_p(E_p) E_p^2$.

$$\eta_\pi = 1 - e^{-\left(\frac{\tau_{res}}{\tau_{pp}}\right)} \longrightarrow \frac{\tau_{res}}{\tau_{pp}} \quad \tau_{pp} \gg \tau_{res}$$
$$\longrightarrow 1 \quad \tau_{pp} \ll \tau_{res}$$

Simple order-of-magnitude calculations

gamma rays

valid at large energies

$$Q_\gamma(E_\gamma)E_\gamma^2 = f_\gamma \eta_\pi Q_p(E_p)E_p^2$$

emitted power in gamma rays

fraction of the energy converted into pions...

number of protons of energy E_p produced per s

...and into gamma rays ($1/3 \rightarrow \pi^0$)

power (energy per unit time)

$$\eta_\pi = 1 - e^{-\left(\frac{\tau_{res}}{\tau_{pp}}\right)} \rightarrow \frac{\tau_{res}}{\tau_{pp}}$$

$\tau_{pp} \gg \tau_{res}$

$$\rightarrow 1$$

$\tau_{pp} \ll \tau_{res}$

Simple order-of-magnitude calculations

gamma rays

valid at large energies

$$Q_\gamma(E_\gamma)E_\gamma^2 = f_\gamma \eta_\pi Q_p(E_p)E_p^2$$

emitted power in gamma rays

fraction of the energy converted into pions...

number of protons of energy E_p produced per s

$E_\gamma \sim E_p/10$

...and into gamma rays ($1/3 \rightarrow \pi^0$)

power (energy per unit time)

$$\eta_\pi = 1 - e^{-\left(\frac{\tau_{res}}{\tau_{pp}}\right)} \rightarrow \frac{\tau_{res}}{\tau_{pp}} \quad \tau_{pp} \gg \tau_{res}$$

$$\rightarrow 1 \quad \tau_{pp} \ll \tau_{res}$$

Simple order-of-magnitude calculations

neutrinos

valid at large energies

The same as for gammas but:

$$\rightarrow f_\gamma \longrightarrow f_\nu = \frac{2}{3} \times \frac{3}{4} \times \frac{1}{3}$$

↑ ↑ ↑
fraction of fraction of per flavour (after
charged pions pion energy -> oscillations)

$$\rightarrow E_\gamma \longrightarrow E_\nu \sim 0.05 \times E_p$$

Simple order-of-magnitude calculations

valid at large energies

gamma rays

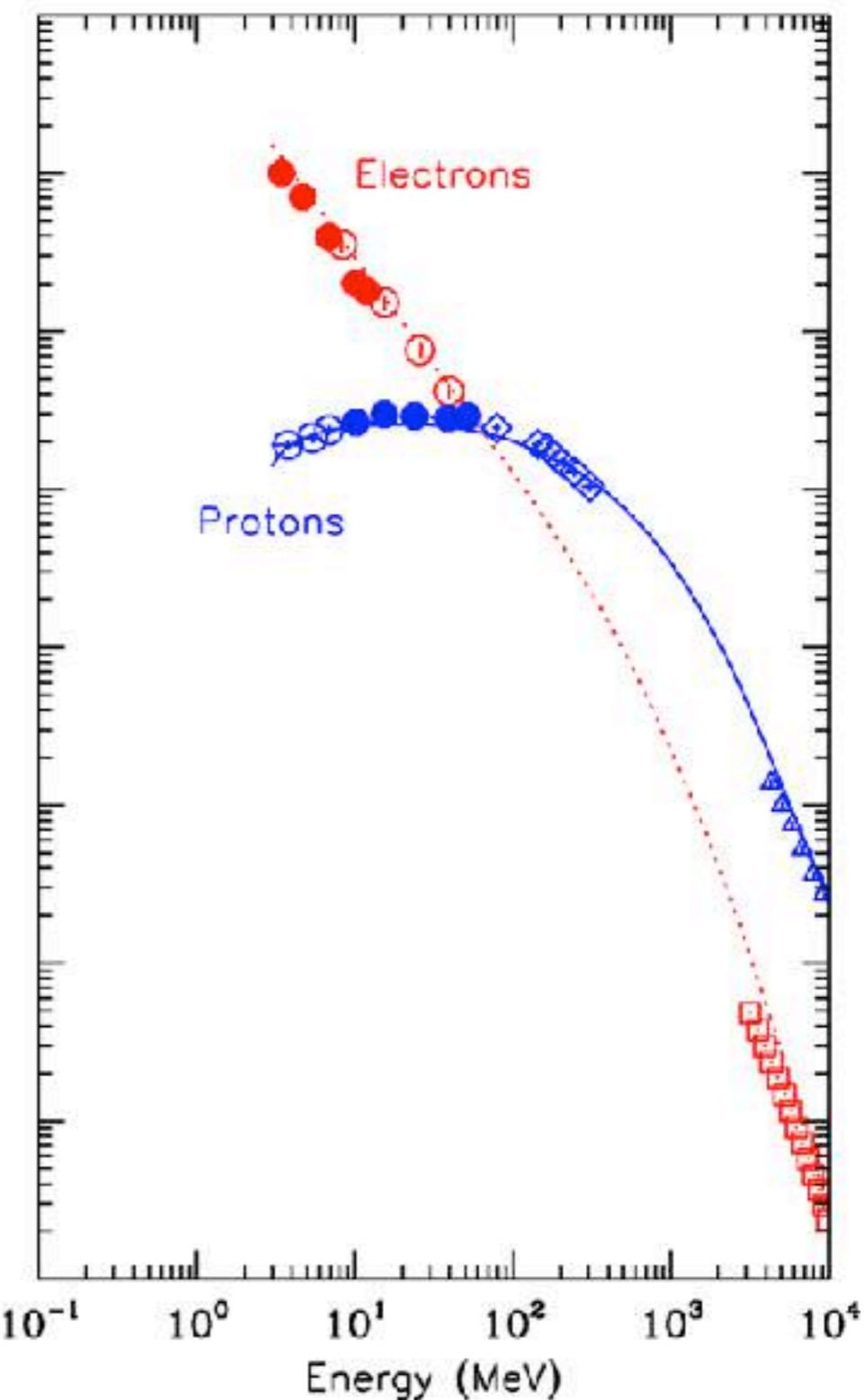
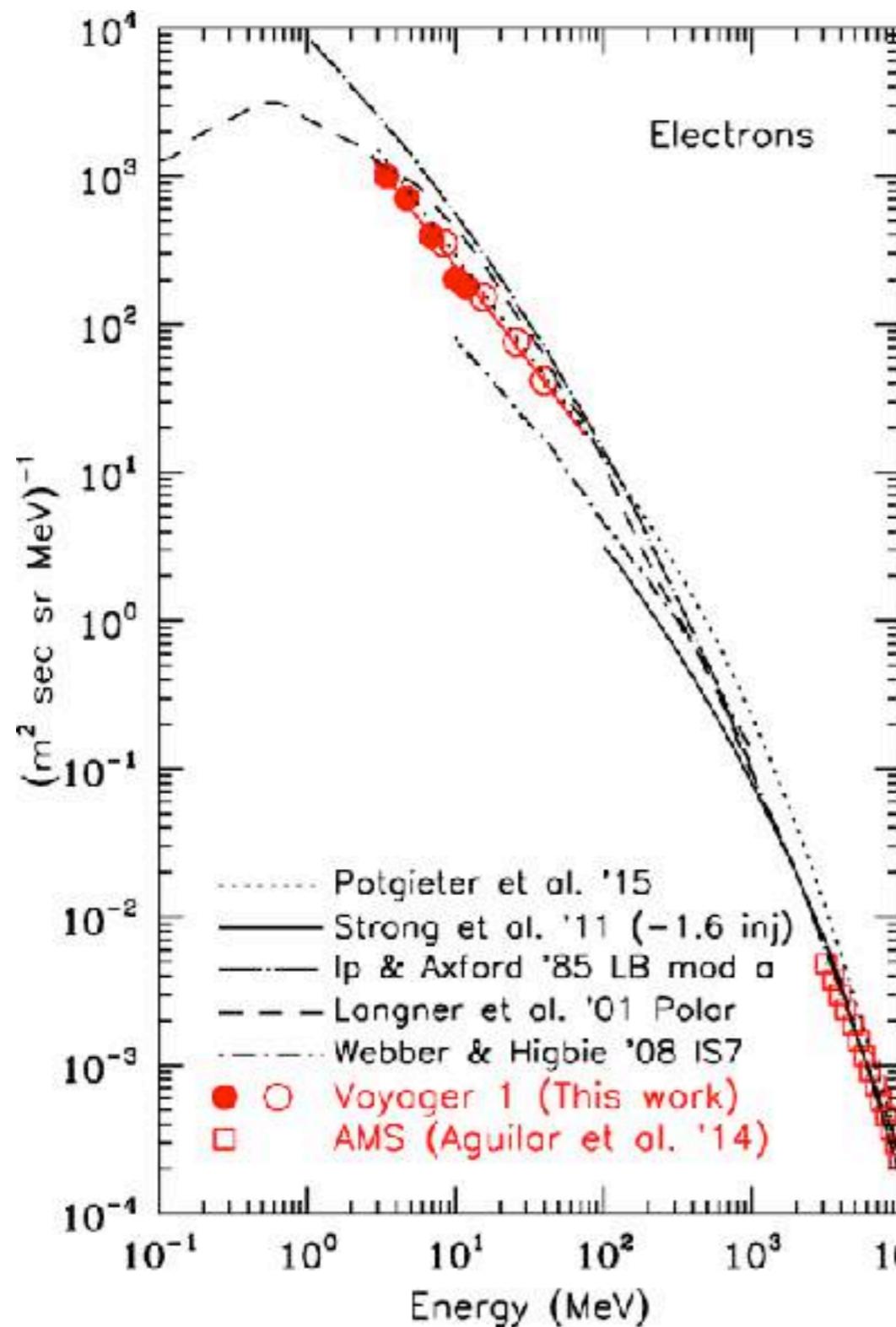
$$Q_\gamma(E_\gamma)E_\gamma^2 = \frac{\eta_\pi}{3}Q_p(E_p)E_p^2$$

neutrinos
(per flavour)

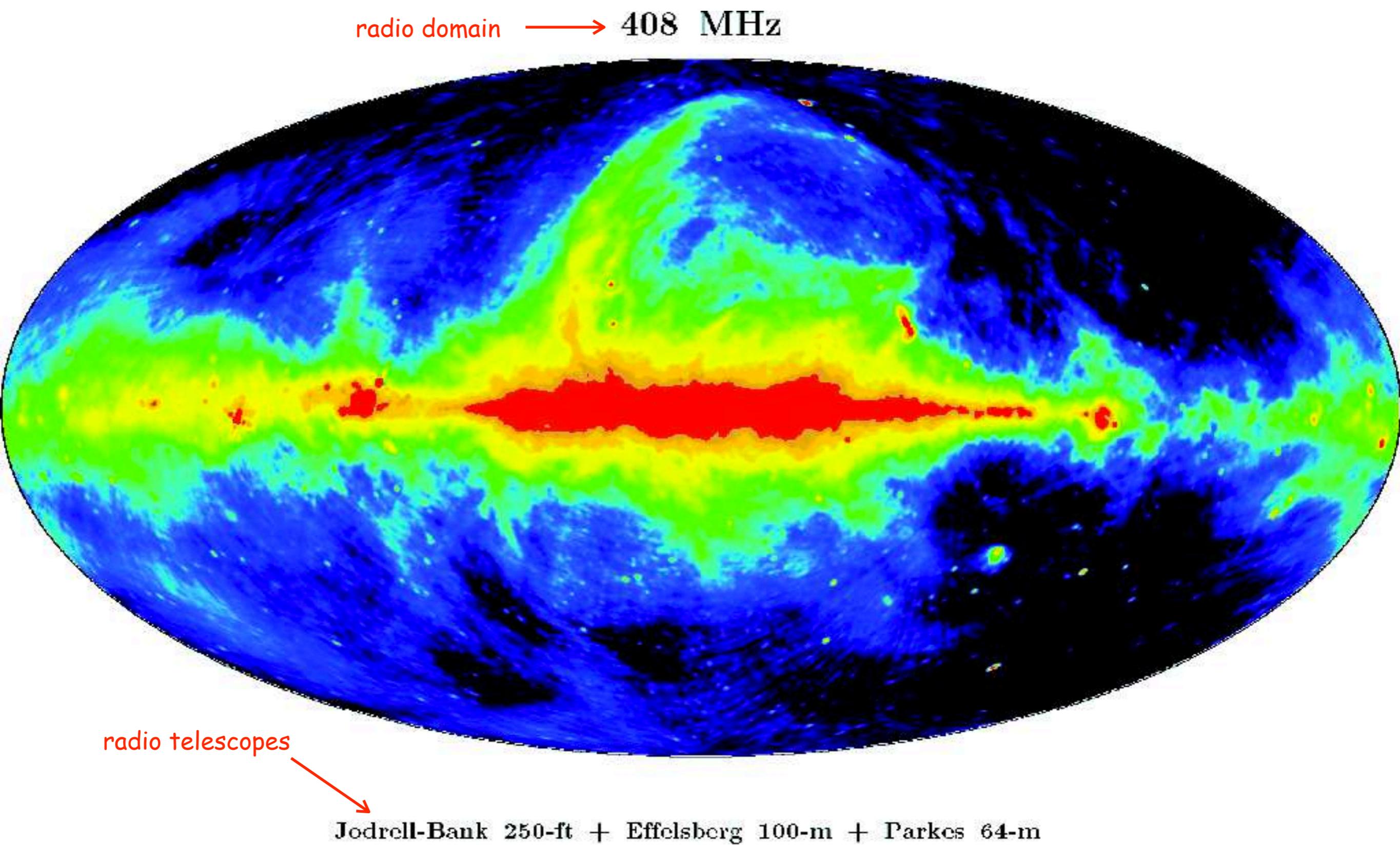
$$Q_\nu(E_\nu)E_\nu^2 = \frac{1}{2}Q_\gamma(E_\gamma)E_\gamma^2$$

What about electrons?

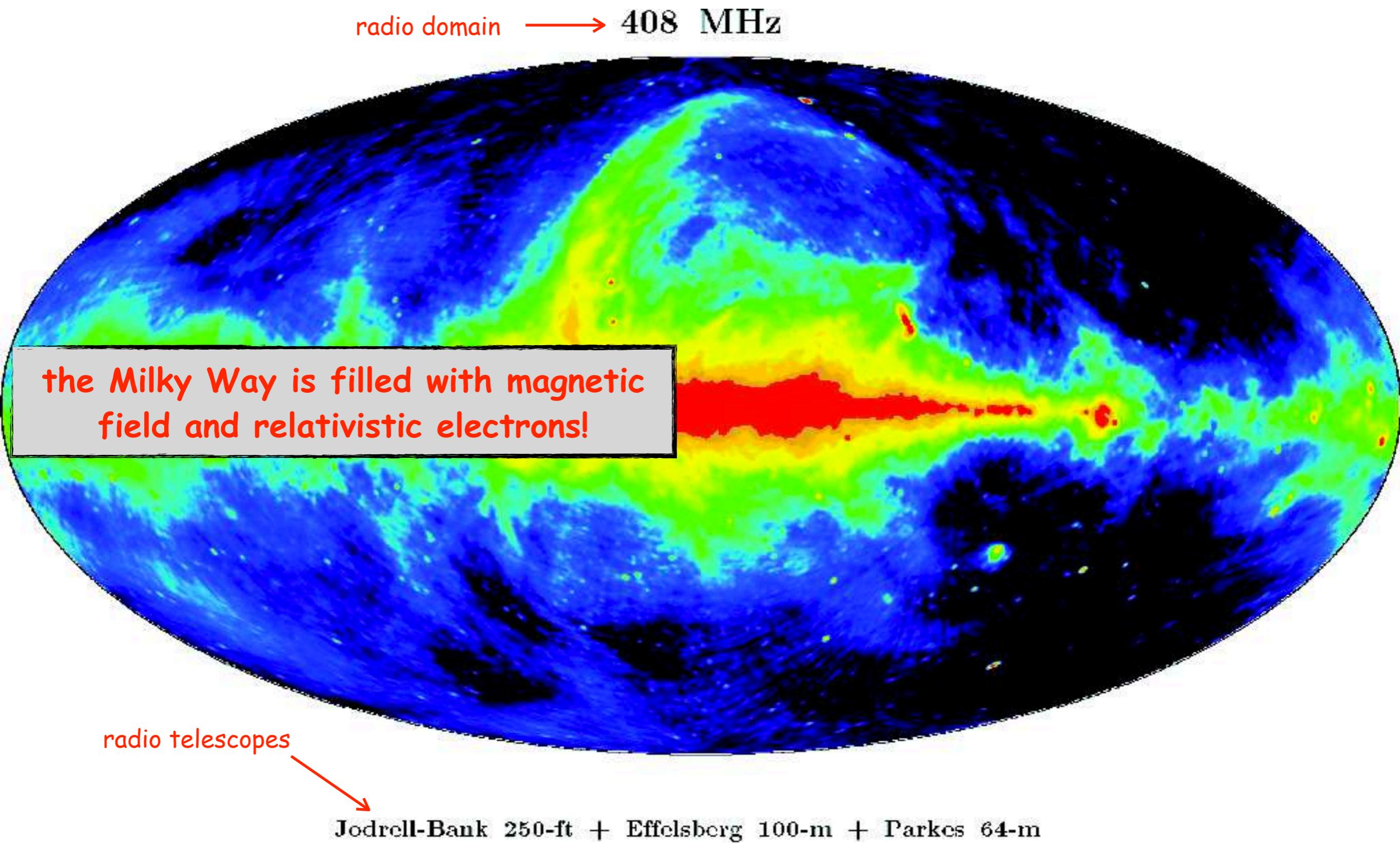
false myths: the proton-to-electron ratio is ~ 100 in Galactic cosmic rays



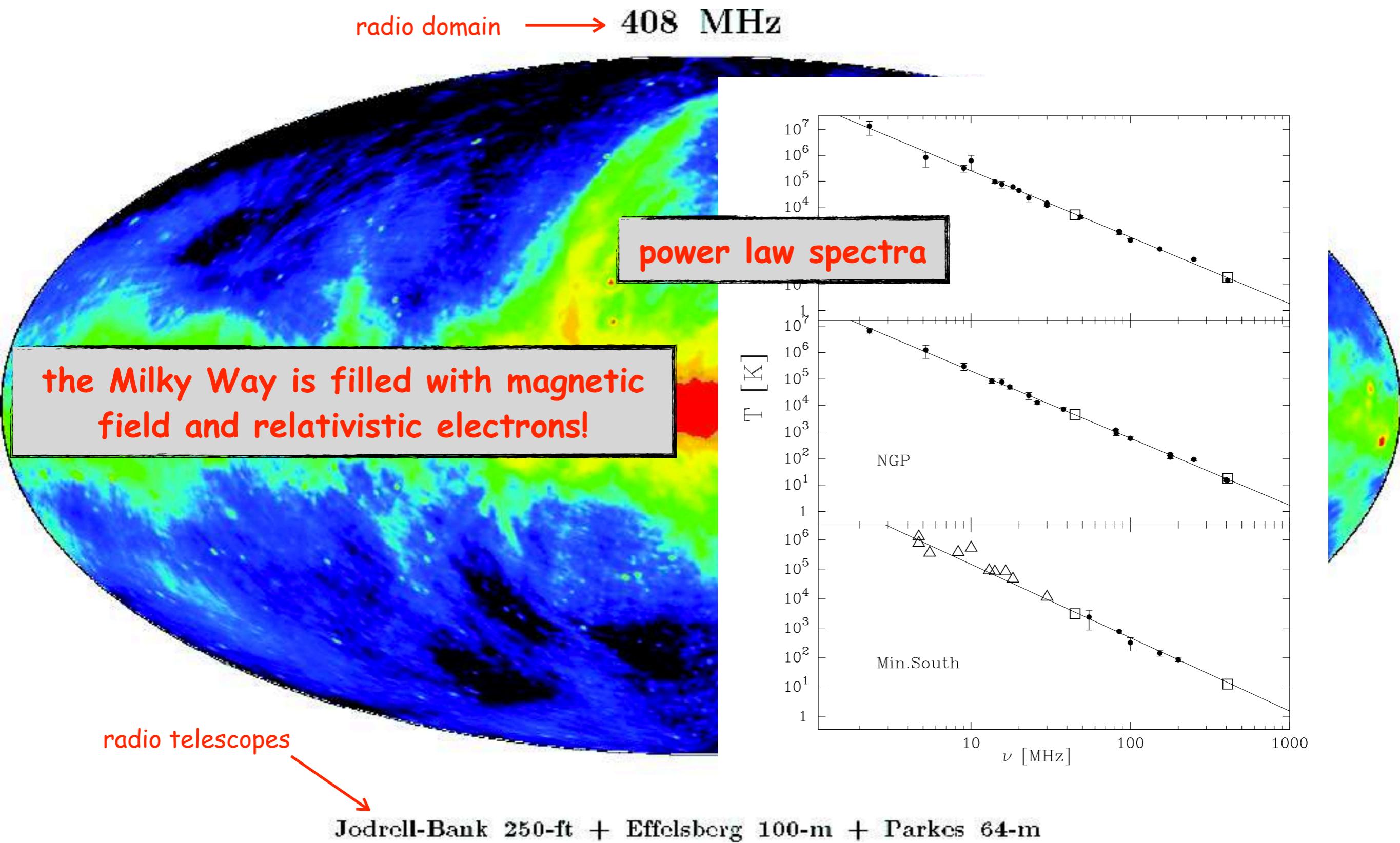
Synchrotron emission from the Milky Way



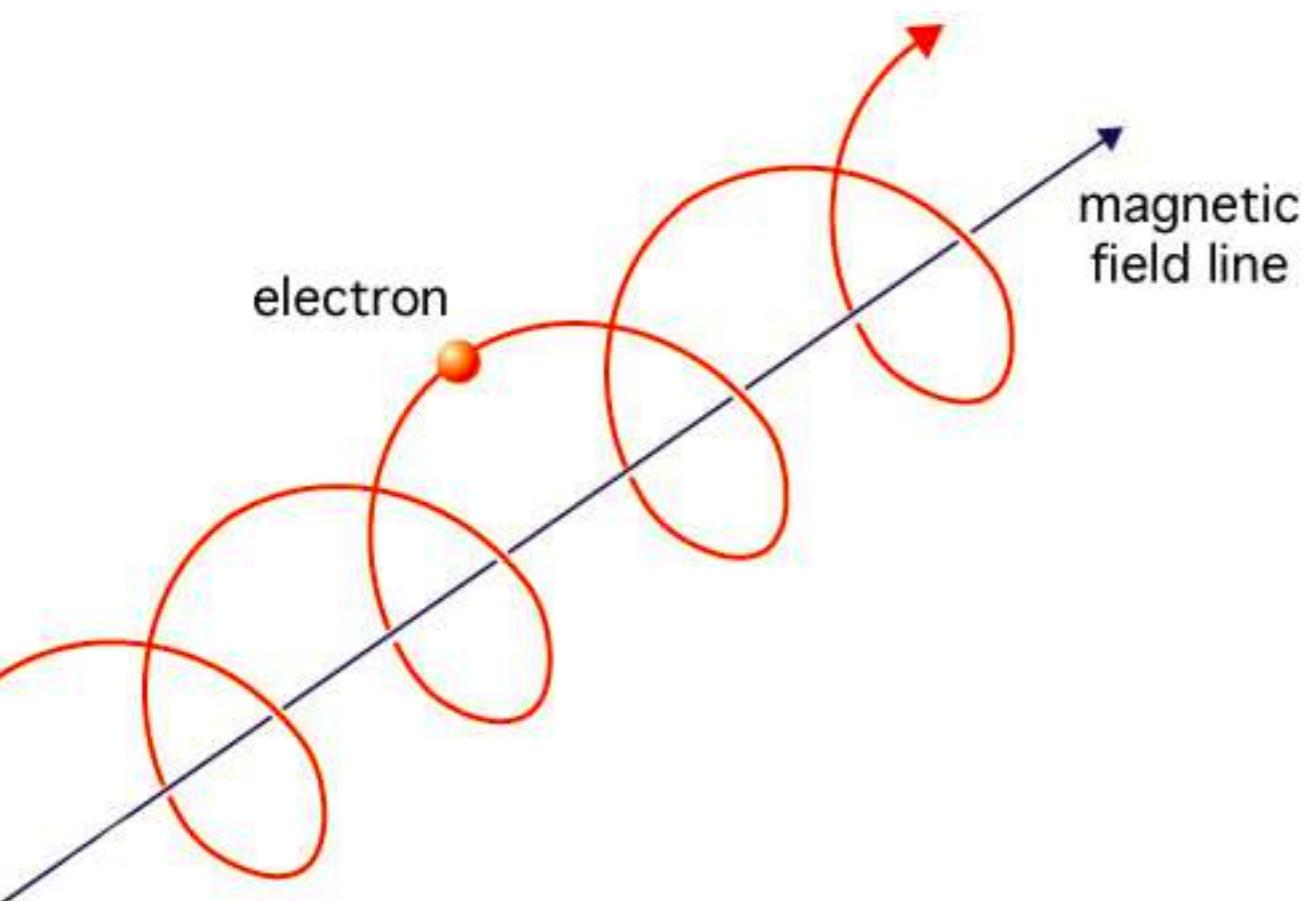
Synchrotron emission from the Milky Way



Synchrotron emission from the Milky Way



Motion of a particle in a magnetic field

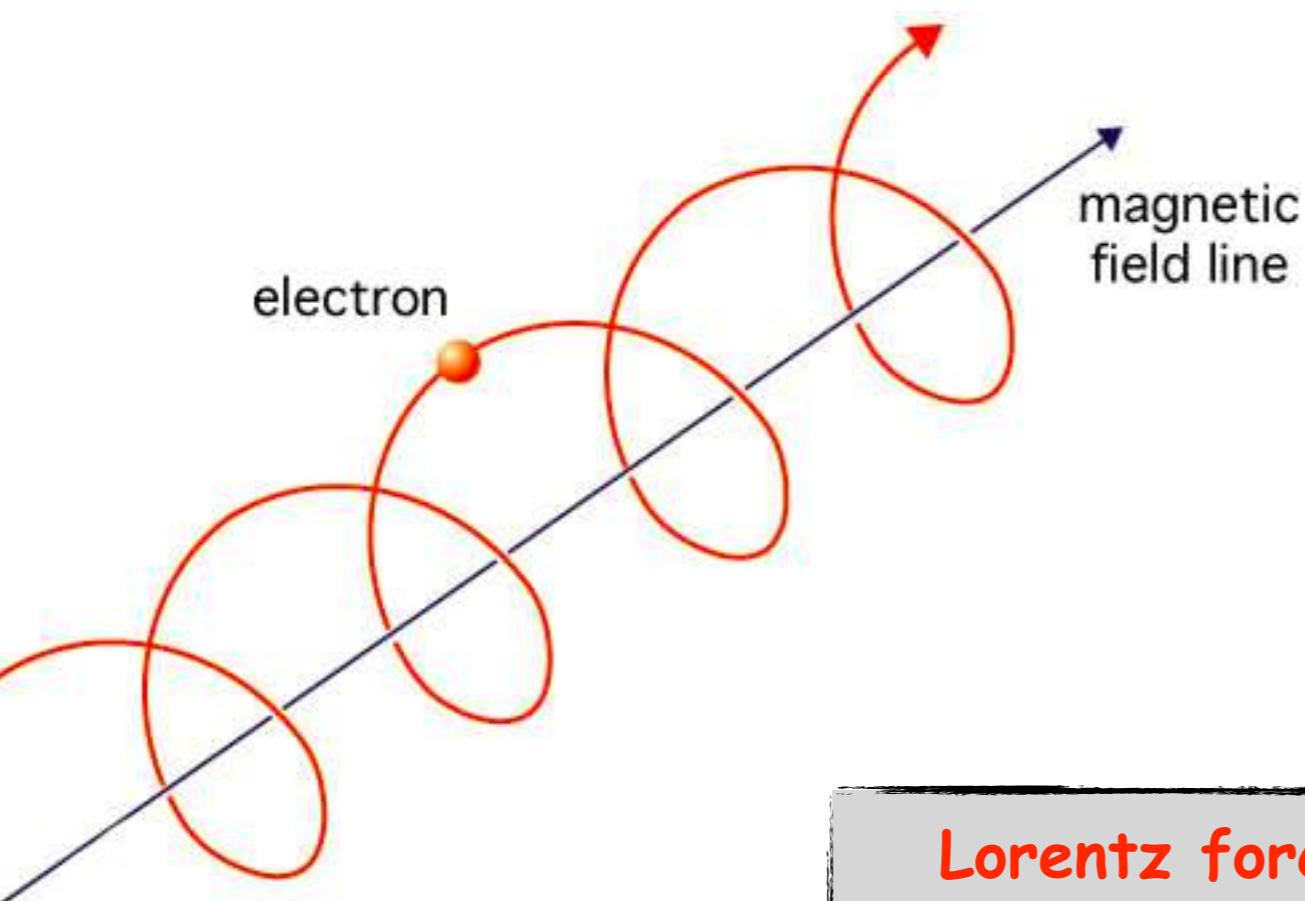


$$\mu = \cos \vartheta$$

pitch angle = angle between v and B

$$\begin{cases} v_{\parallel} &= \mu v \\ v_{\perp} &= (1 - \mu^2)^{1/2}v \end{cases}$$

Motion of a particle in a magnetic field



$$\mu = \cos \vartheta$$

pitch angle = angle between v and B

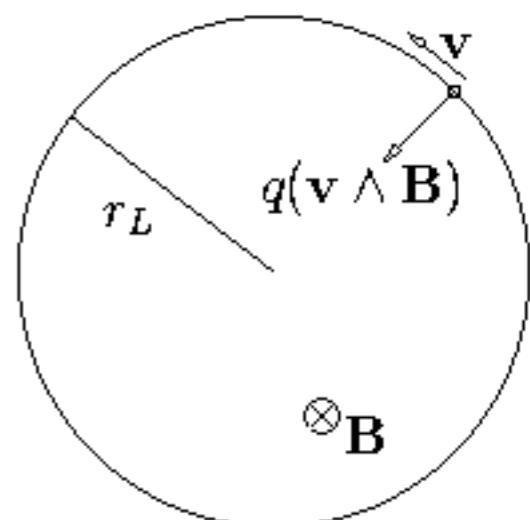
$$\begin{cases} v_{\parallel} &= \mu v \\ v_{\perp} &= (1 - \mu^2)^{1/2}v \end{cases}$$

Lorentz force

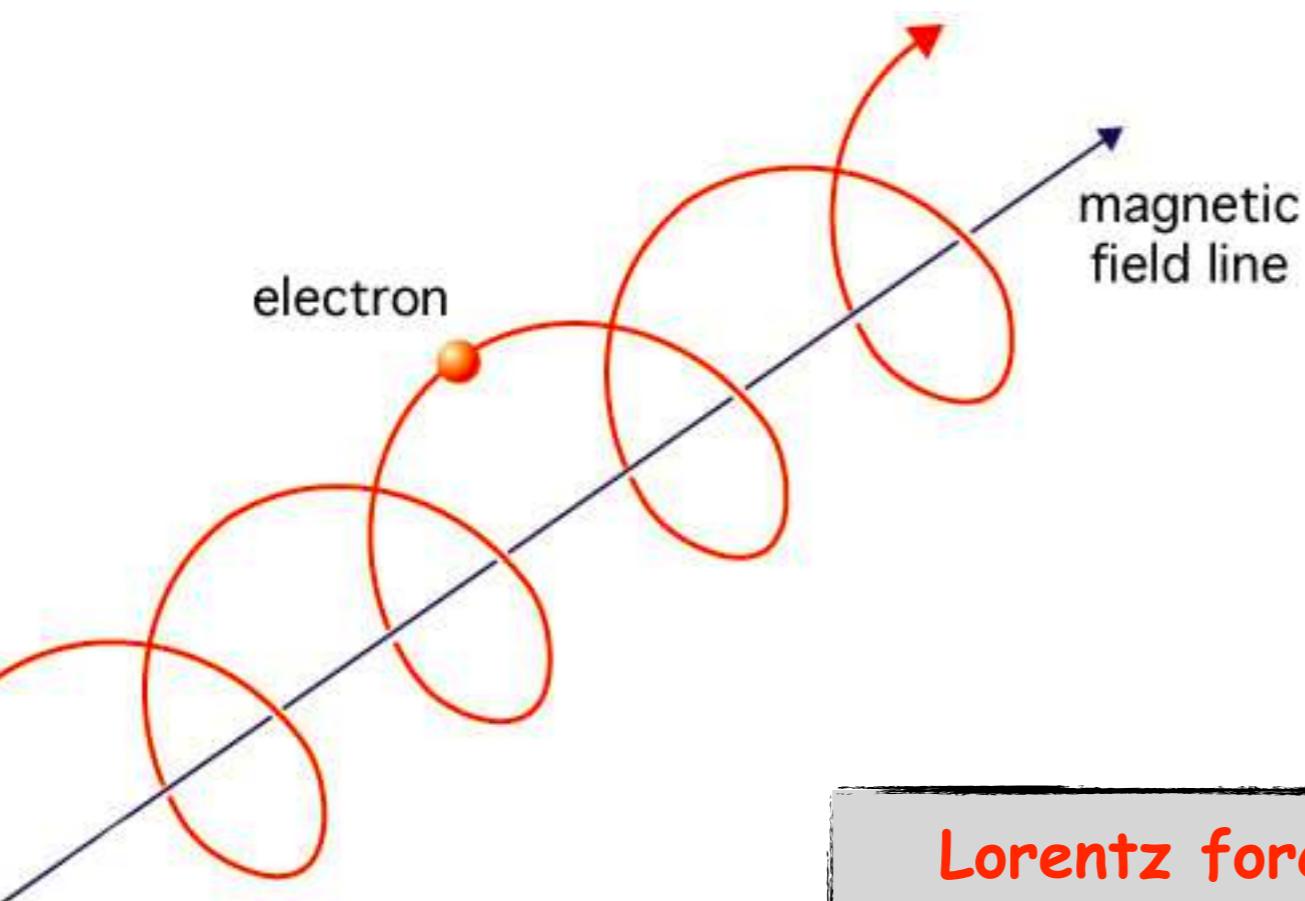
$$F_L = \frac{q}{c} \vec{v} \times \vec{B}$$

Larmor radius

$$R_L = \frac{p_{\perp} c}{q B}$$



Motion of a particle in a magnetic field



$$\mu = \cos \vartheta$$

pitch angle = angle between v and B

$$\begin{cases} v_{\parallel} &= \mu v \\ v_{\perp} &= (1 - \mu^2)^{1/2}v \end{cases}$$

Lorentz force

$$F_L = \frac{q}{c} \vec{v} \times \vec{B}$$

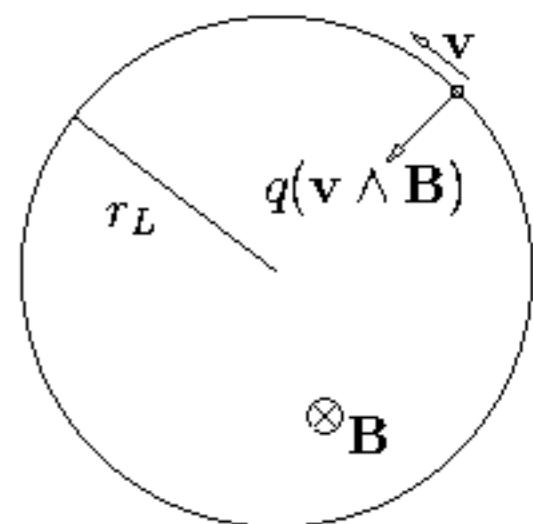
Larmor radius

$$R_L = \frac{p_{\perp} c}{q B}$$

gyration frequency

$$\nu_B = \frac{1}{t_g} = \frac{v_{\perp}}{2\pi R_L} = \frac{qB}{2\pi\gamma mc}$$

Lorentz factor



Power emitted by an electron*

non-relativistic

$$P = \frac{2e^2}{3c^3} a^2 \longrightarrow P = \frac{2e^2}{3c^3} \gamma^4 \left[\gamma^2 a_{\parallel}^2 + a_{\perp}^2 \right]$$

relativistic

* implicit assumption: the energy of the electron does not change during one gyration around the B-field

Power emitted by an electron*

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$$P = \frac{2e^2}{3c^3} a^2$$

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relativistic

Lorentz force is orthogonal to v

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$$P = \frac{2e^2}{3c^3} a^2$$

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relativistic

Lorentz force is orthogonal to v

$$F_L = F_{L,\perp} = \frac{ev_{\perp}B}{c} \equiv \gamma m \frac{dv_{\perp}}{dt} \rightarrow a_{\perp} = \frac{ev_{\perp}B}{\gamma mc}$$

* implicit assumption: the energy of the electron does not change during one gyration around the B-field

Power emitted by an electron*

non-relativistic

$$P = \frac{2e^2}{3c^3} a^2 \rightarrow P = \frac{2e^2}{3c^3} \gamma^4 \left[\gamma^2 a_{\parallel}^2 + a_{\perp}^2 \right]$$

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Lorentz force is orthogonal to v

$$F_L = F_{L,\perp} = \frac{ev_{\perp}B}{c} \equiv \gamma m \frac{dv_{\perp}}{dt} \rightarrow a_{\perp} = \frac{ev_{\perp}B}{\gamma mc}$$



$$P = \frac{4}{3} \sigma_T c U_B \gamma^2$$

■ Thomson cross section

$$\sigma_T = \frac{8\pi}{3} \left(\frac{e^2}{mc^2} \right)^2 = 6.65 \times 10^{-25} \text{ cm}^2$$

■ magnetic field energy density $U_B = B^2/8\pi$

■ ultra relativistic electrons $\beta \rightarrow 1$

■ isotropic distribution of particles $\langle \sin^2 \vartheta \rangle = 2/3$

* implicit assumption: the energy of the electron does not change during one gyration around the B-field

Power emitted by an electron*

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* implicit assumption: the energy of the electron does not change during one gyration around the B-field

Characteristic frequency

as done for Bremsstrahlung: characteristic time ----> characteristic frequency

gyration frequency?

$$\nu_B = \frac{1}{t_g} = \frac{v_{\perp}}{2\pi R_L} = \frac{qB}{2\pi\gamma mc}$$

Characteristic frequency

as done for Bremsstrahlung: characteristic time ----> characteristic frequency

gyration frequency?

$$\nu_B = \frac{1}{t_g} = \frac{v_{\perp}}{2\pi R_L} = \cancel{\frac{qB}{2\pi\gamma c}}$$

Beaming

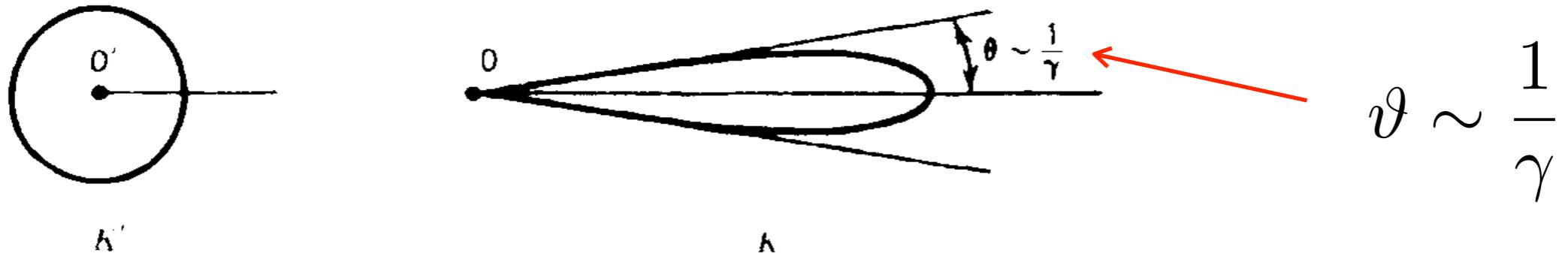


Figure 4.3 Relativistic beaming of radiation emitted isotropically in the rest frame K' .

the radiation emitted by a relativistic particle is concentrated within a cone of opening angle $1/\gamma$ entered along the particle velocity

Characteristic frequency

as done for Bremsstrahlung: characteristic time \longrightarrow characteristic frequency

gyration frequency?

$$\nu_B = \frac{1}{t_g} = \frac{v_{\perp}}{2\pi R_L} = \cancel{\frac{qP}{2\pi\gamma c}}$$

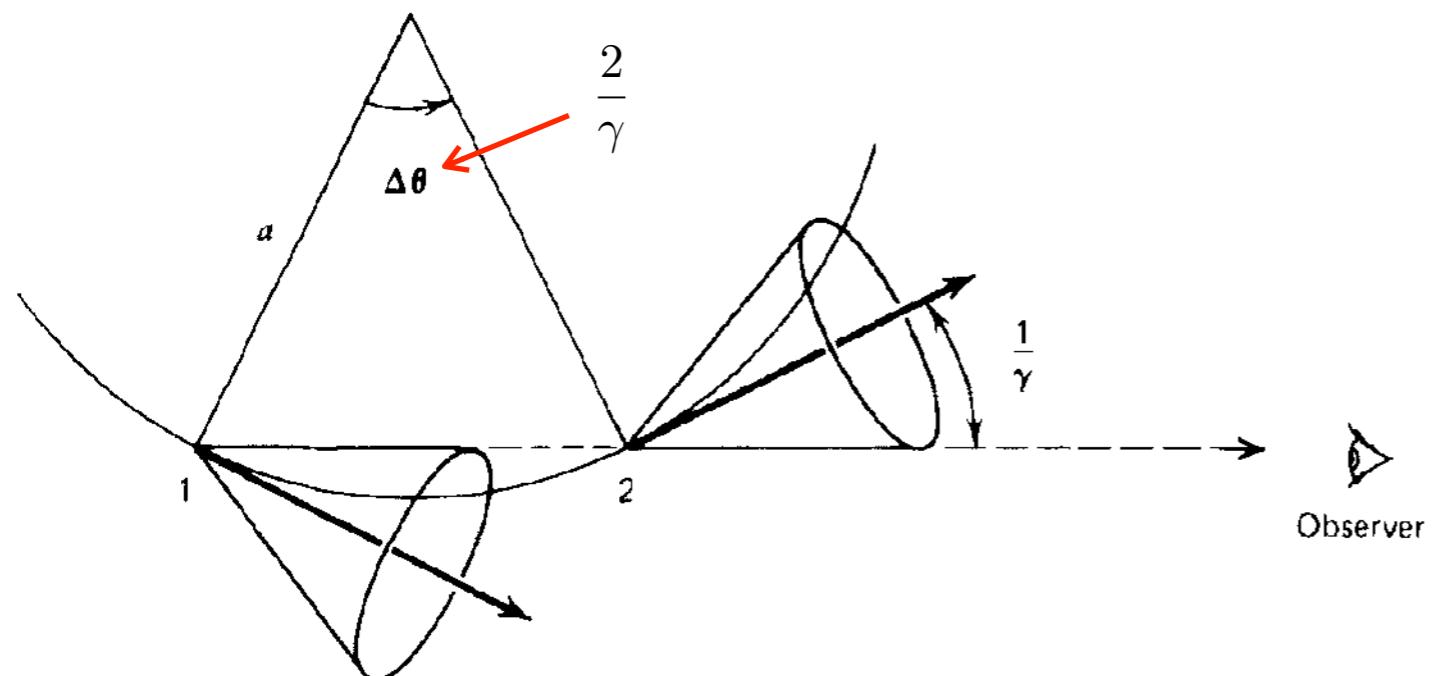


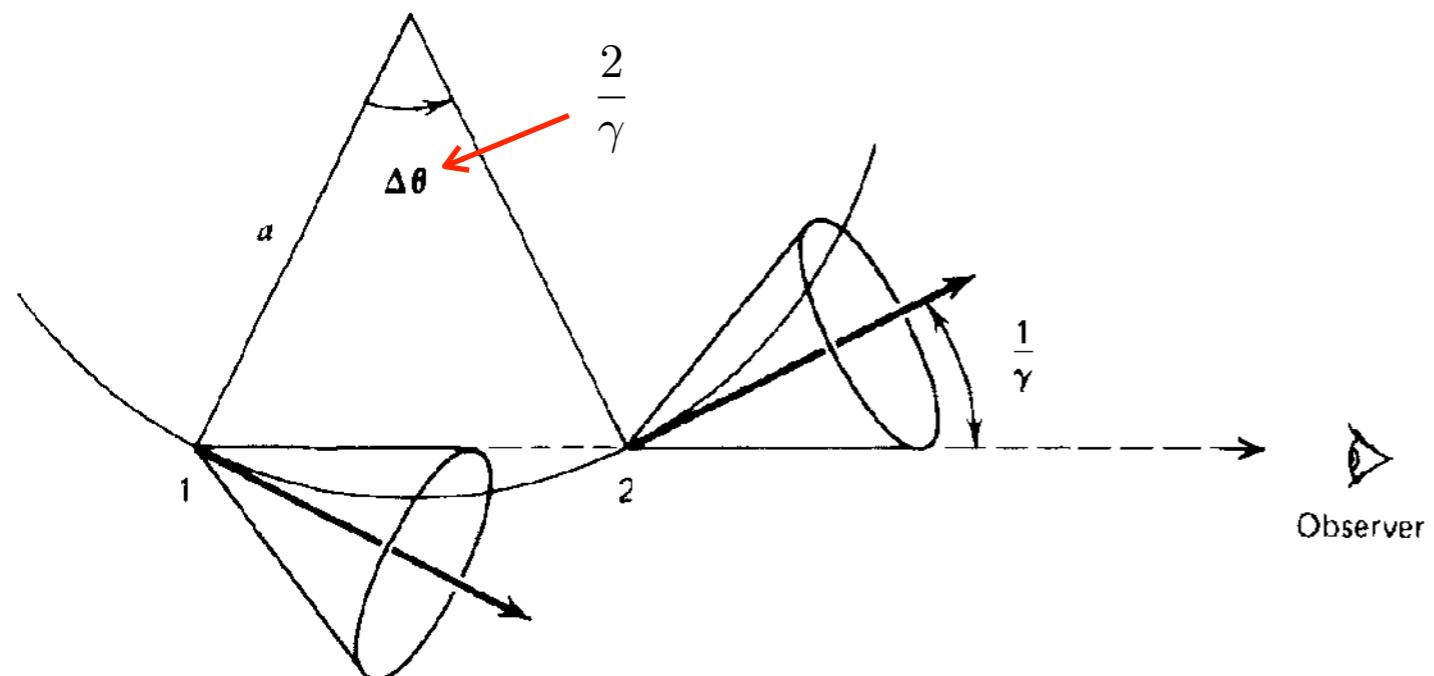
Figure 6.2 Emission cones at various points of an accelerated particle's trajectory.

Characteristic frequency

as done for Bremsstrahlung: characteristic time \longrightarrow characteristic frequency

gyration frequency?

$$\nu_B = \frac{1}{t_g} = \frac{v_{\perp}}{2\pi R_L} = \cancel{\frac{qP}{2\pi\gamma c}}$$



photons that reach us are emitted in the time interval:

$$\Delta t_e = \frac{R_L \Delta\vartheta}{v_{\perp}} = \frac{1}{\pi\gamma\nu_B}$$

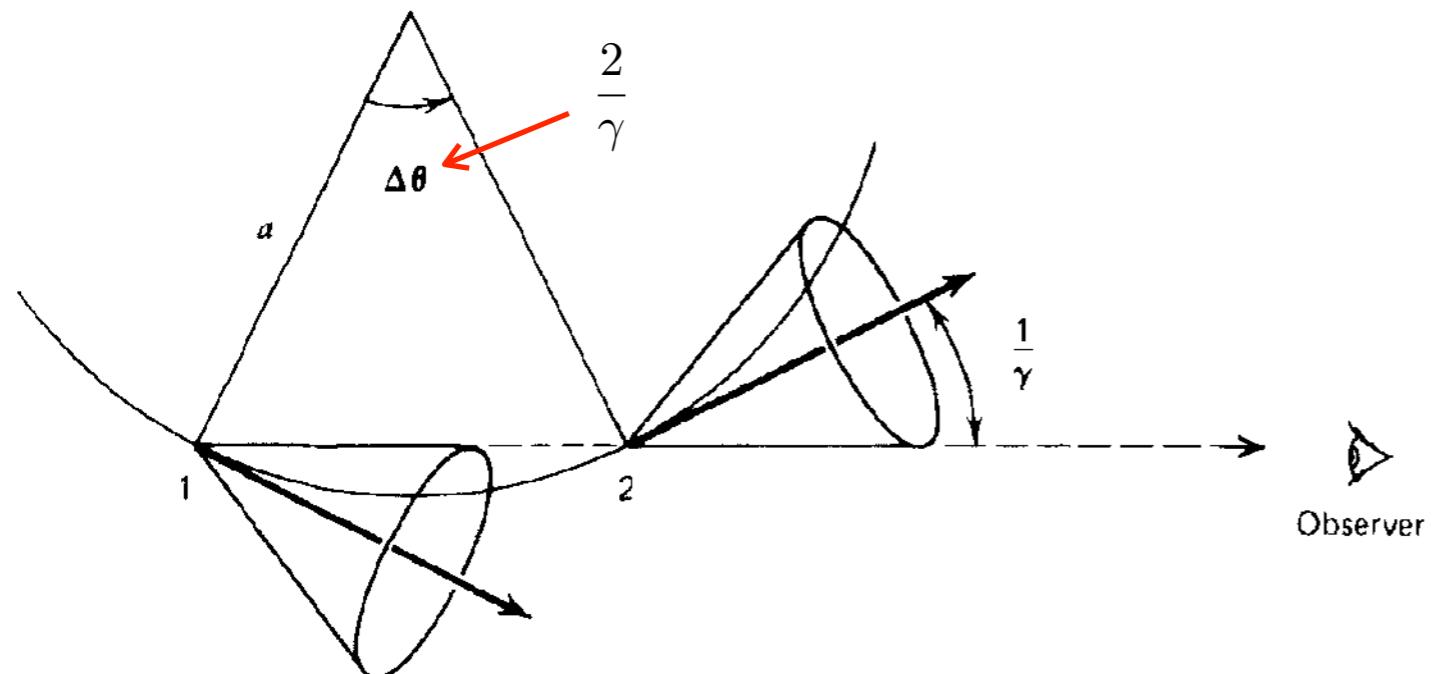
Figure 6.2 Emission cones at various points of an accelerated particle's trajectory.

Characteristic frequency

as done for Bremsstrahlung: characteristic time \longrightarrow characteristic frequency

gyration frequency?

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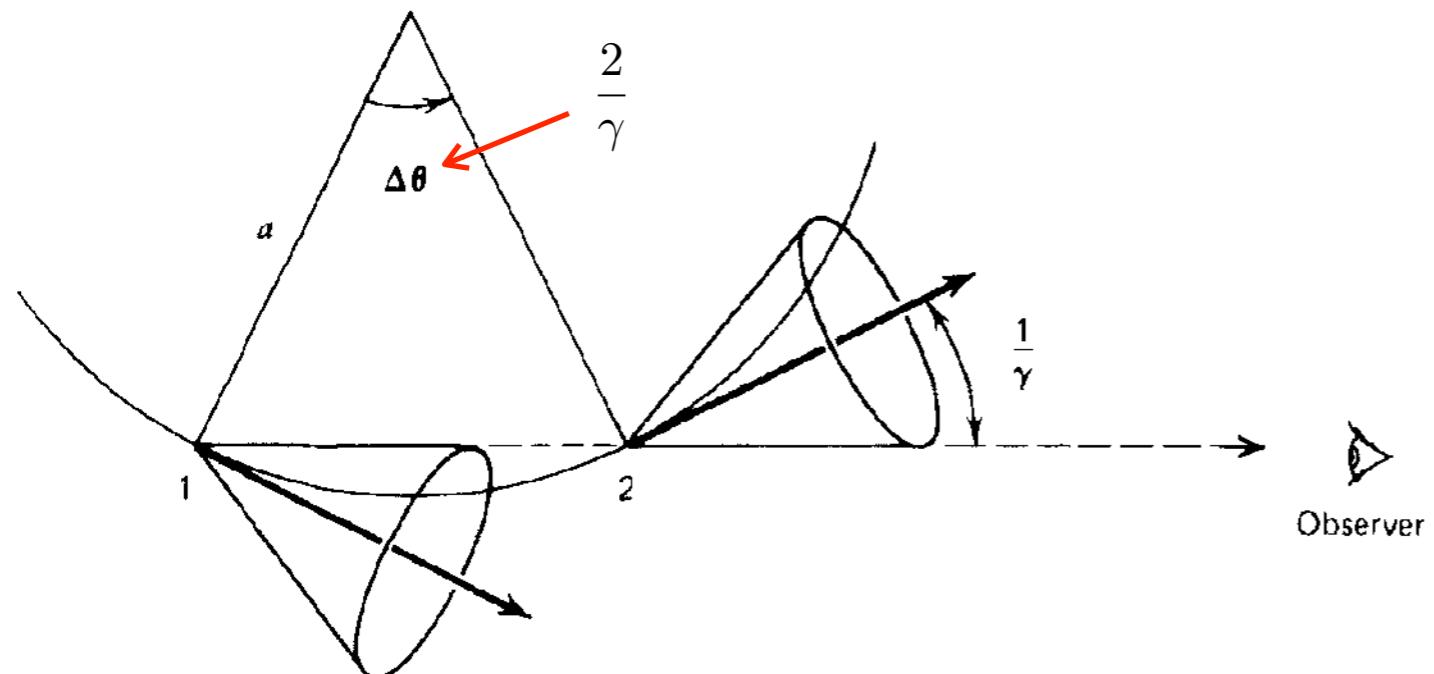
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$$\Delta t_a = \frac{c \Delta t_e - v_{\perp} \Delta t_e}{c} \approx \Delta t_e (1 - \beta) = \Delta t_e \frac{1 - \beta^2}{1 + \beta} \approx \frac{\Delta t_e}{2\gamma^2}$$

Emission from one and many electrons

duration of the
received pulse

$$\Delta t_a \approx \frac{\Delta t_e}{2\gamma^2} = \frac{1}{2\pi\gamma^3\nu_B} = \frac{1}{\gamma^2} \frac{mc}{qB}$$

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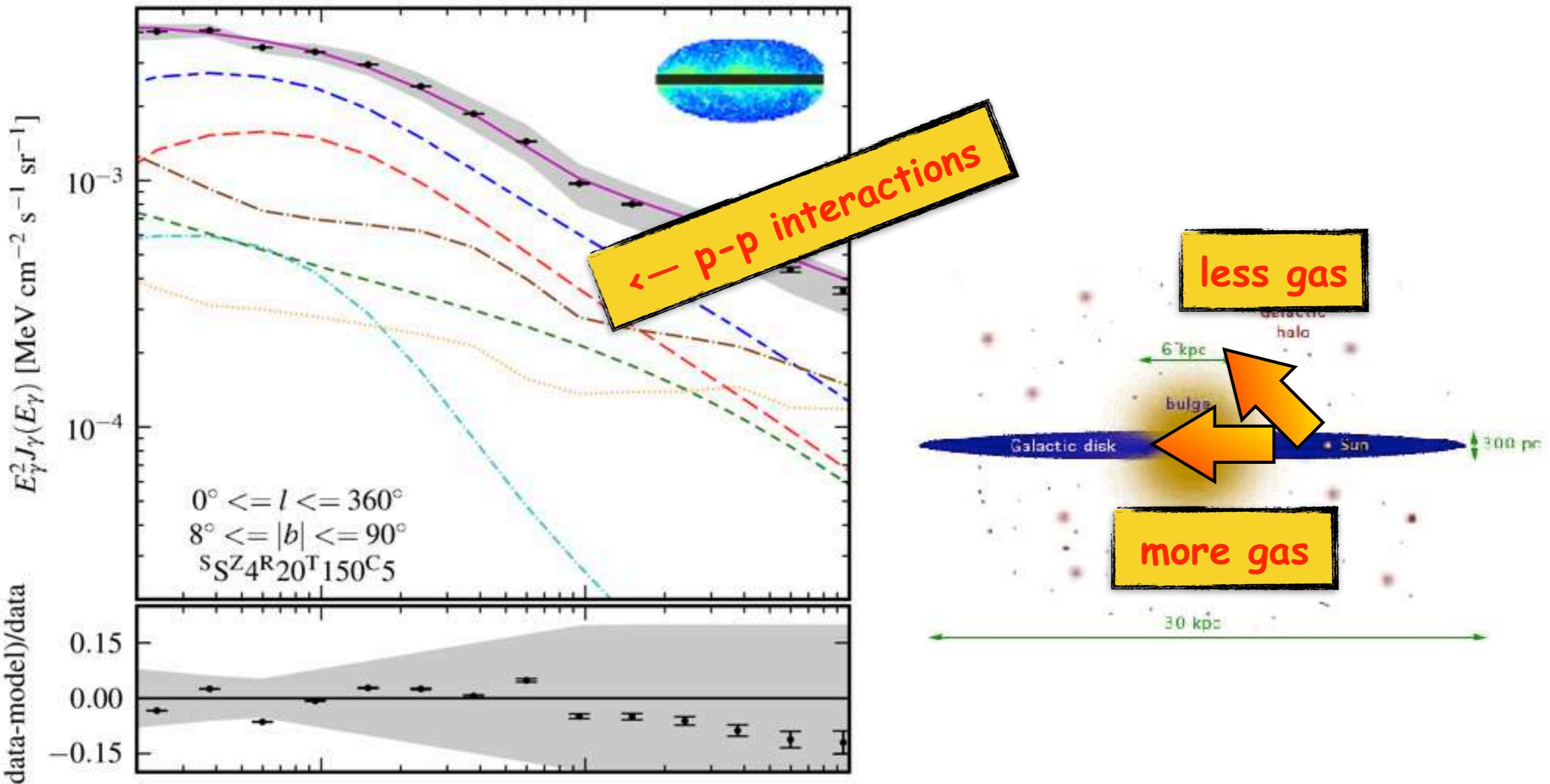
several ways to measure B exist, and they indicate $B \sim 3 \mu G$ in the Milky Way

$$\nu_s = \gamma^2 \frac{qB}{2\pi mc} \left\{ \begin{array}{l} E_e = 10 \text{ GeV} \rightarrow \nu_s \sim 3 \text{ GHz} \\ E_e = 100 \text{ TeV} \rightarrow \nu_s \sim 1 \text{ keV} \end{array} \right.$$

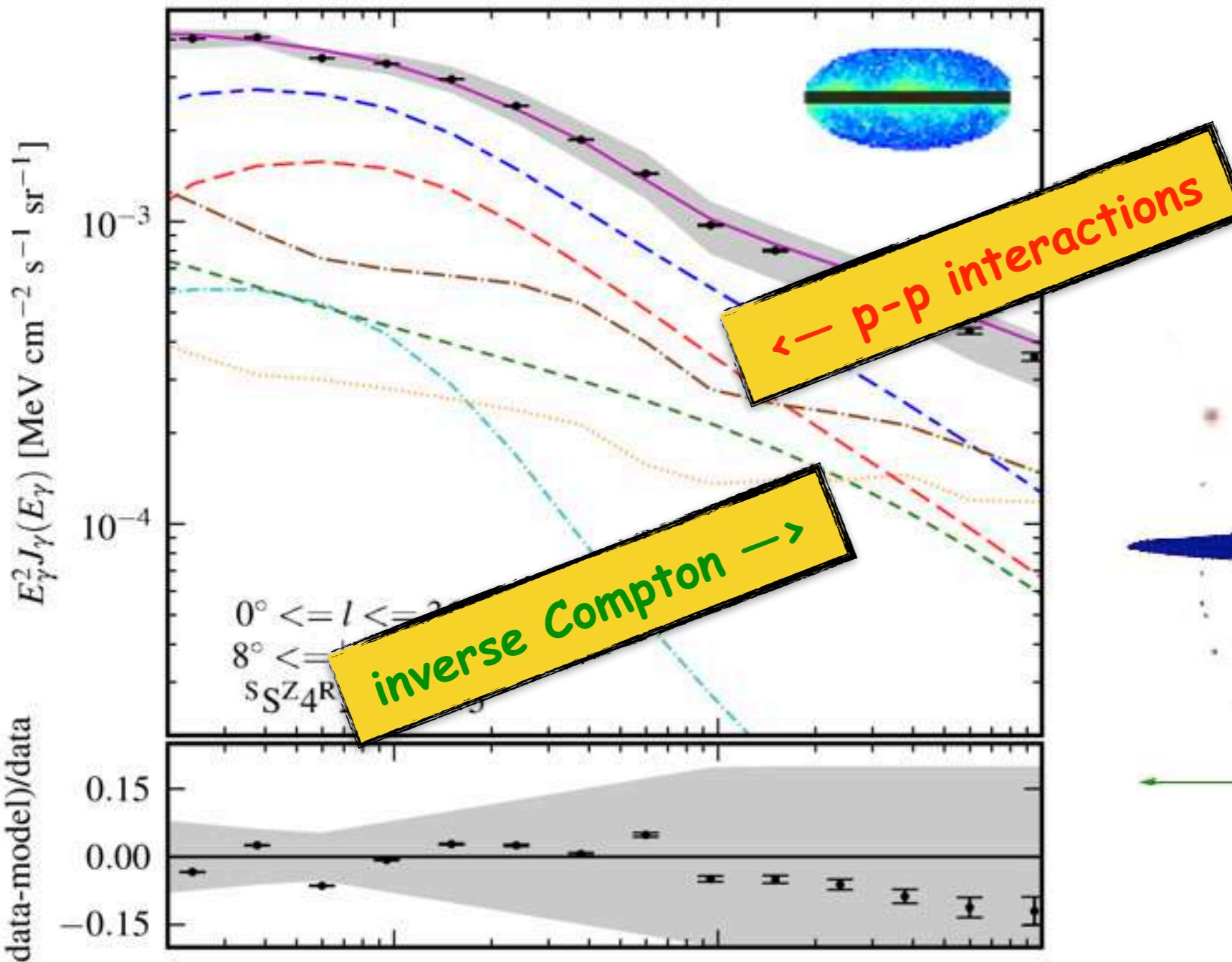
radio

X-rays

Diffuse emission, large Galactic latitudes

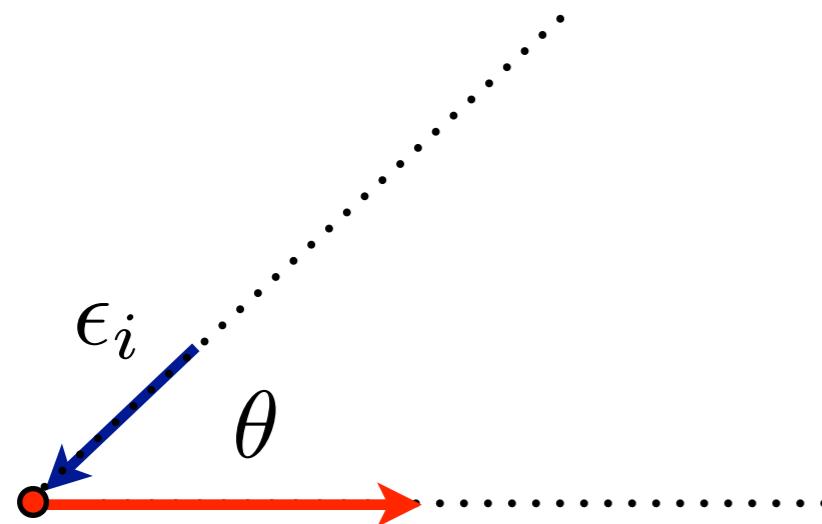


Diffuse emission, large Galactic latitudes



Leptonic Gamma-Rays: Inverse Compton

Relativistic **electrons** can interact with soft background photons
(Cosmic Microwave Background, IR and Optical galactic background...)



Leptonic Gamma-Rays: Inverse Compton

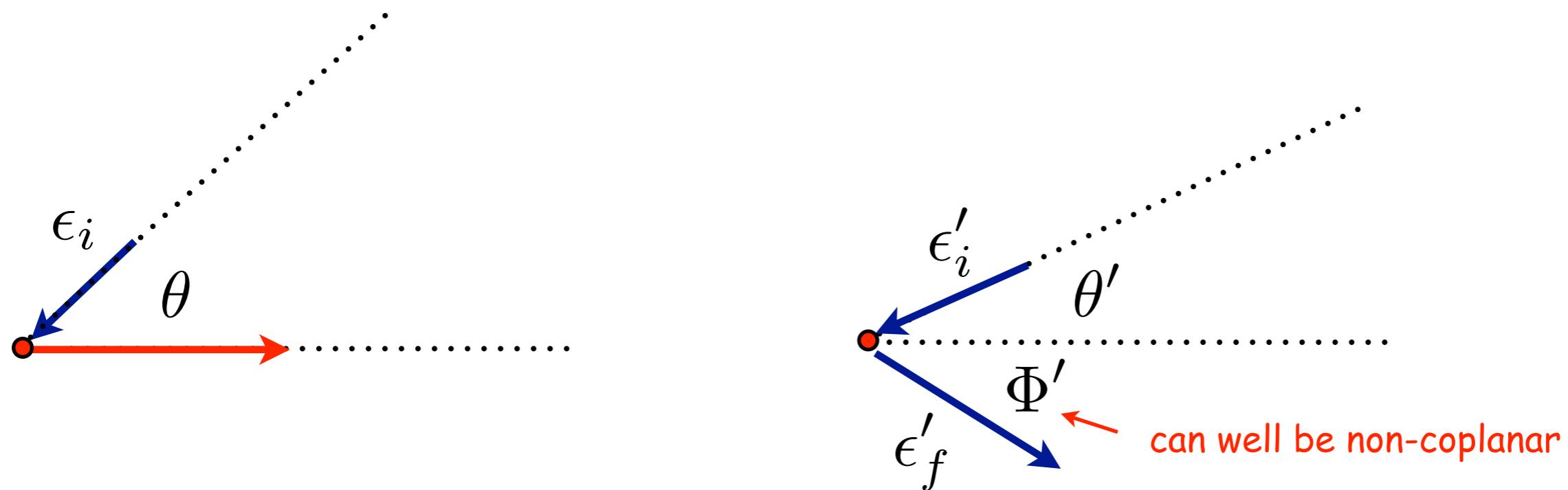
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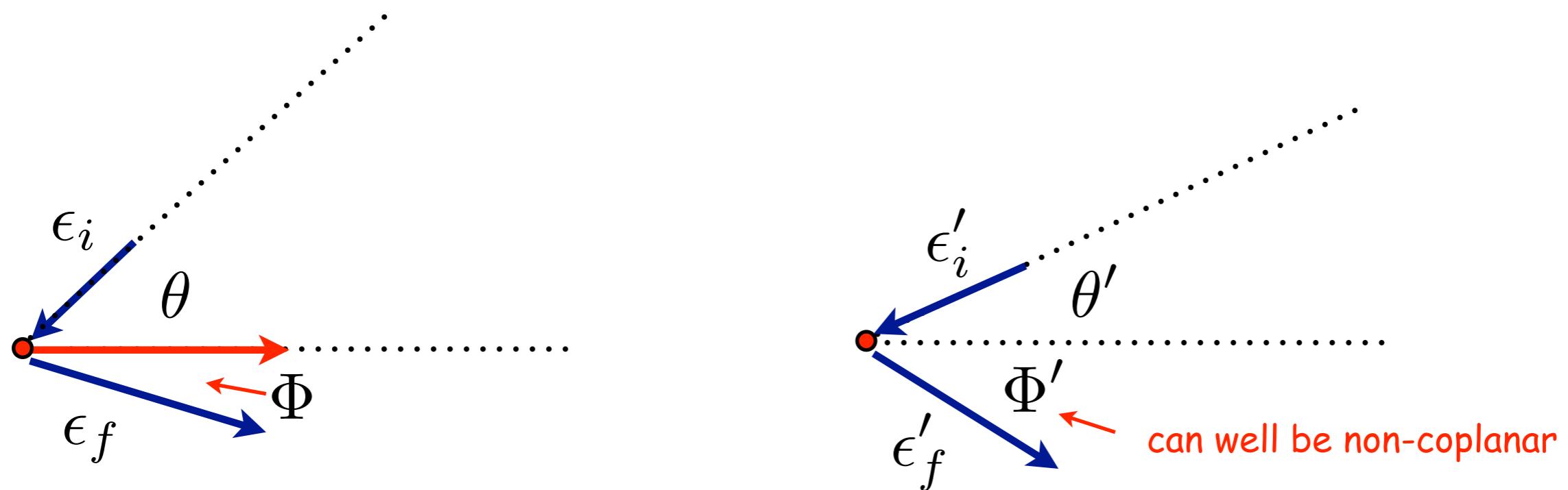


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In the lab rest frame the (final) photon energy is: $\epsilon_f = \epsilon'_f \gamma (1 + \beta \cos \Phi)$

Leptonic Gamma-Rays: Inverse Compton

$$\epsilon_f = \gamma^2 \epsilon_i G(\theta, \Phi)$$

After averaging over angles (tedious...):

$$\epsilon_f = \frac{4}{3} \gamma^2 \epsilon_i$$

Leptonic Gamma-Rays: Inverse Compton

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Example:

Cosmic Microwave Background $\rightarrow T \sim 3 \text{ K}$ $kT \approx 3 \times 10^{-4} \text{ eV}$

- $E_e = 1 \text{ GeV} \rightarrow \epsilon_\gamma = 1,5 \text{ keV}$ X-rays
- $E_e = 1 \text{ TeV} \rightarrow \epsilon_\gamma = 1,5 \text{ GeV}$ gamma rays (FERMI)
- $E_e = 25 \text{ TeV} \rightarrow \epsilon_\gamma = 1 \text{ TeV}$ gamma rays (Cherenkov Telescopes)

Leptonic Gamma-Rays: Inverse Compton

is there a maximum energy for
the up-scattered photons?

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energy conservation...

above a given energy Inverse Compton scattering becomes ineffective

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if $\gamma \epsilon_i \sim mc^2$ we must use the quantum relativistic (Klein-Nishina) cross section

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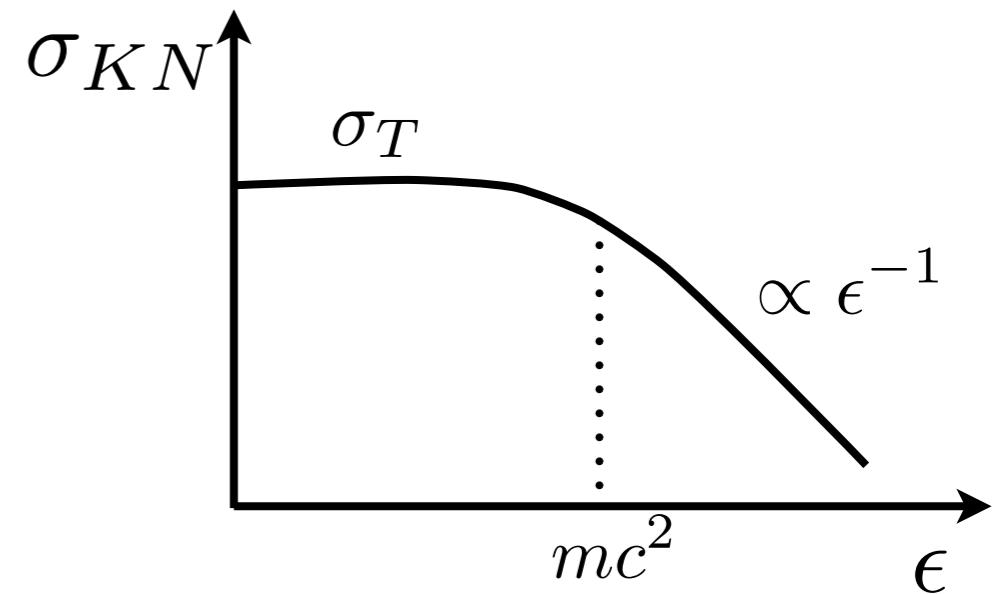
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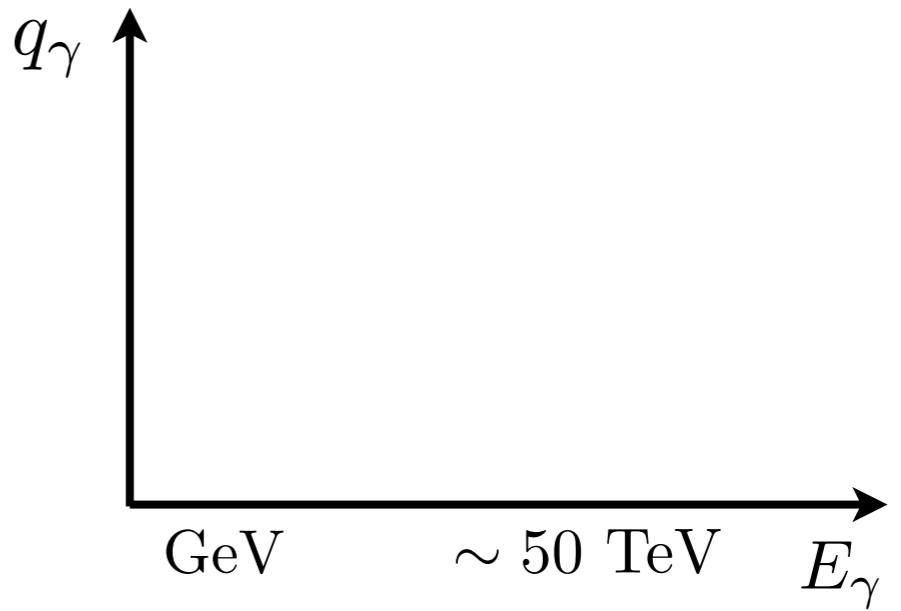
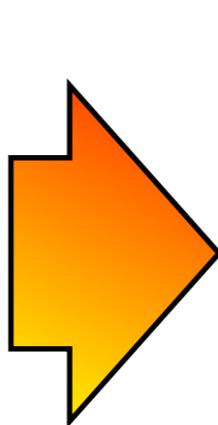
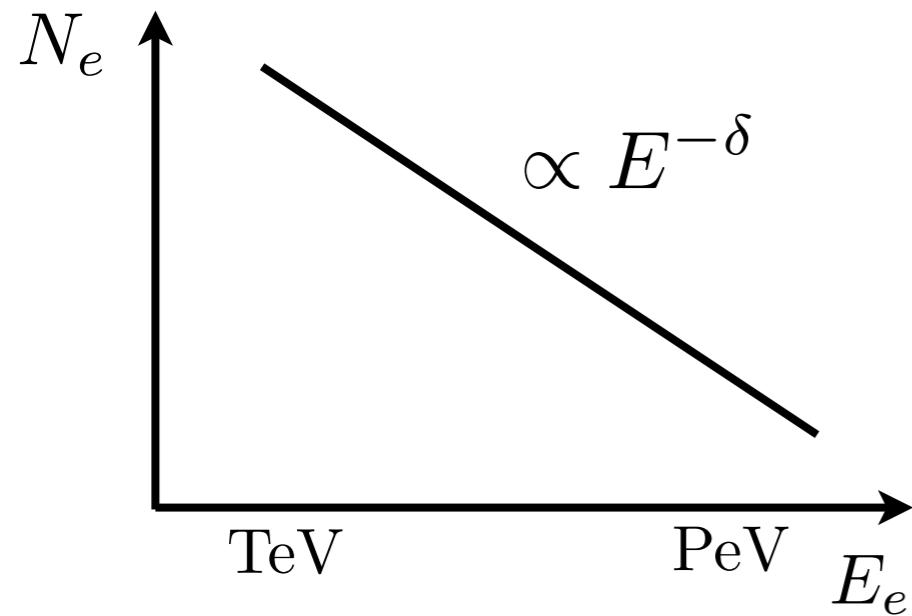
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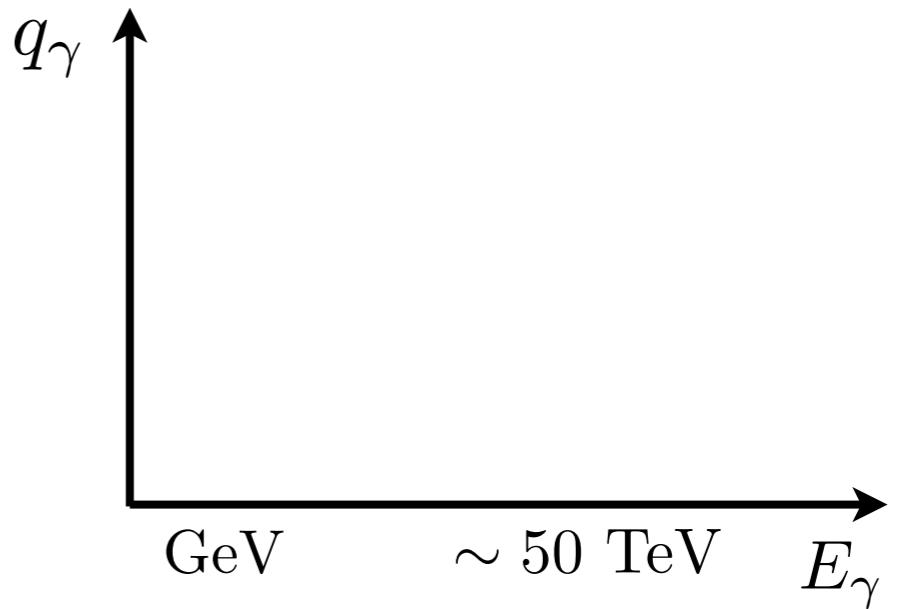
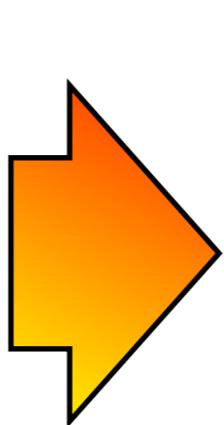
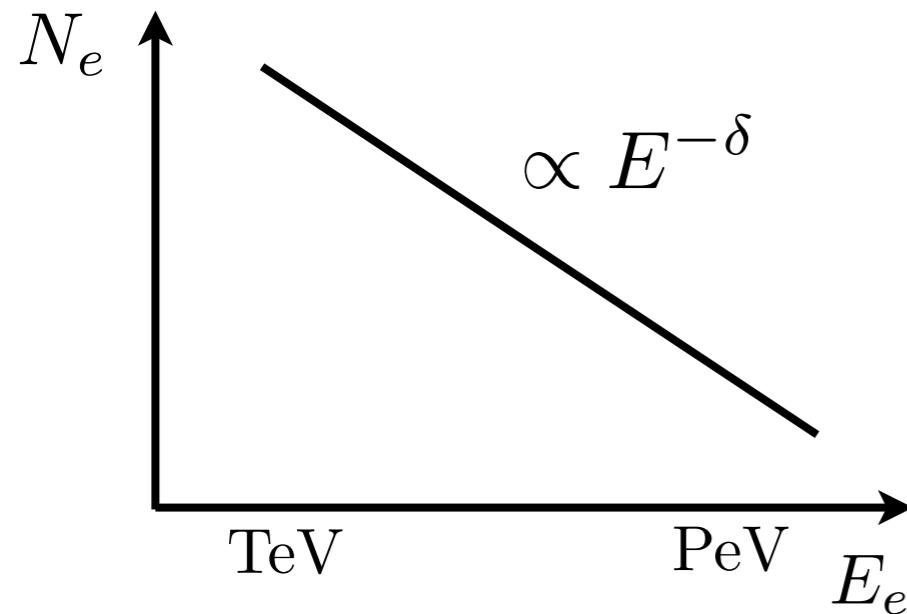
Photon spectrum:



$$q_\gamma(E_\gamma) = \int dE_e N_e(E_e) \delta(E_\gamma - \frac{4}{3}\gamma^2 \epsilon_{CMB})(n_{CMB} \sigma_T c)$$

Leptonic Gamma-Rays: Inverse Compton

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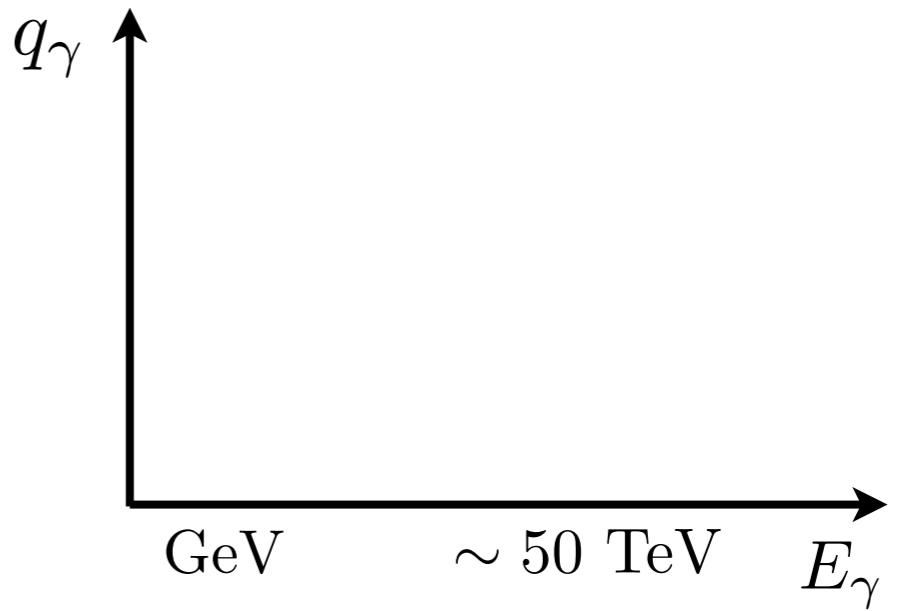
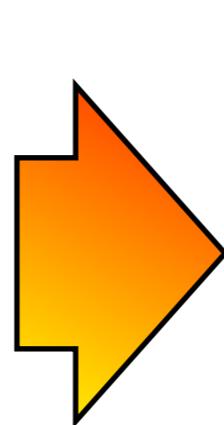
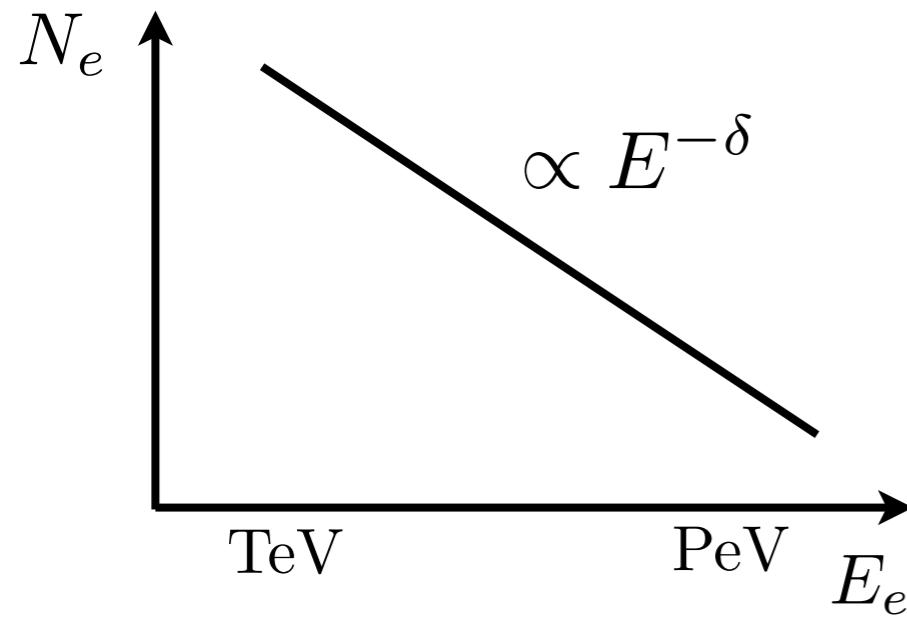
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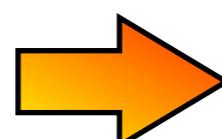
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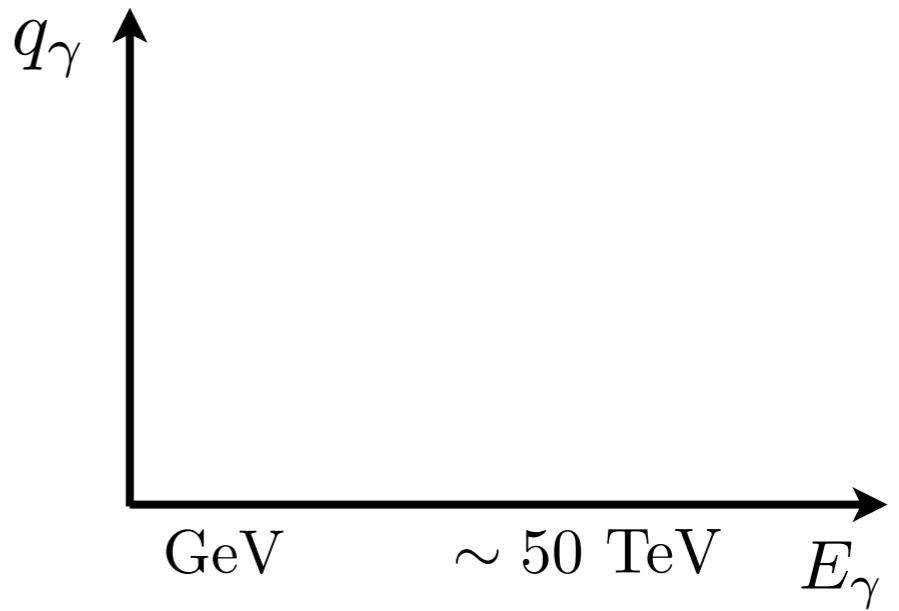
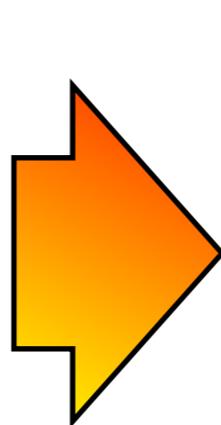
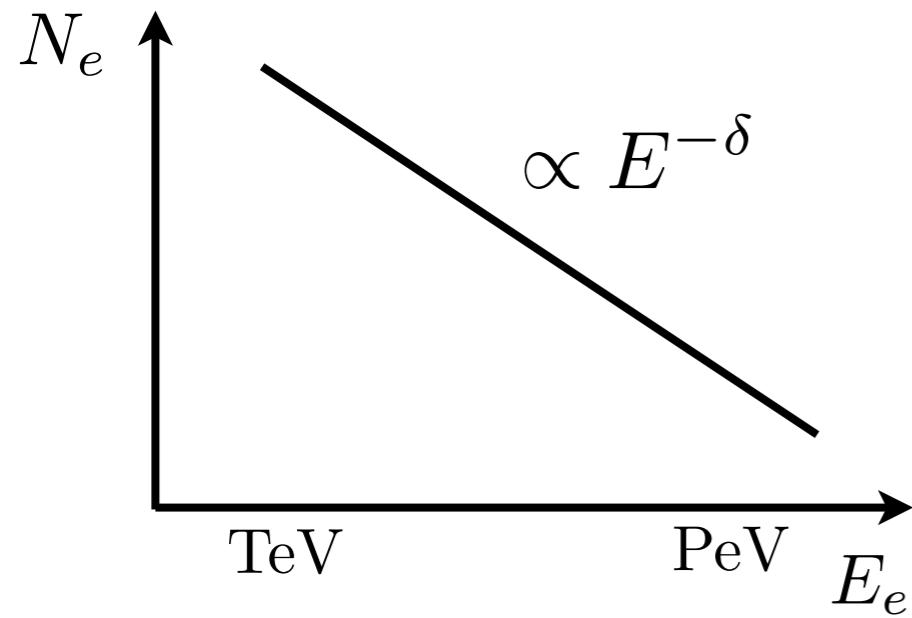
$$\delta \left(E_\gamma - \frac{4}{3} \left(\frac{E_e}{mc^2} \right)^2 \epsilon_{CMB} \right)$$

Annotations below the equation:

- A red bracket above the term $\left(\frac{E_e}{mc^2} \right)^2$ is labeled $g(x)$.
- A red arrow points from the term $x_0 \propto E_\gamma^{1/2}$ to the term $\left(\frac{E_e}{mc^2} \right)^2$.
- A red arrow points from the term $g' \propto E_e$ to the term ϵ_{CMB} .

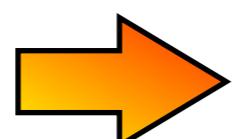
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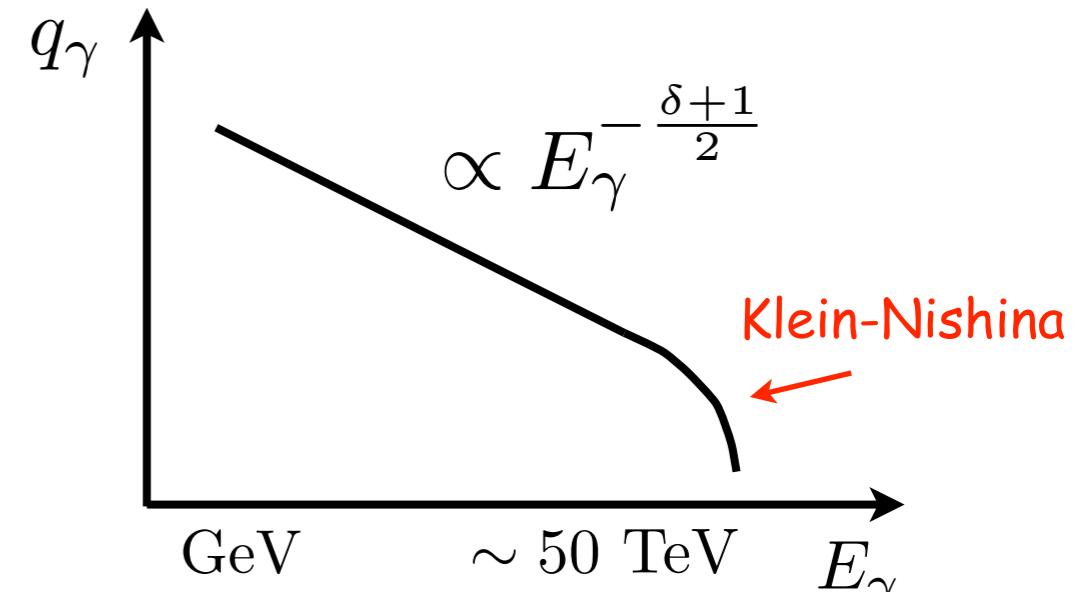
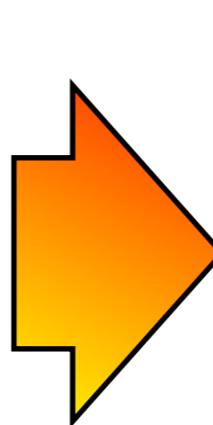
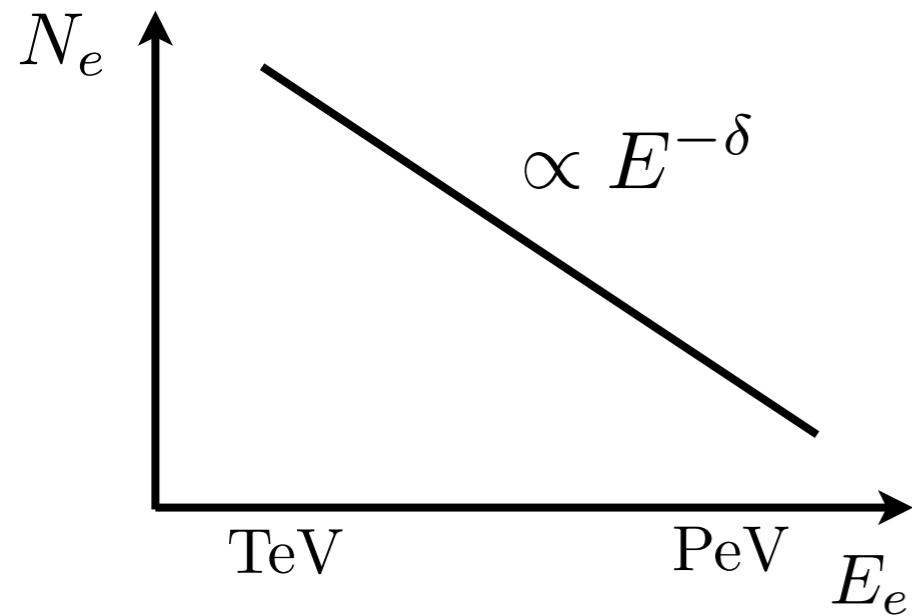
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Annotations below the equation:

- $x_0 \propto E_\gamma^{1/2}$ (red arrow pointing to $a E_\gamma^{1/2}$)
- $g' \propto E_e$ (red arrow pointing to $\frac{2}{a} E_\gamma^{1/2}$)

Leptonic Gamma-Rays: Inverse Compton

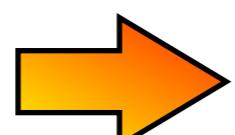
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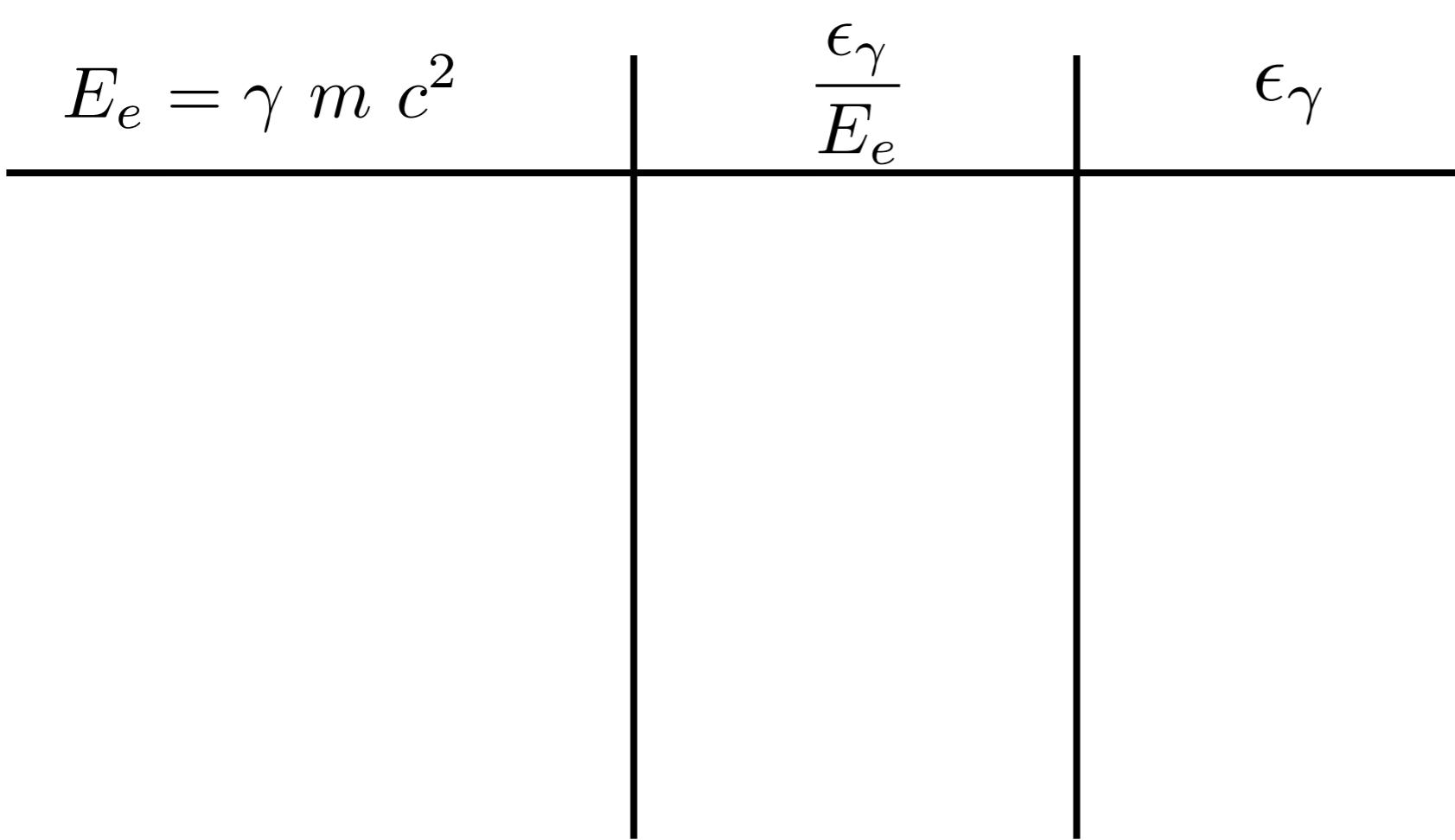
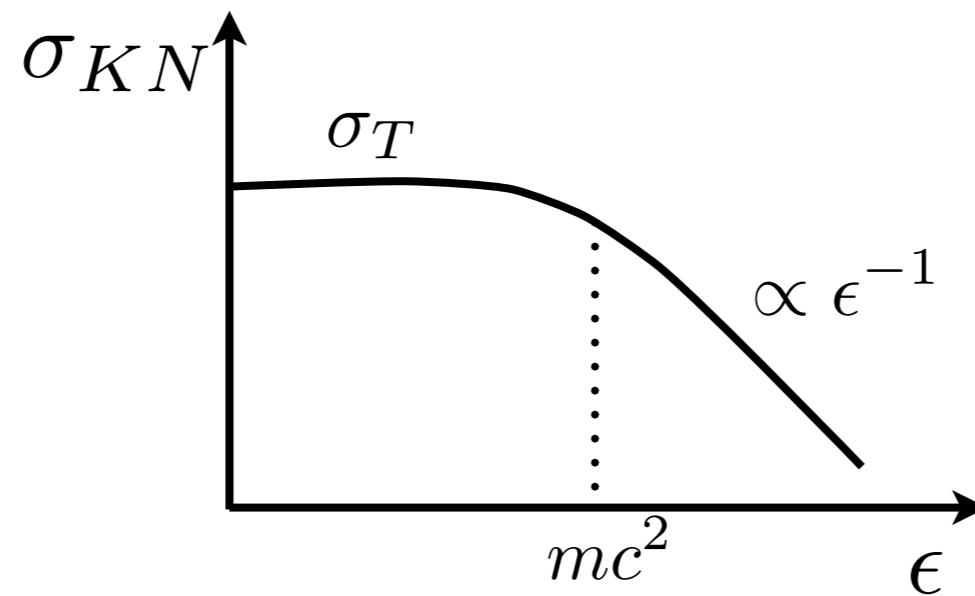


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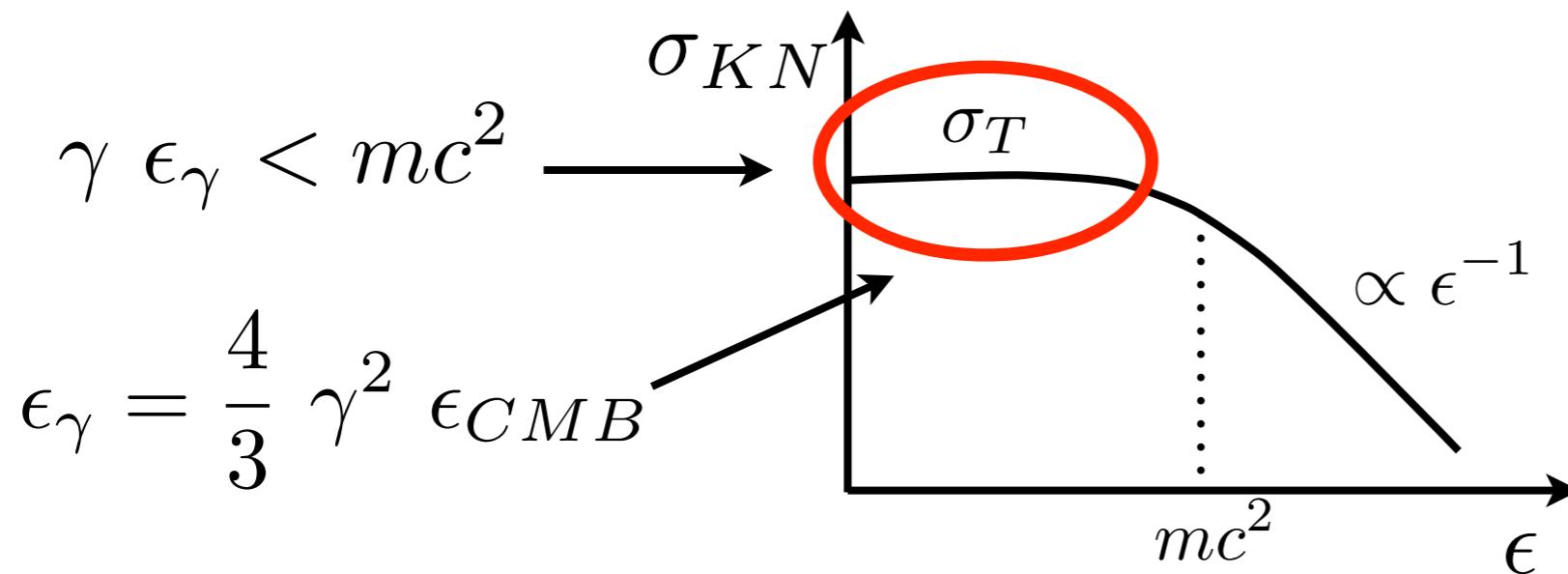
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Leptonic Gamma-Rays: Inverse Compton



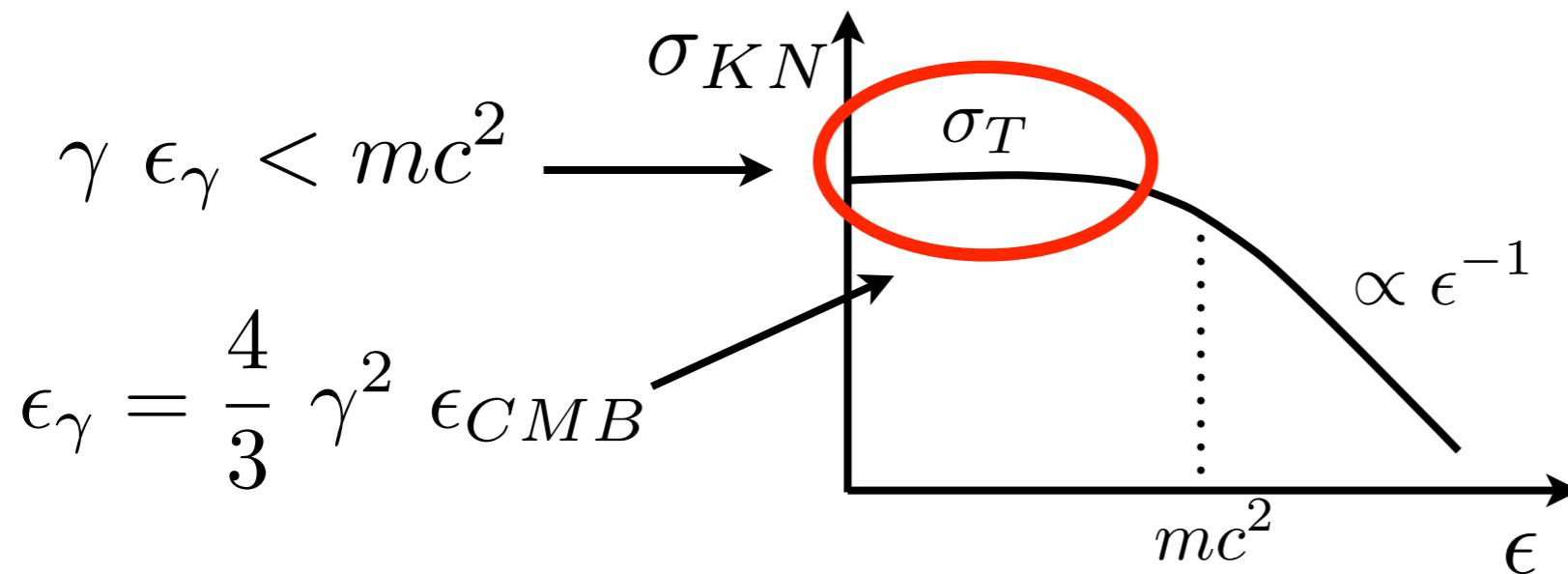
Leptonic Gamma-Rays: Inverse Compton



$E_e = \gamma m c^2$	$\frac{\epsilon_\gamma}{E_e}$	ϵ_γ
1 TeV	~0.2%	~1.5 GeV
25 TeV	~4%	~1 TeV
100 TeV	~15%	~15 TeV

Thomson

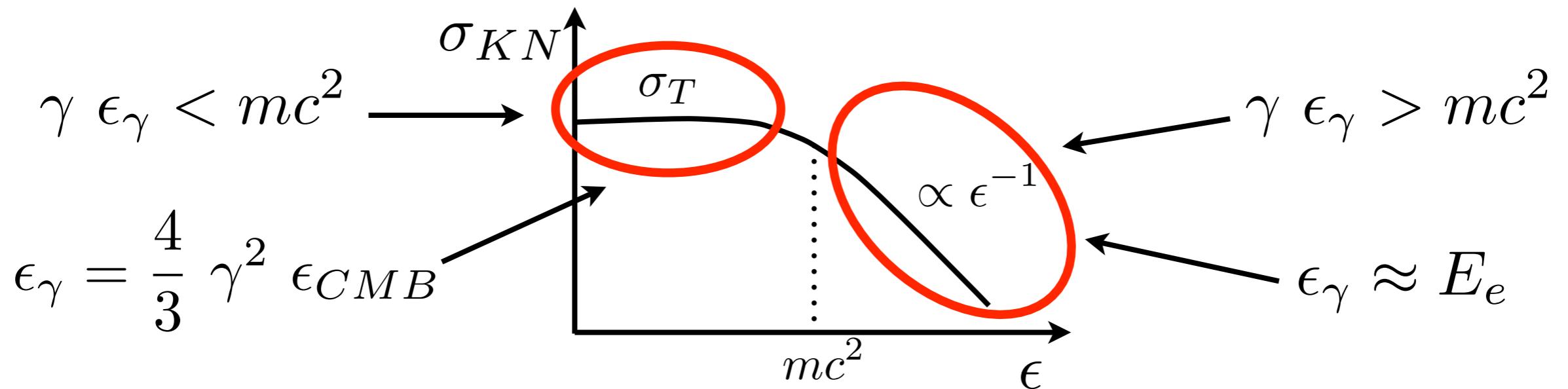
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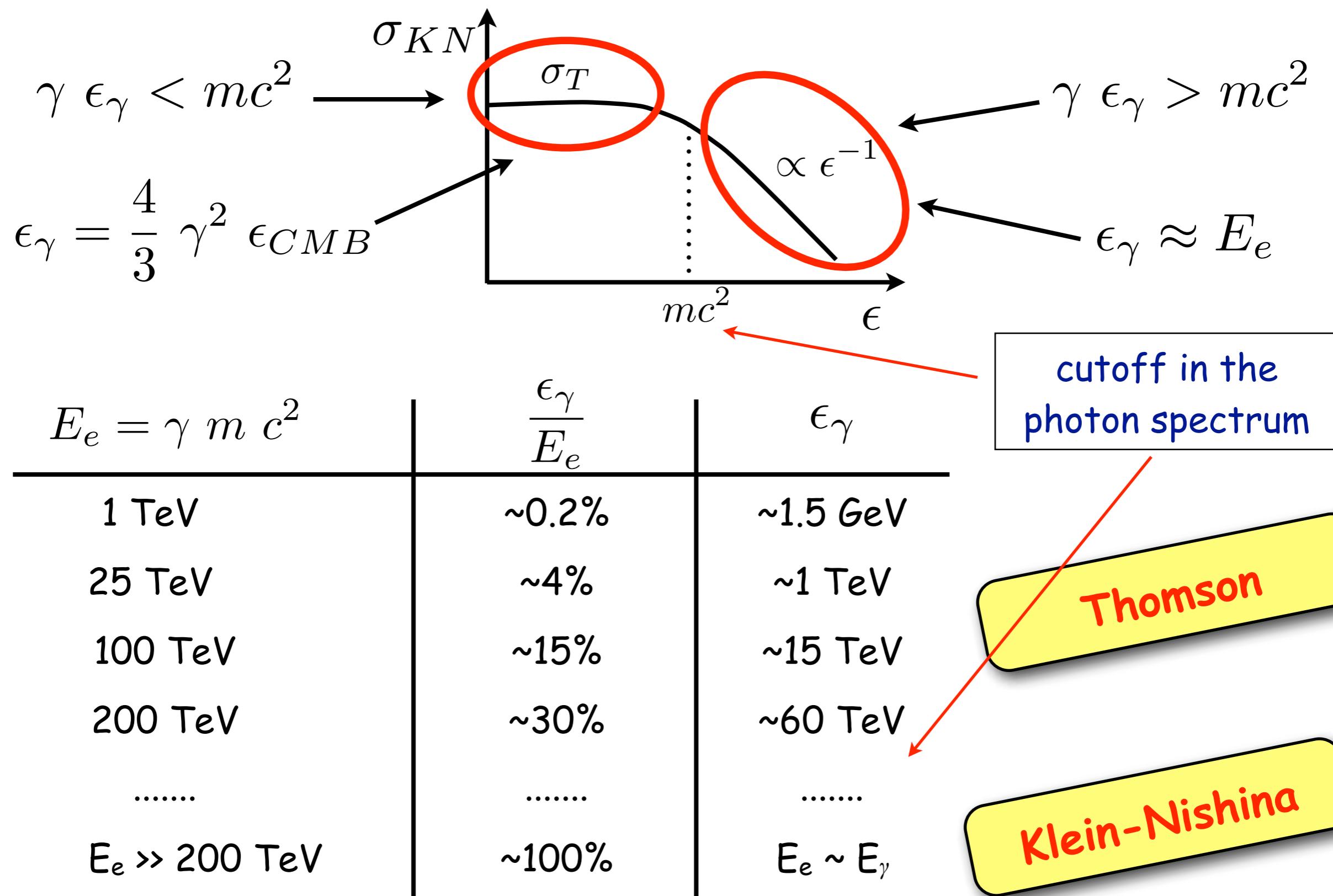


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$E_e \gg 200$ TeV	~100%	$E_e \sim E_\gamma$

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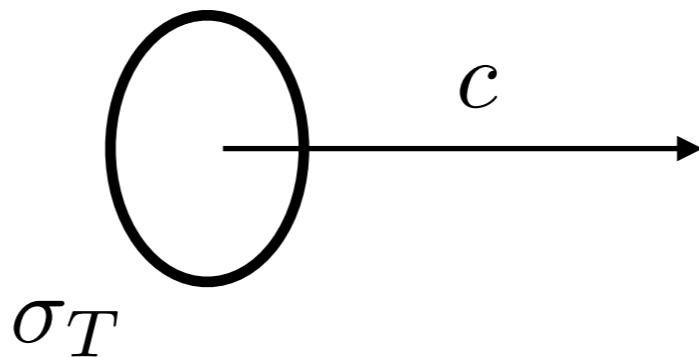
Klein-Nishina

Leptonic Gamma-Rays: Inverse Compton



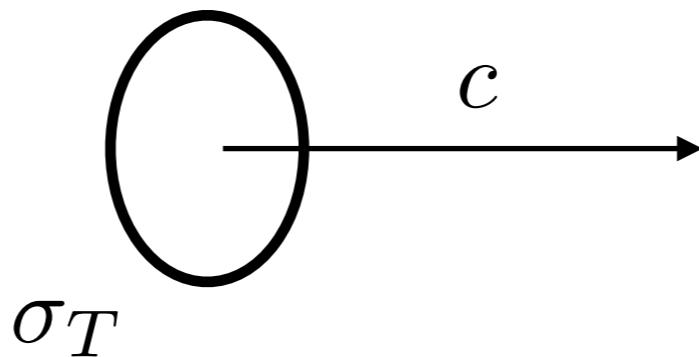
Inverse Compton: energy loss rate

Scattering rate



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Scattering rate

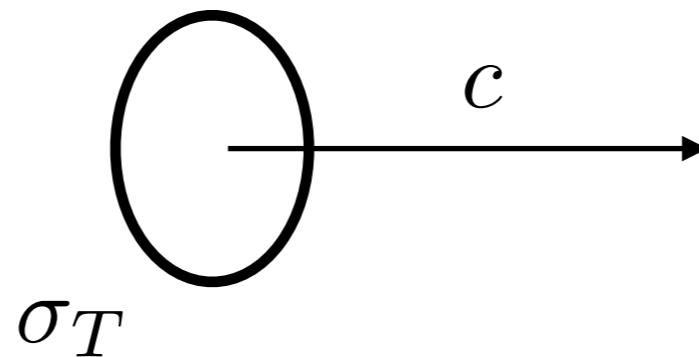


σ_T

Volume swept per second $\sigma_T c$

Inverse Compton: energy loss rate

Scattering rate



σ_T

Volume swept per second $\sigma_T c$

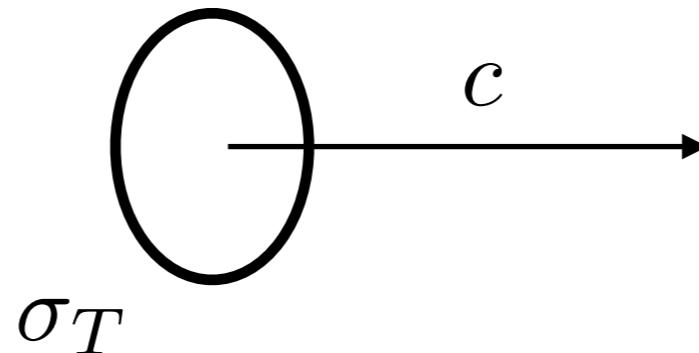
Interaction rate

$$\sigma_T c n_{CMB}$$

$$n_{CMB} = \frac{\omega_{CMB}}{\langle \epsilon \rangle}$$

Inverse Compton: energy loss rate

Scattering rate



σ_T

Volume swept per second $\sigma_T c$

Interaction rate

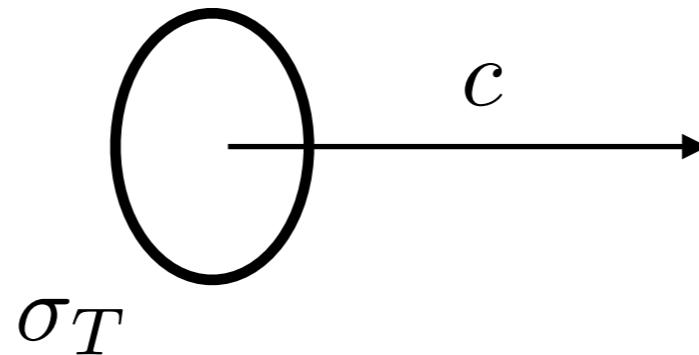
$$\sigma_T c n_{CMB}$$

$$n_{CMB} = \frac{\omega_{CMB}}{\langle \epsilon \rangle}$$

Radiated power $P_{IC} = \left(\sigma_T c \frac{\omega_{CMB}}{\langle \epsilon \rangle} \right) \left(\frac{4}{3} \gamma^2 \langle \epsilon \rangle \right) = \frac{4}{3} \sigma_T c \gamma^2 \omega_{CMB}$

Inverse Compton: energy loss rate

Scattering rate



$$\sigma_T$$

Volume swept per second $\sigma_T c$

Interaction rate

$$\sigma_T c n_{CMB}$$

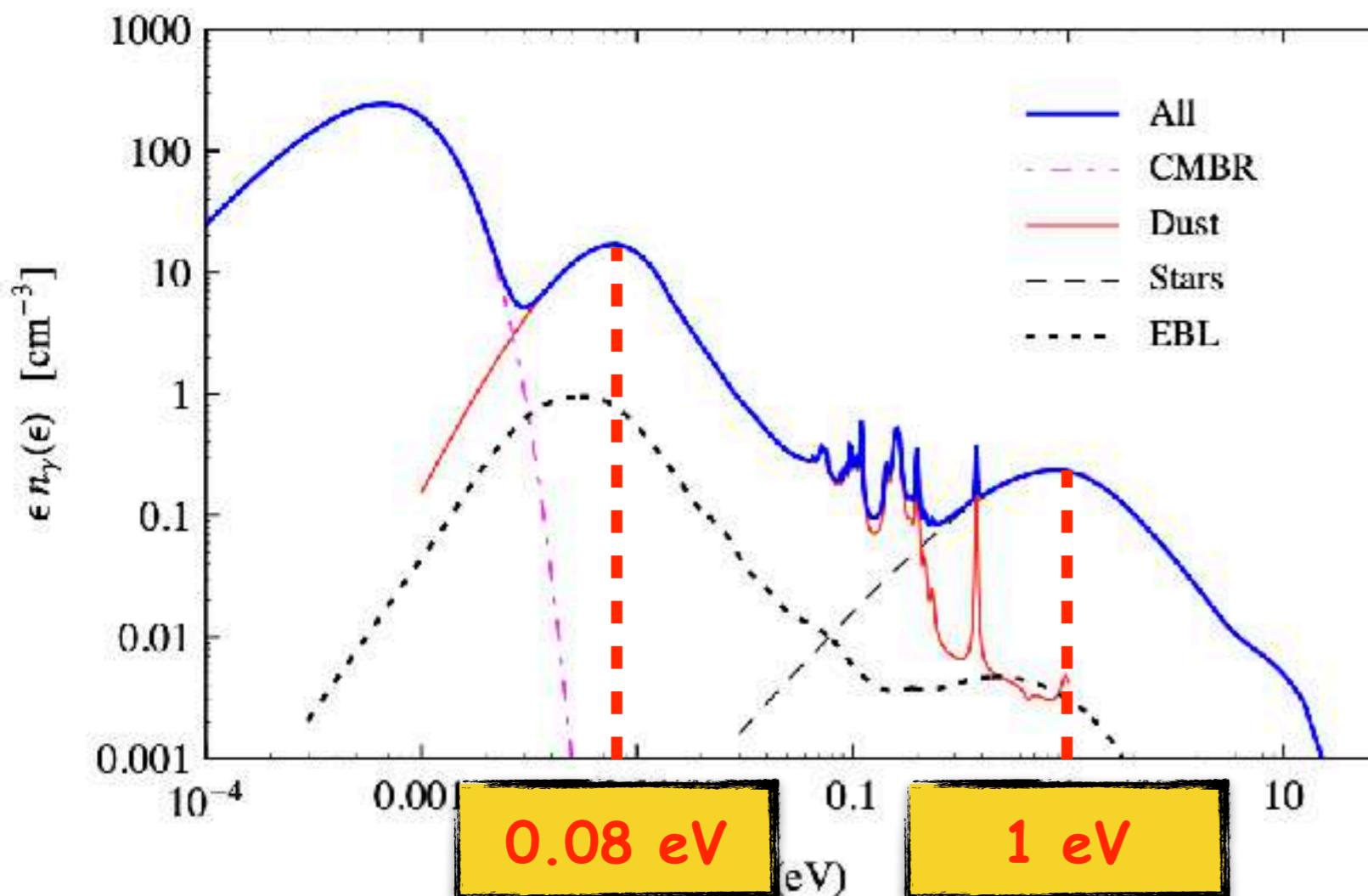
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Same expression as synchrotron!

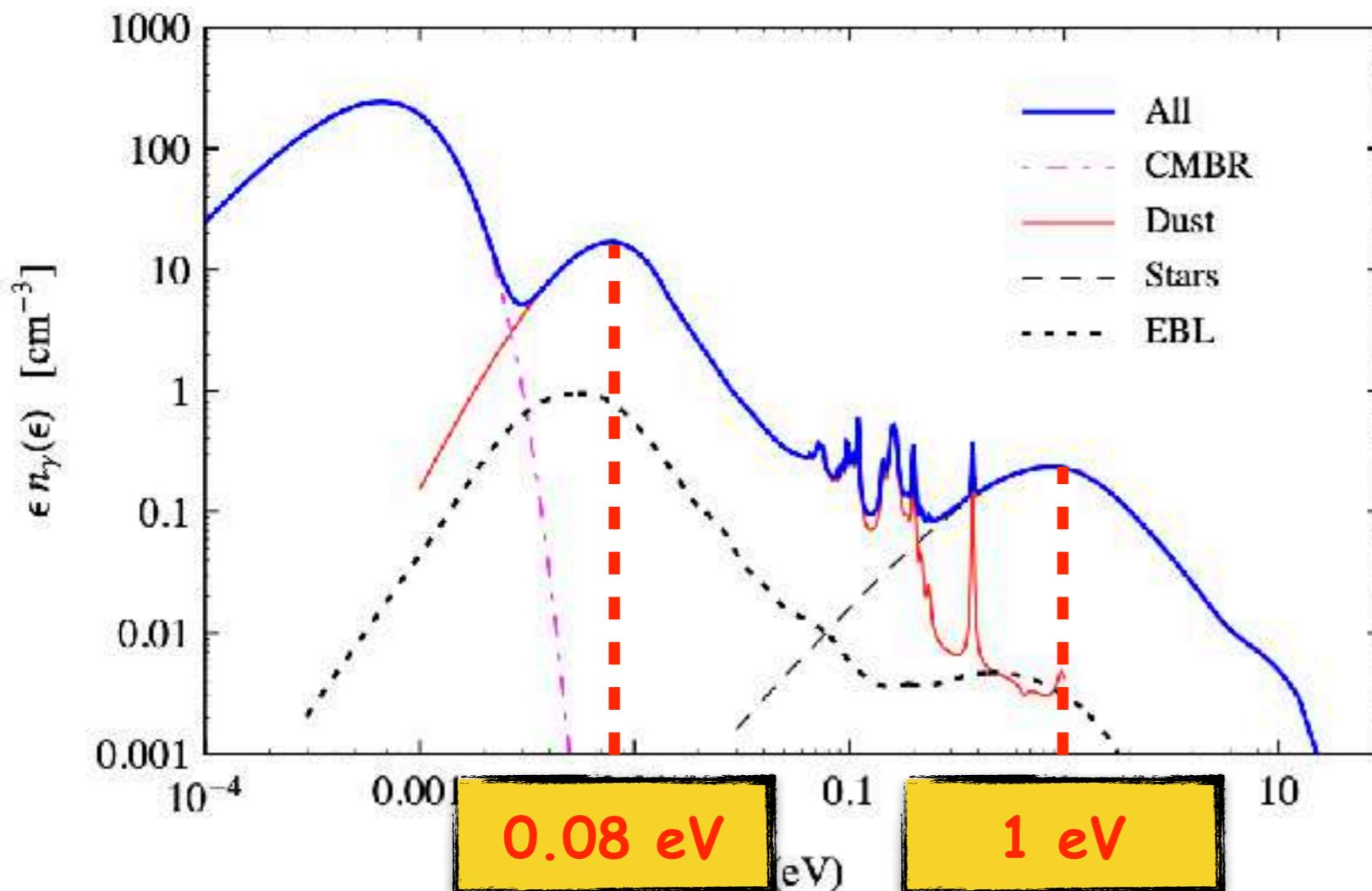
Note on the K-N regime

Lipari & Vernetto 2018



Note on the K-N regime

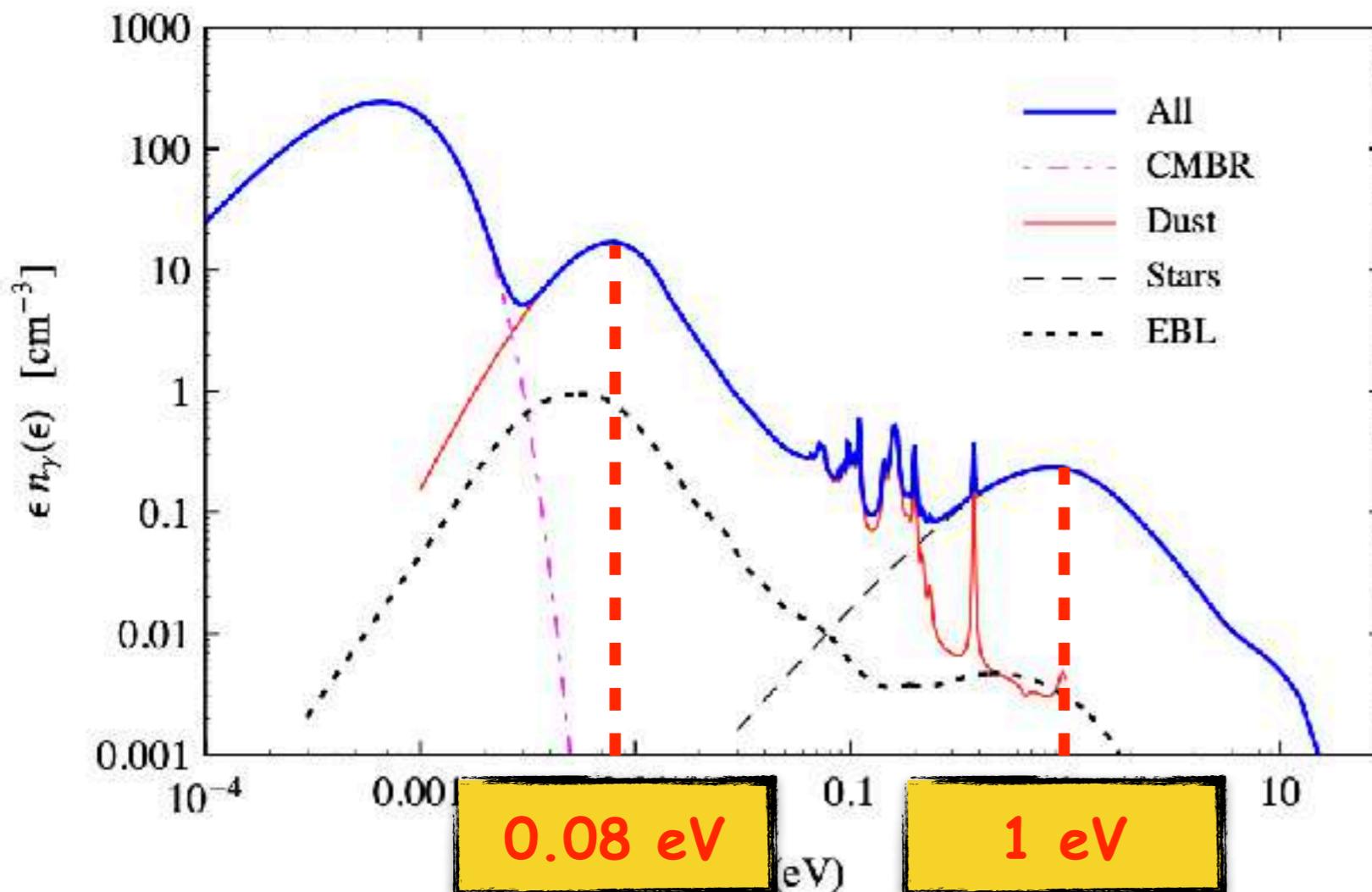
Lipari & Vernetto 2018



$$\gamma \epsilon_\gamma > mc^2$$

Note on the K-N regime

Lipari & Vernetto 2018



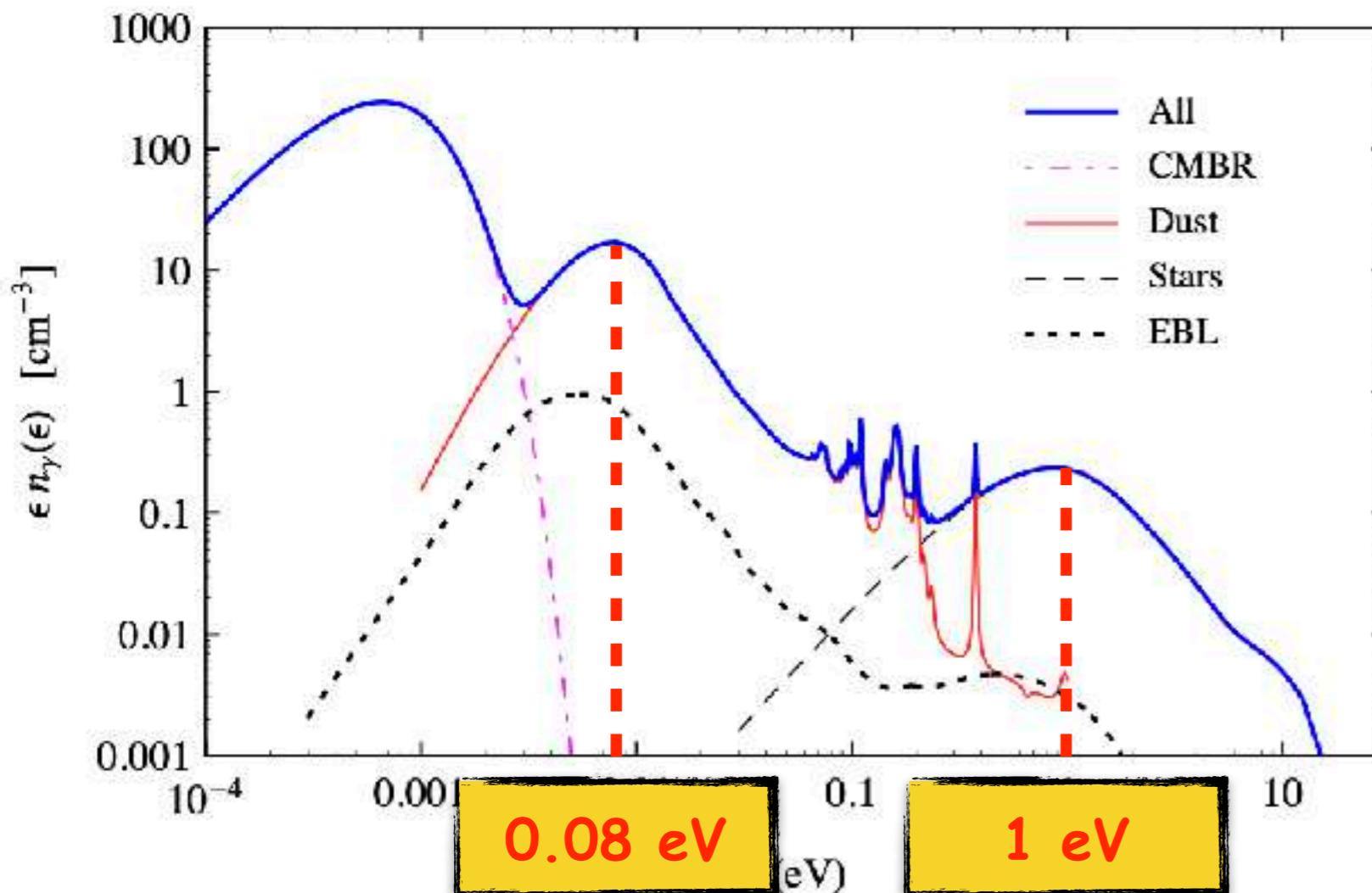
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$$E_e > \frac{(mc^2)^2}{\epsilon_\gamma}$$

Note on the K-N regime

Lipari & Vernetto 2018



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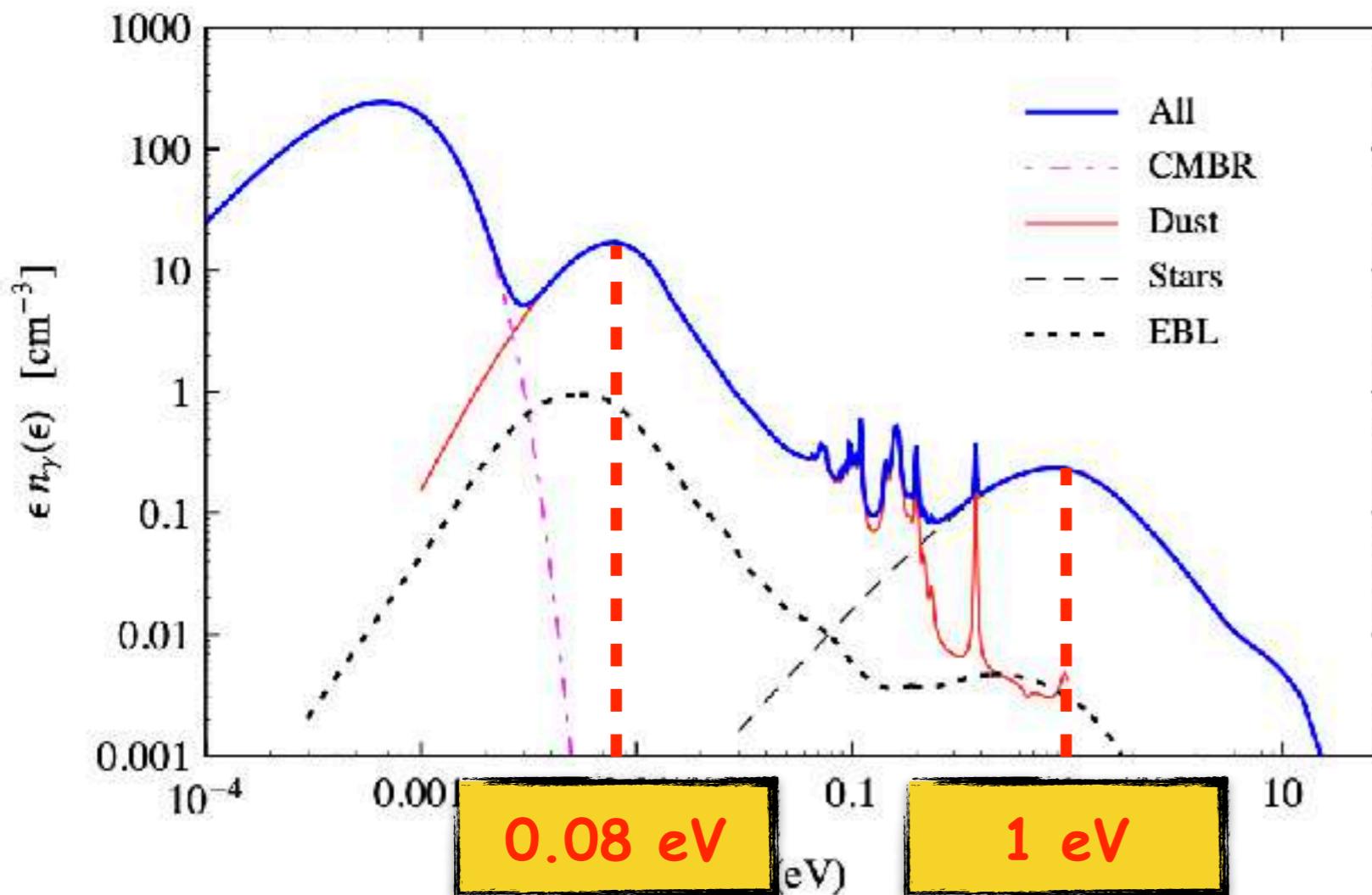
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$$E_\gamma = \frac{4}{3} \gamma^2 \epsilon_\gamma \quad \rightarrow \quad E_\gamma > \frac{4}{3} \frac{(mc^2)^2}{\epsilon_\gamma} \sim 0.3 \left(\frac{\epsilon_\gamma}{\text{eV}} \right) \text{TeV}$$

Note on the K-N regime

Lipari & Vernetto 2018



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\downarrow

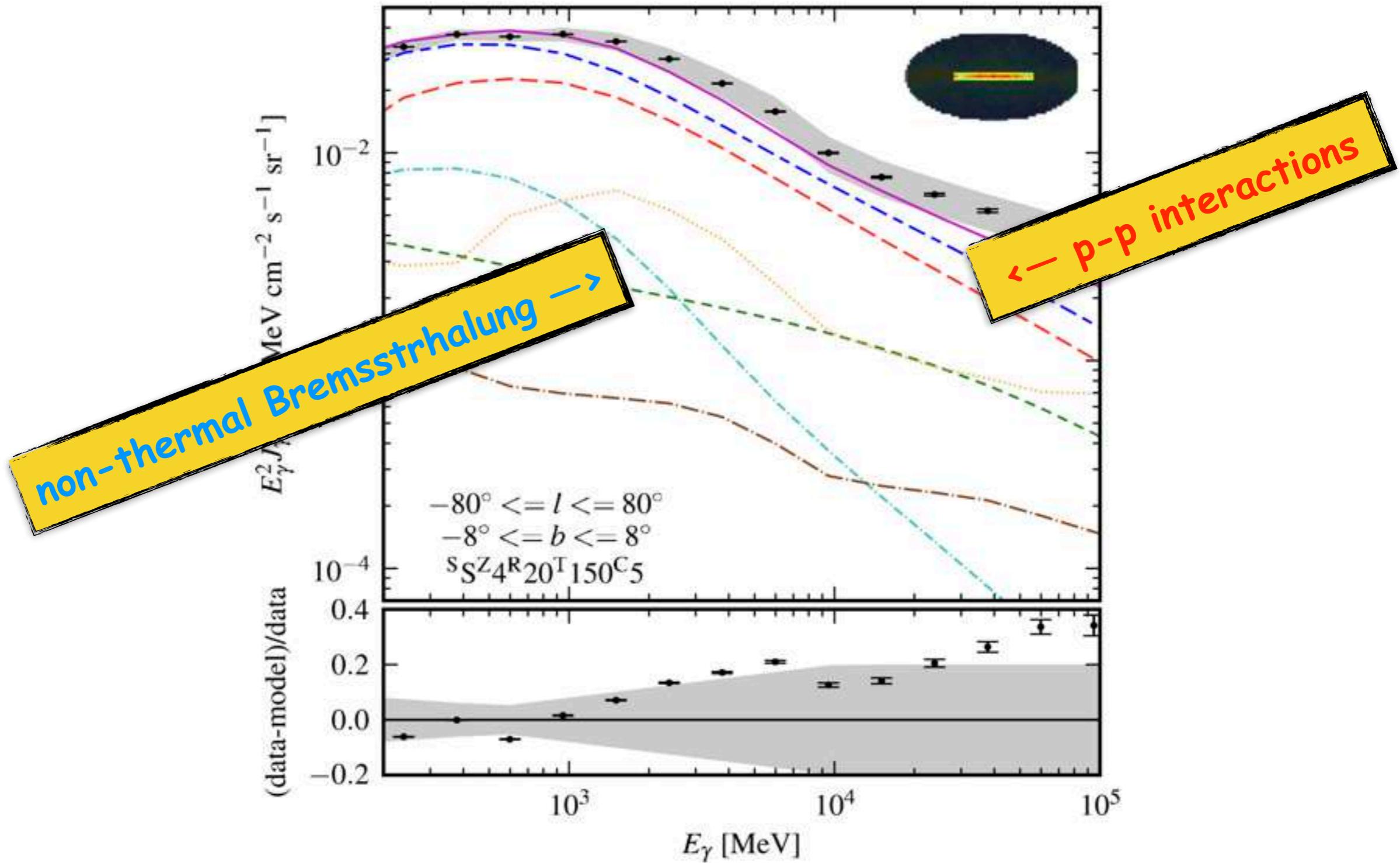
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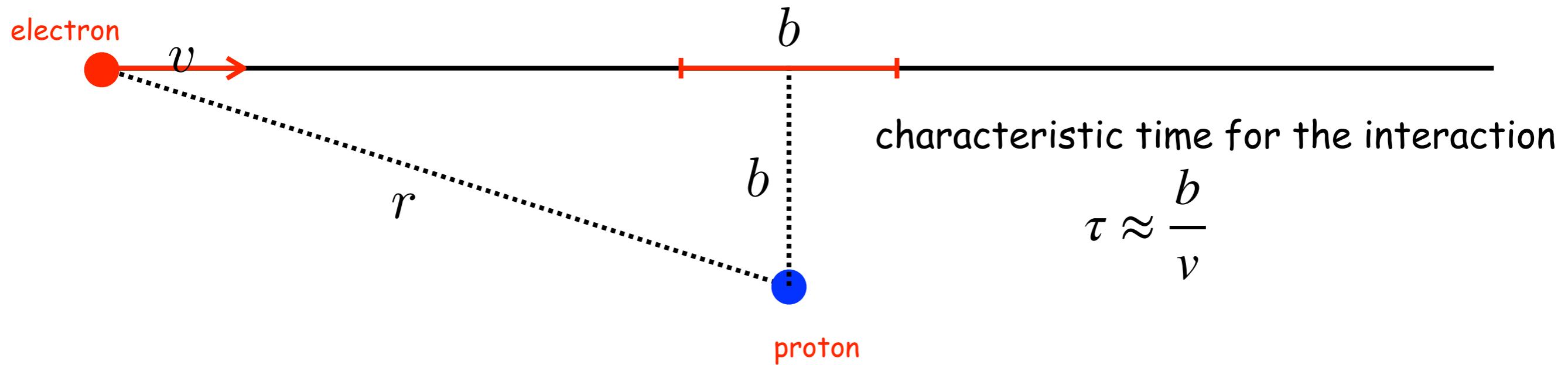
in first approximation: if you are a TeV astronomer just CMB matters for ICS

Non-thermal bremsstrahlung



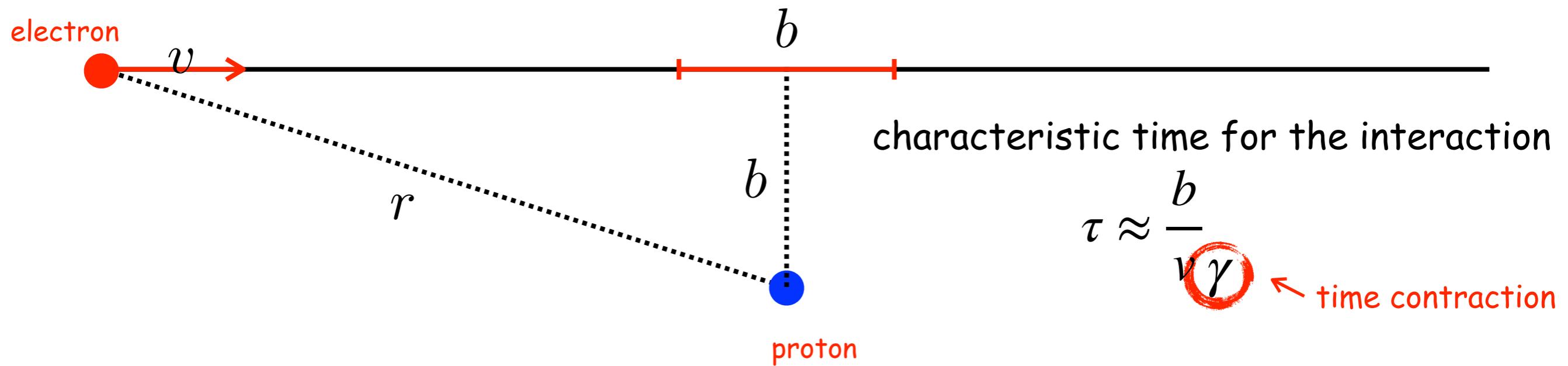
Non-thermal Bremsstrahlung

remember what we did for the thermal Bremsstrahlung



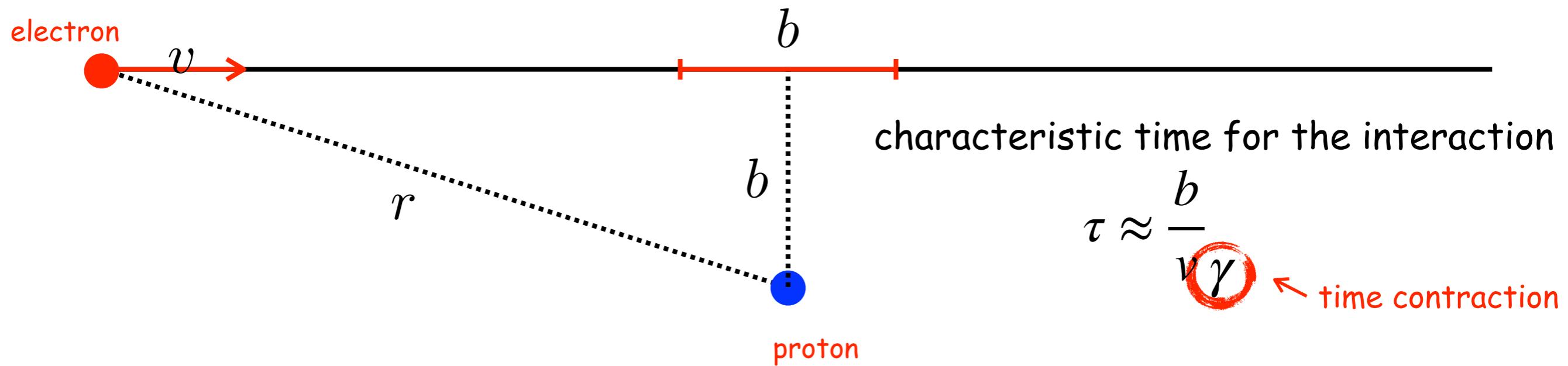
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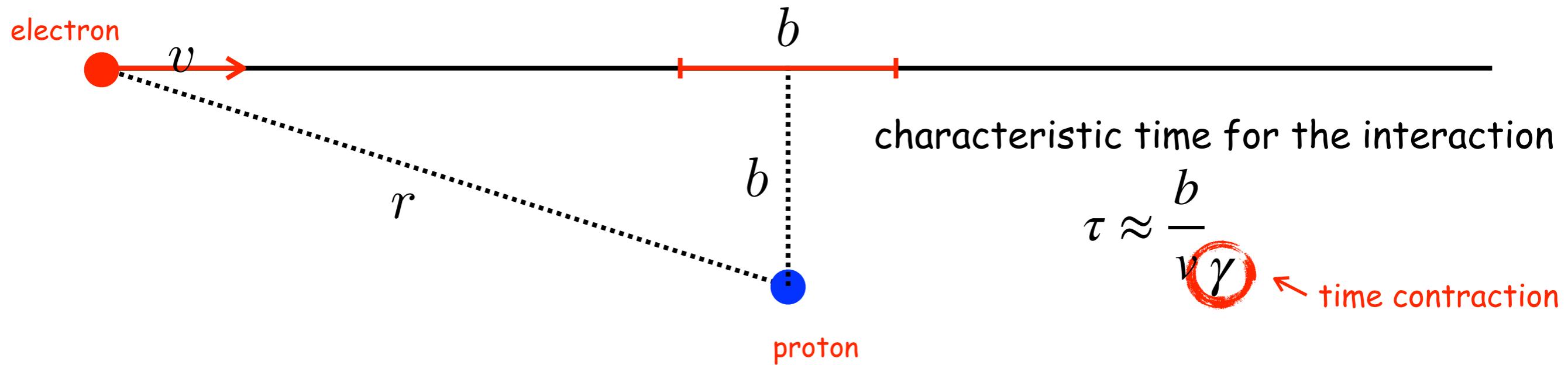
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→ in its rest frame, the e^- sees the proton's E field contracted into a very fast pulse

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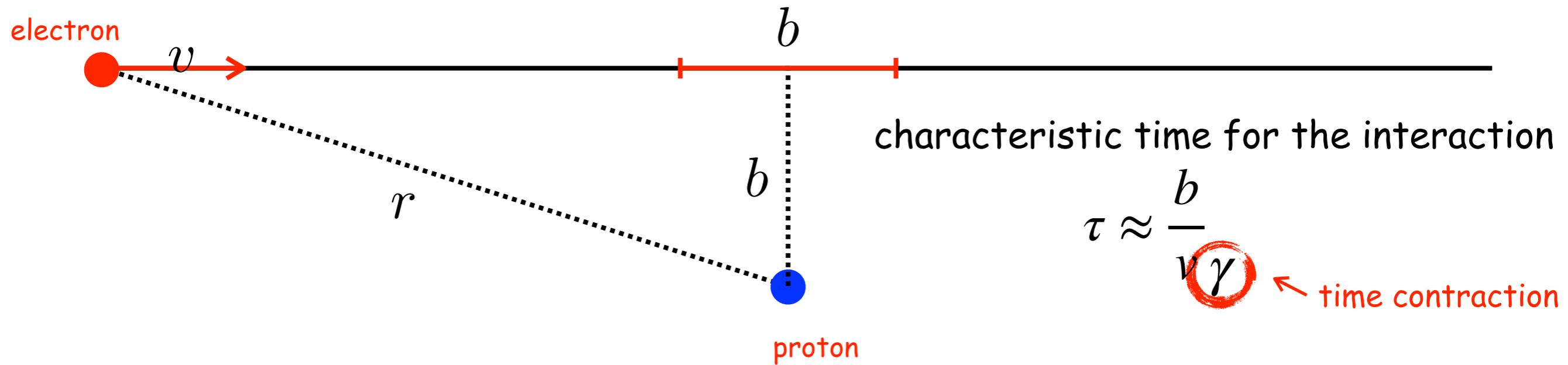
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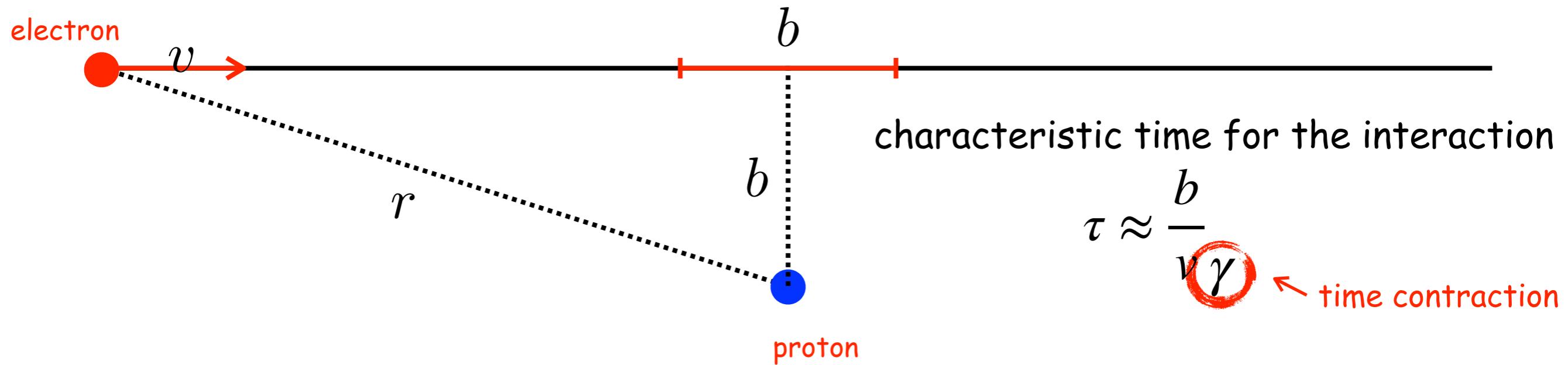
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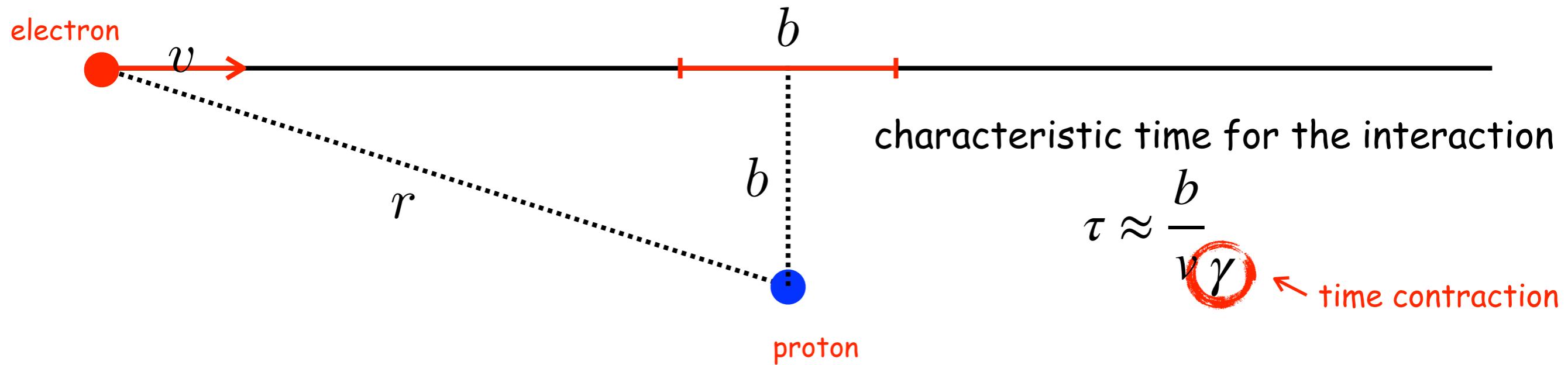
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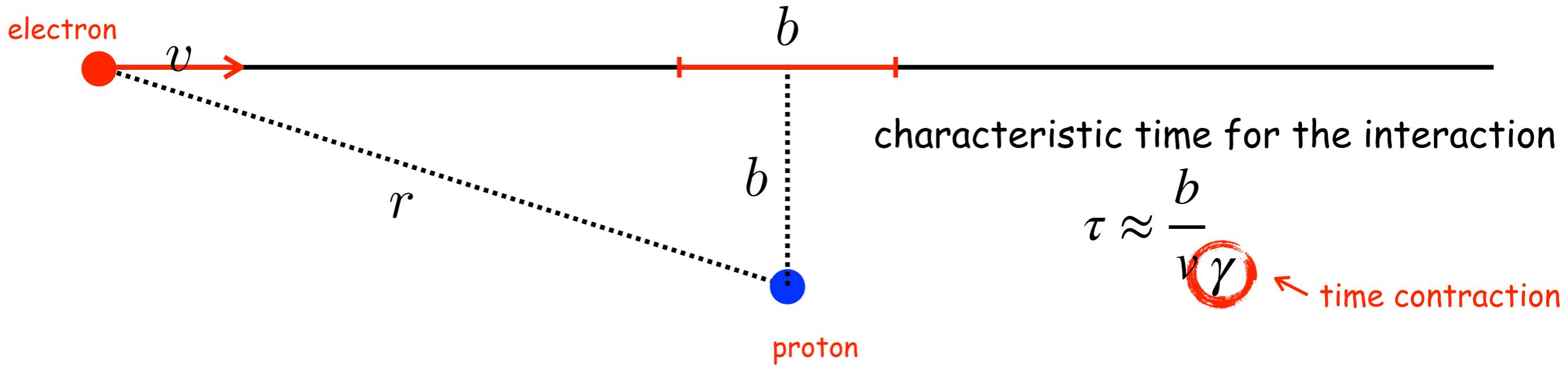
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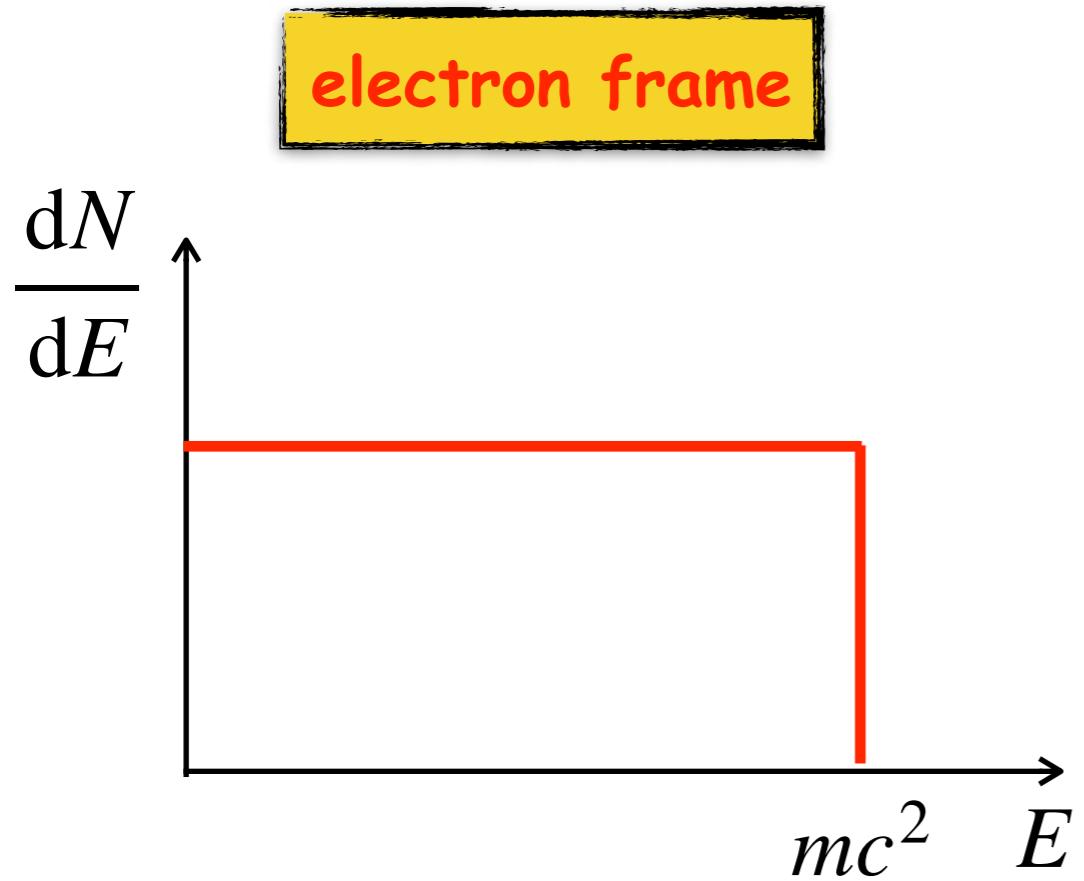
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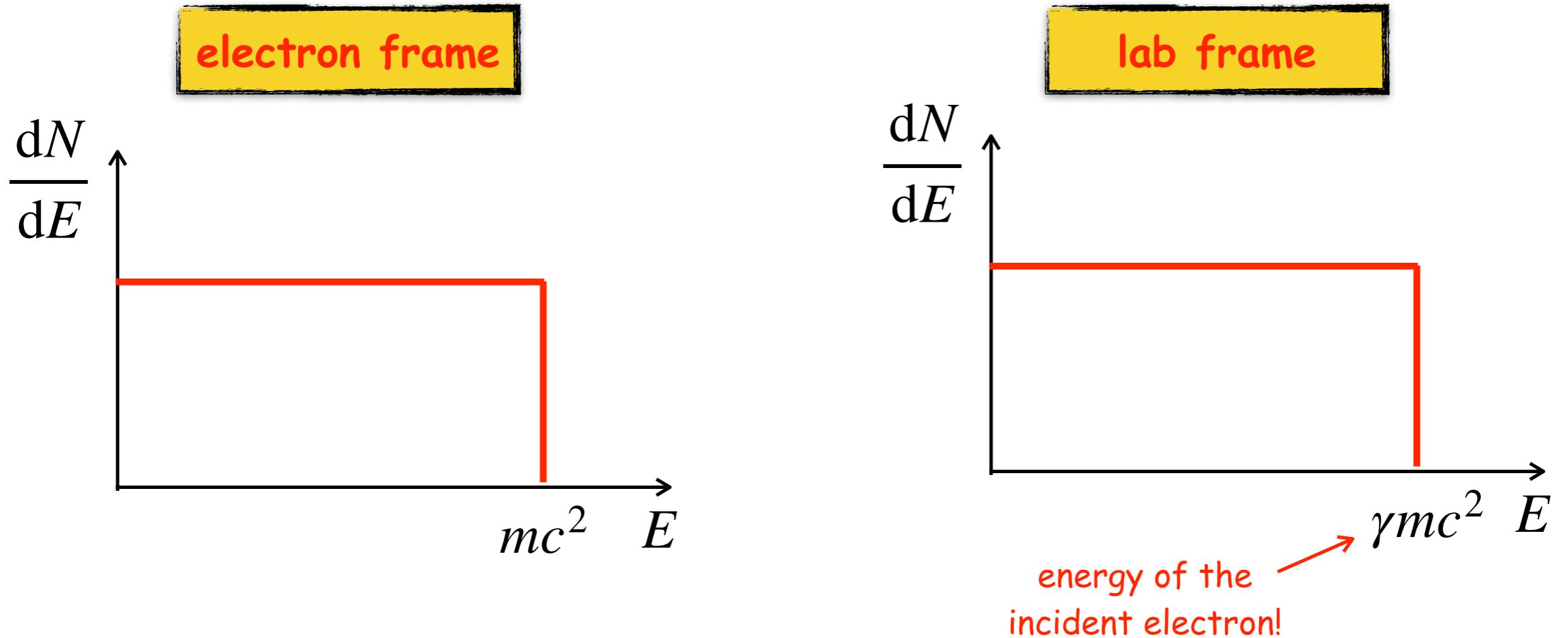


- in its rest frame, the e^- sees the proton's E field contracted into a very fast pulse
- the pulse can be approximated as a spectrum of plane waves, i.e., "virtual" photons
 - the e^- will Compton scatter the virtual photon
 - Compton cross section drops beyond photon energies equal to mc^2
- the spectrum of scattered photons (in the e^- rest frame) extends up to mc^2
 - in the lab frame the spectrum extends up to γmc^2 !

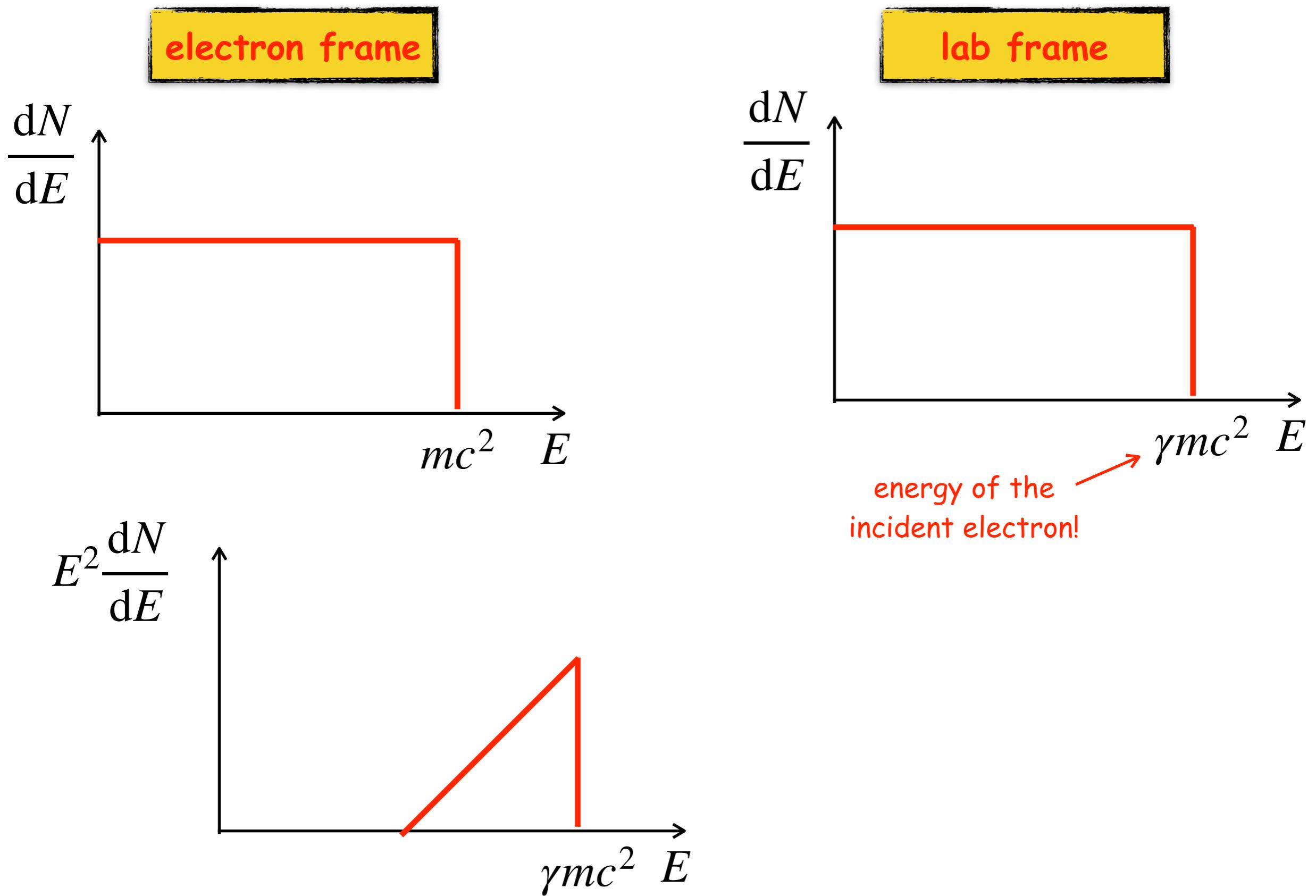
Non-thermal bremsstrahlung



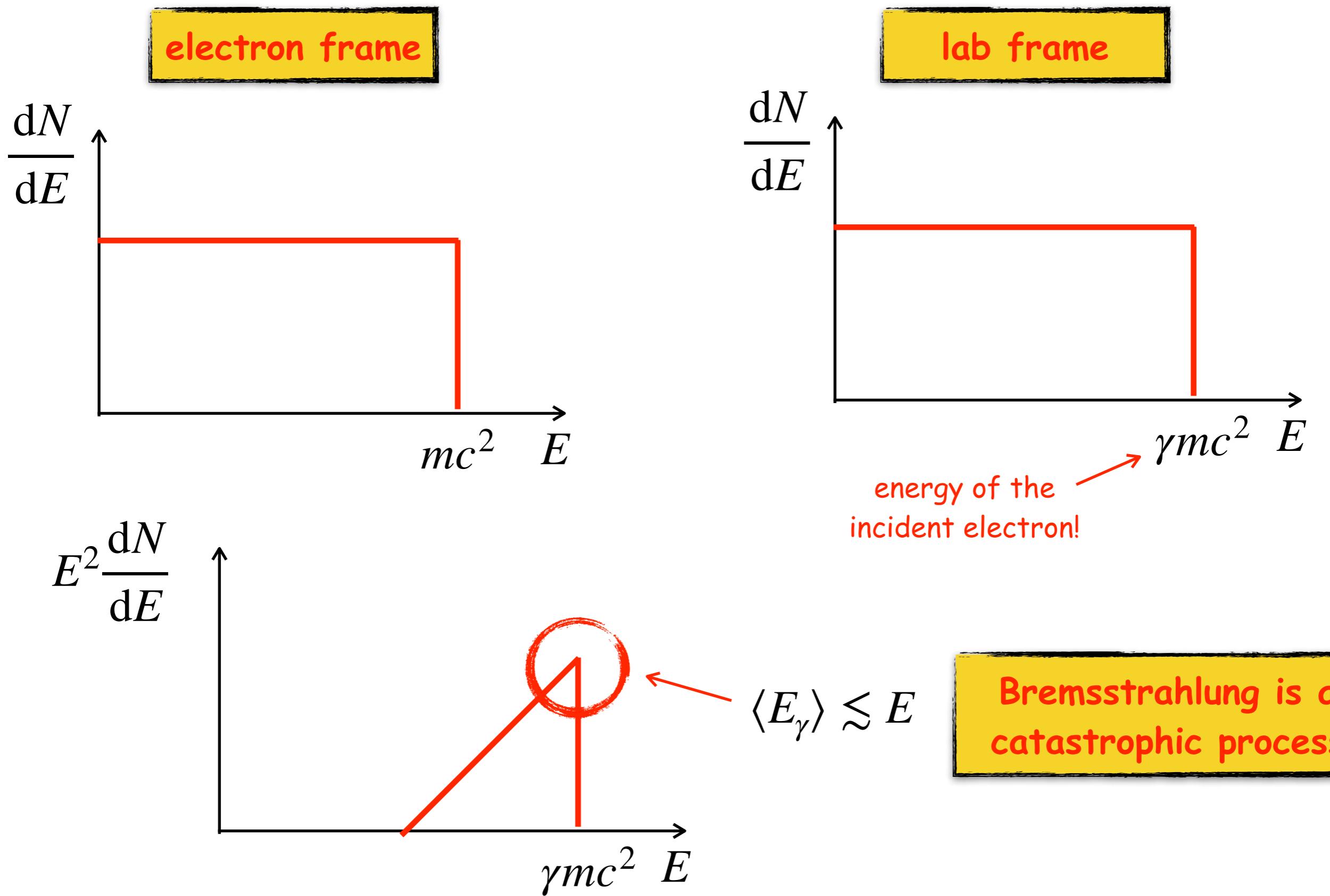
Non-thermal bremsstrahlung



Non-thermal bremsstrahlung



Non-thermal bremsstrahlung



Energy loss times in the ISM...

continuous losses →

$$\frac{dE}{dt} = \frac{4}{3} \sigma_T c \gamma^2 (\omega_B + \omega_{CMB})$$

synchrotron inverse Compton
↓ ↓
same importance
in the ISM !

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$$\tau_{syn/IC} \sim \frac{E}{dE/dt} \approx 1 \left(\frac{\omega_{TOT}}{0.25 \text{ eV/cm}^3} \right)^{-1} \left(\frac{E}{\text{TeV}} \right)^{-1} \text{Myr}$$

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catastrophic losses →

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$$\tau_{pp} \sim 60 \left(\frac{n_{gas}}{\text{cm}^{-3}} \right)^{-1} \text{Myr}$$

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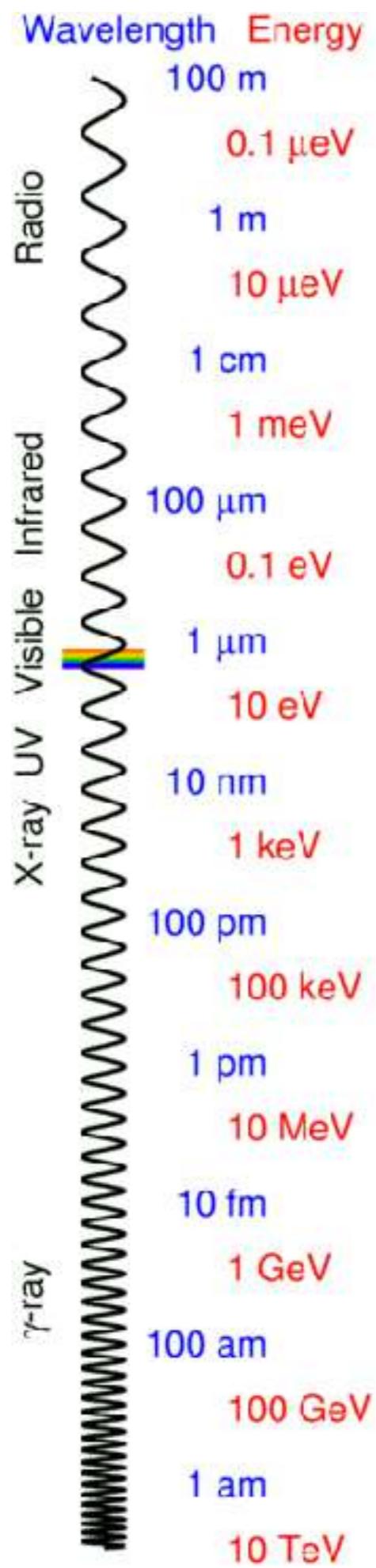
$$\tau_{pp} \sim 60 \left(\frac{n_{gas}}{\text{cm}^{-3}} \right)^{-1} \text{Myr}$$

relativistic Bremsstrahlung →

$$\tau_{Brem} \sim 30 \left(\frac{n_{gas}}{\text{cm}^{-3}} \right)^{-1} \text{Myr}$$

ICS more
 effective in the
 TeV domain

GALACTIC NON-THERMAL EMISSIONS



pp interactions
protons
 $E > 280 \text{ MeV}$

$E \sim 10 \text{ GeV}$ 100 TeV

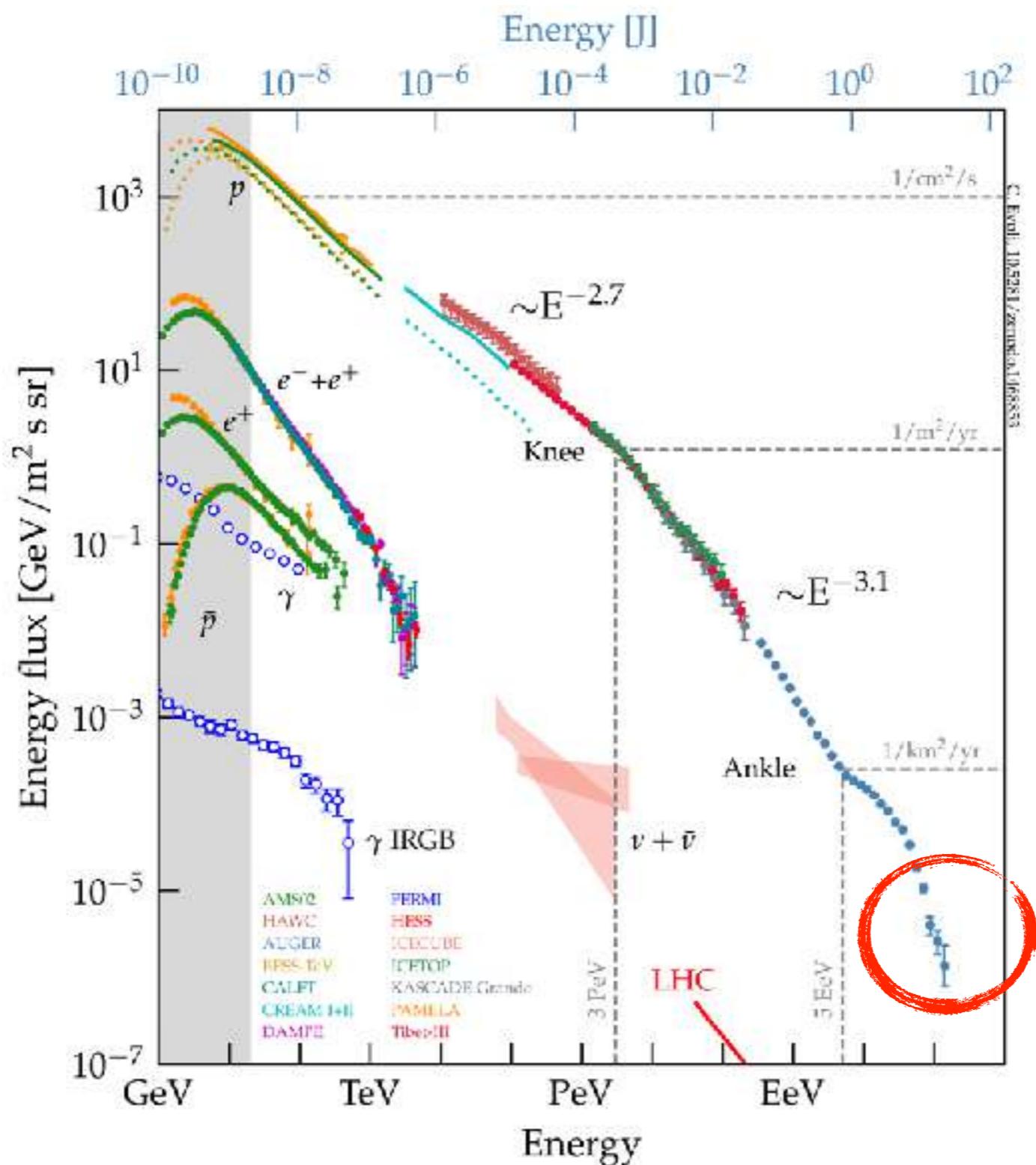
Synchrotron emission

• • • • •

Bremsstrahlung emission
electrons

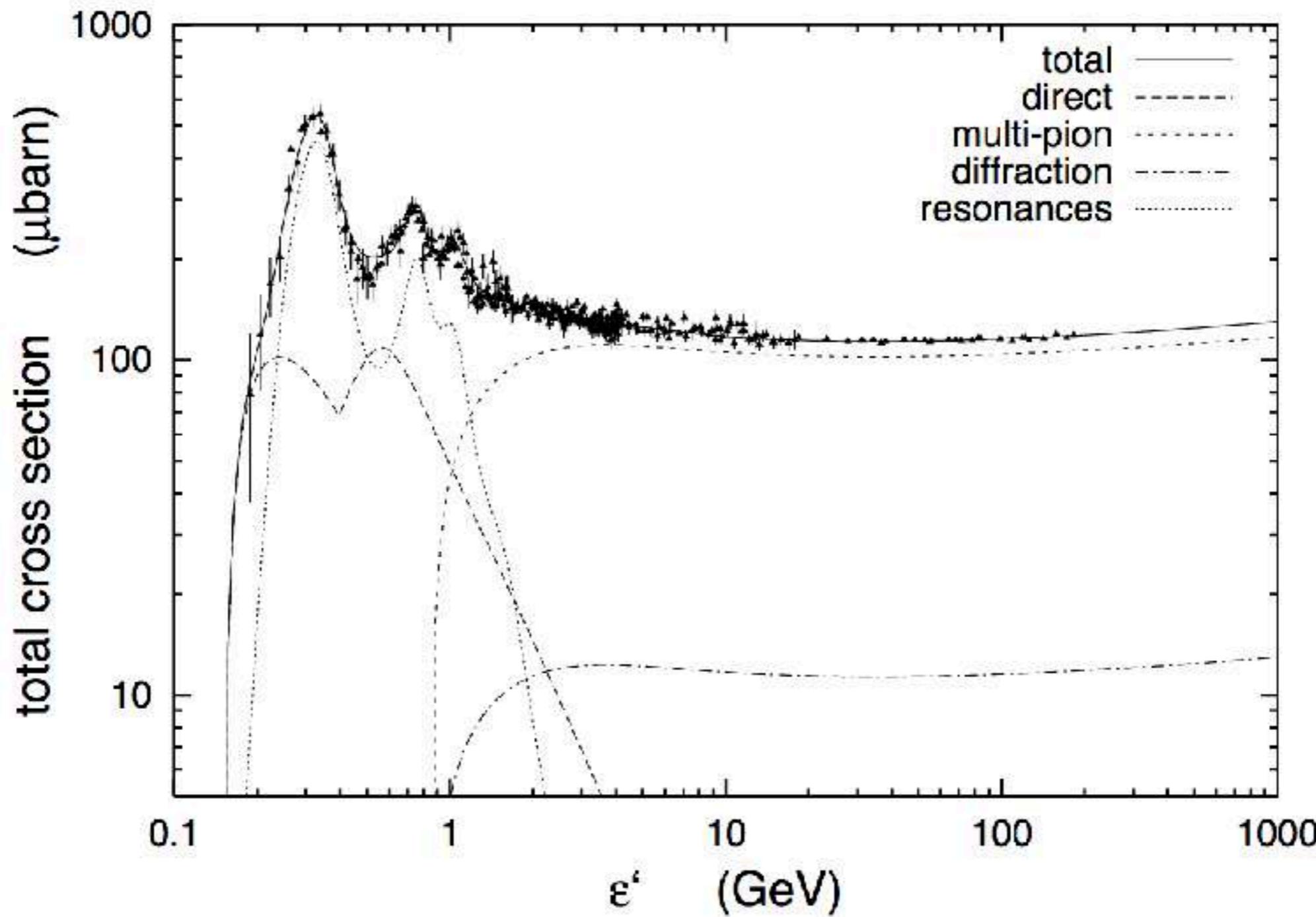
$E \sim \text{keV}$ MeV GeV Tev

Question: where does the cosmic ray spectrum ends?



Photomeson production:

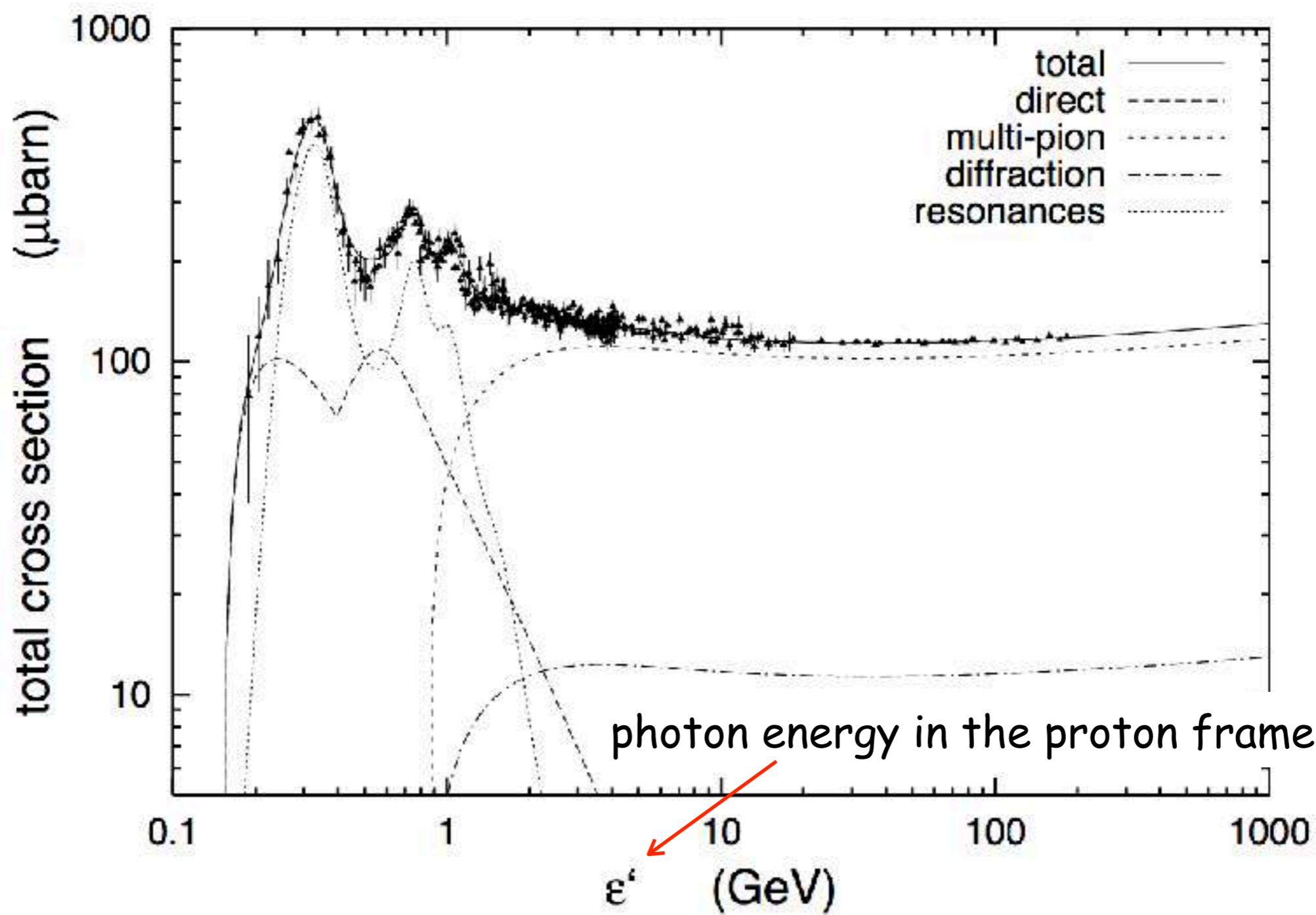
$p\gamma \rightarrow n\pi^+$ or $p\pi^0$



Mucke+ 1999

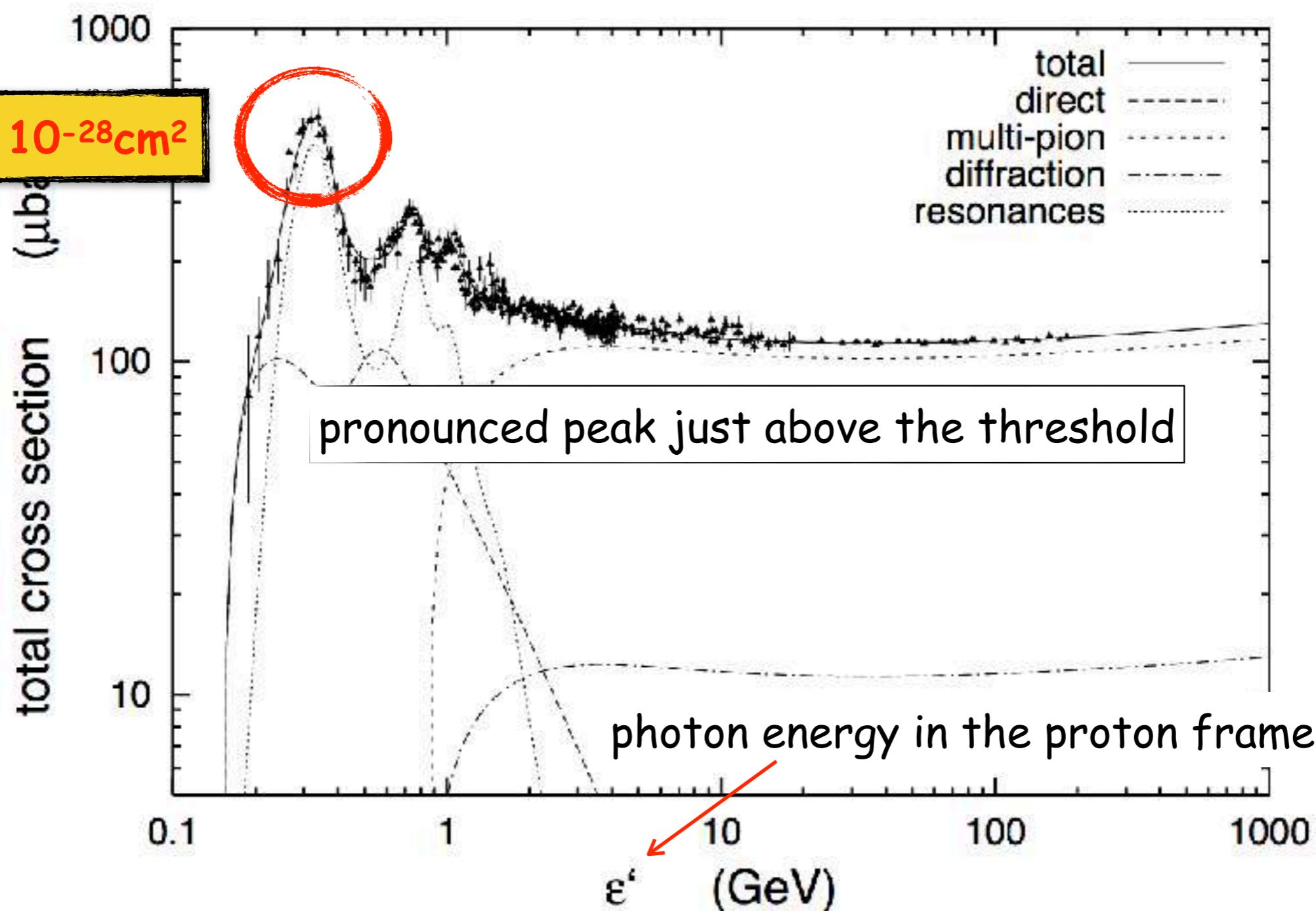
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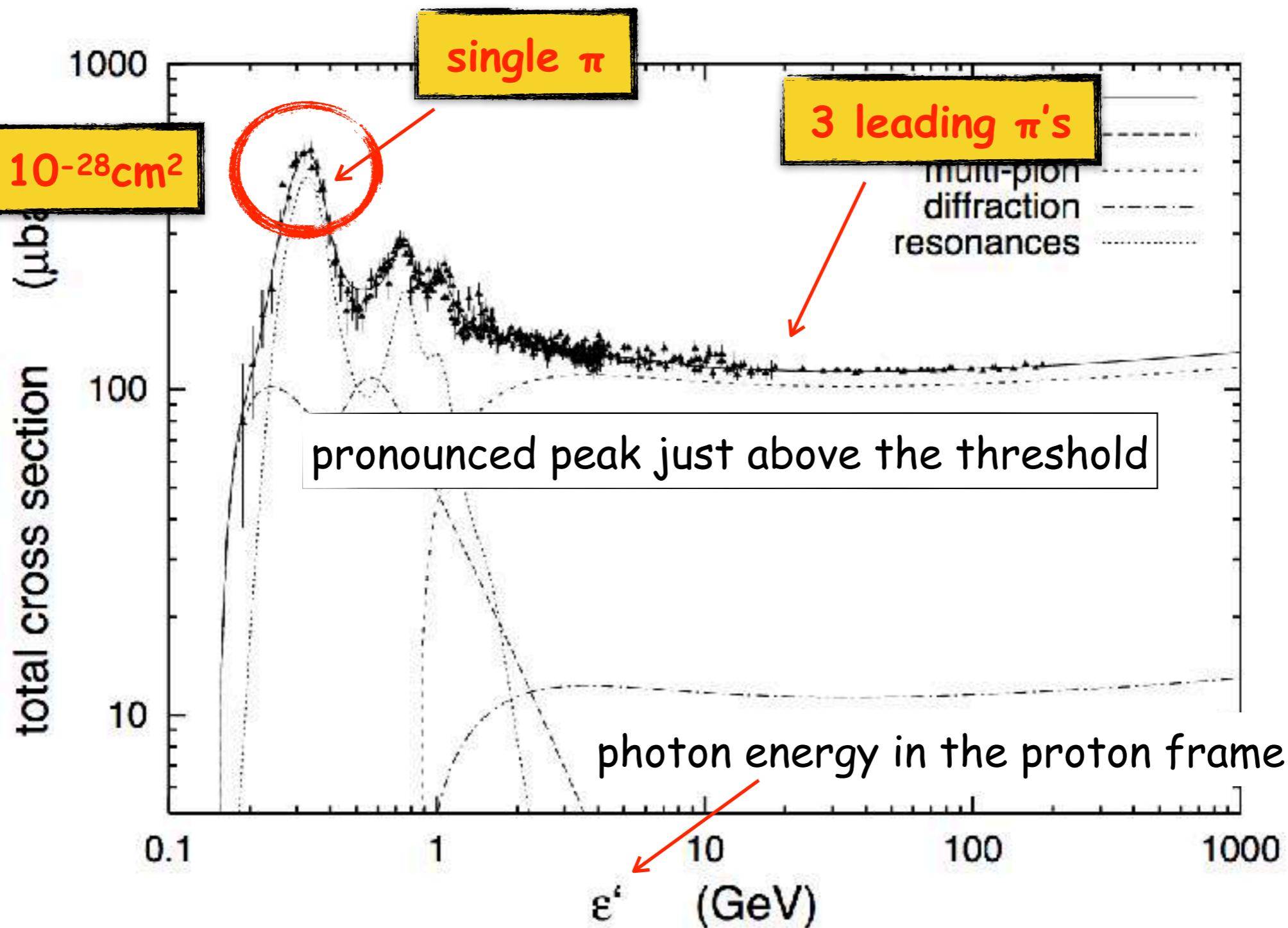


Photomeson production:

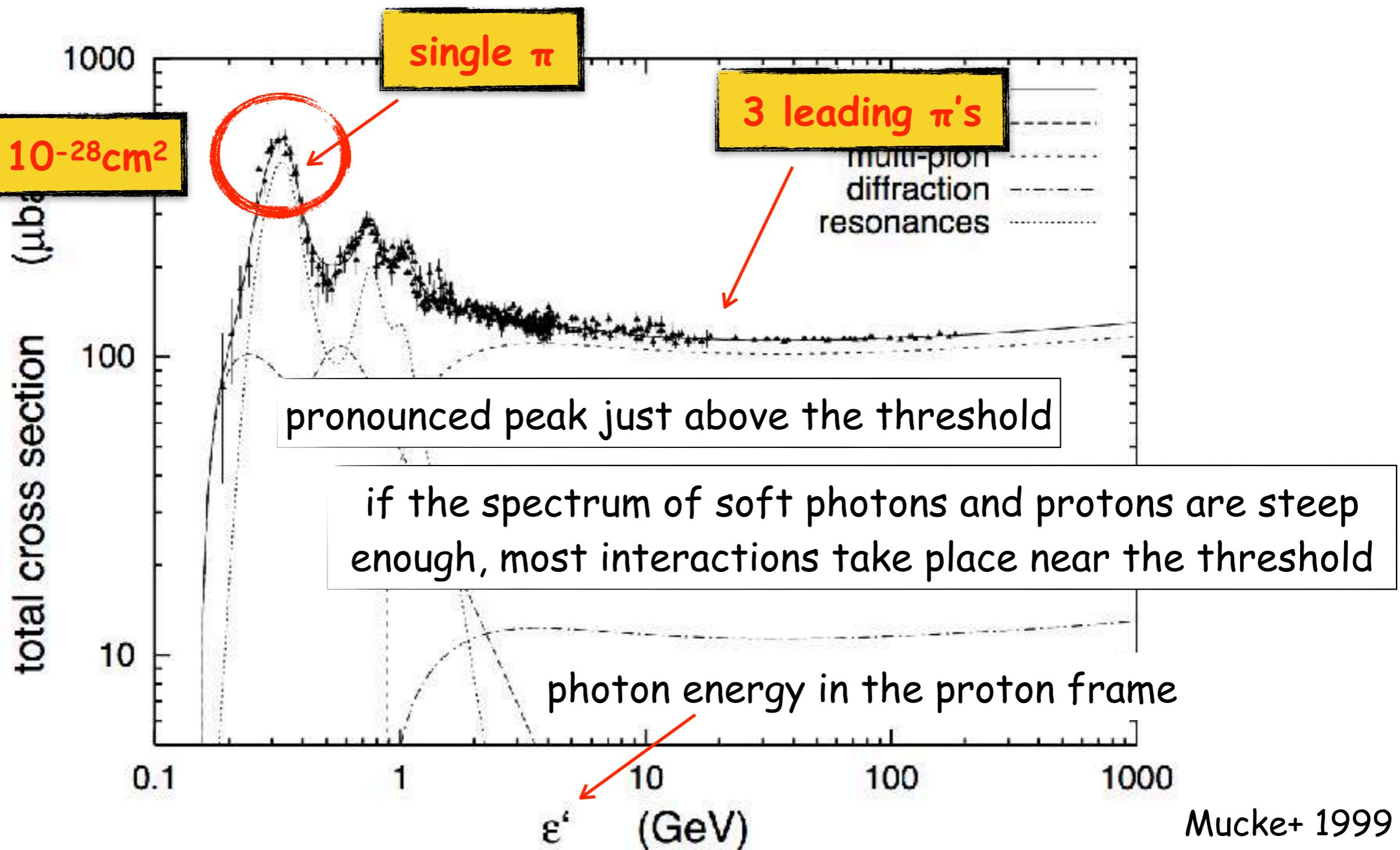
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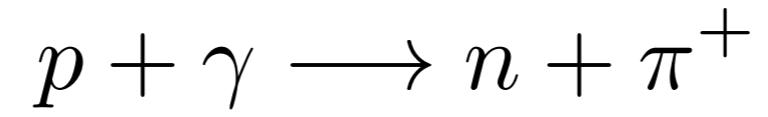
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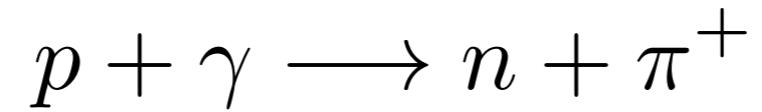
Photomeson production:



Photomeson production: threshold

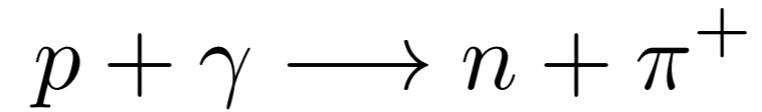


Photomeson production: threshold



energy in the center of mass frame

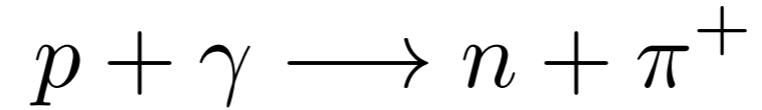
Photomeson production: threshold



energy in the center of mass frame

before the collision \rightarrow $s^2 = (E_p + \epsilon_\gamma)^2 - (\vec{p}_p + \vec{p}_\gamma)^2$

Photomeson production: threshold



energy in the center of mass frame

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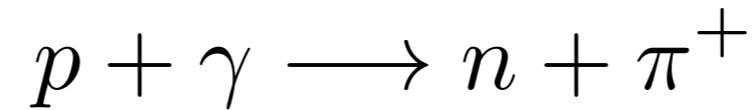
after the collision

and

$$\rightarrow s_{th} = m_p + m_\pi$$

at the threshold

Photomeson production: threshold



energy in the center of mass frame

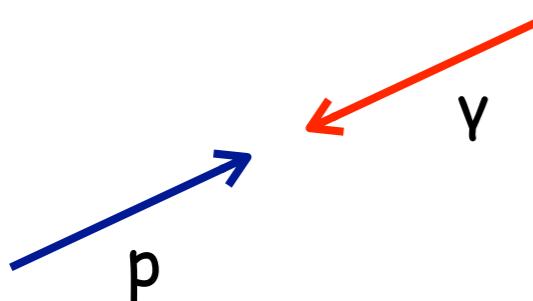
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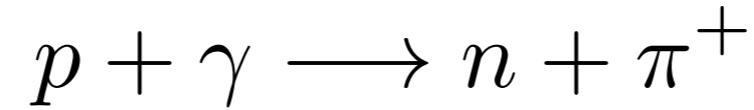
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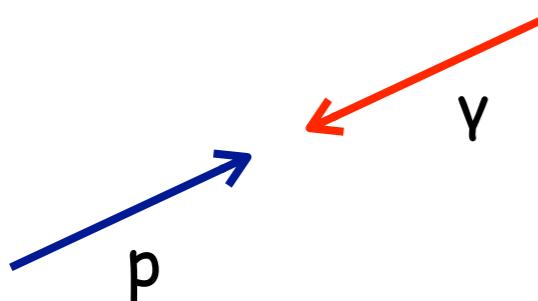
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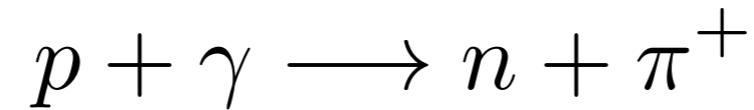
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$$E_p^{min} \approx 60 \left(\frac{\epsilon_\gamma}{\text{eV}} \right)^{-1} \text{PeV}$$

Photomeson production: threshold



energy in the center of mass frame

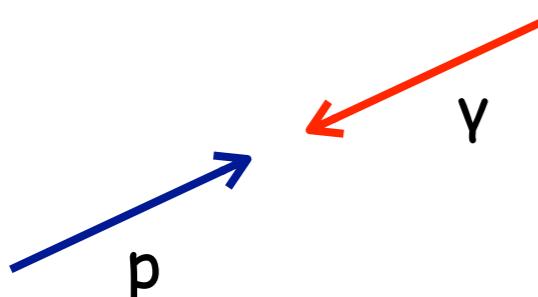
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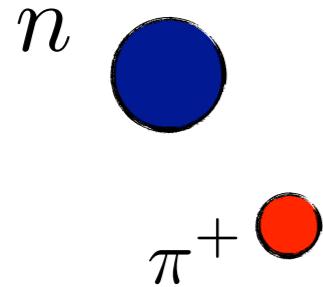
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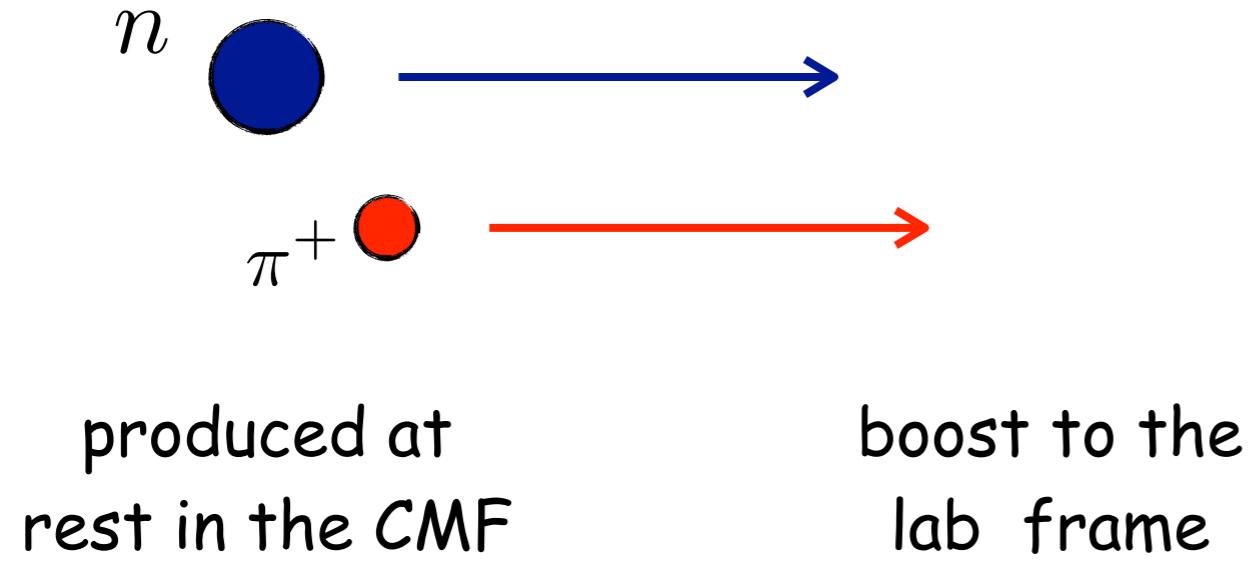
for CMB photons this is $\sim 10^{20}$ eV !

Photomeson production: threshold

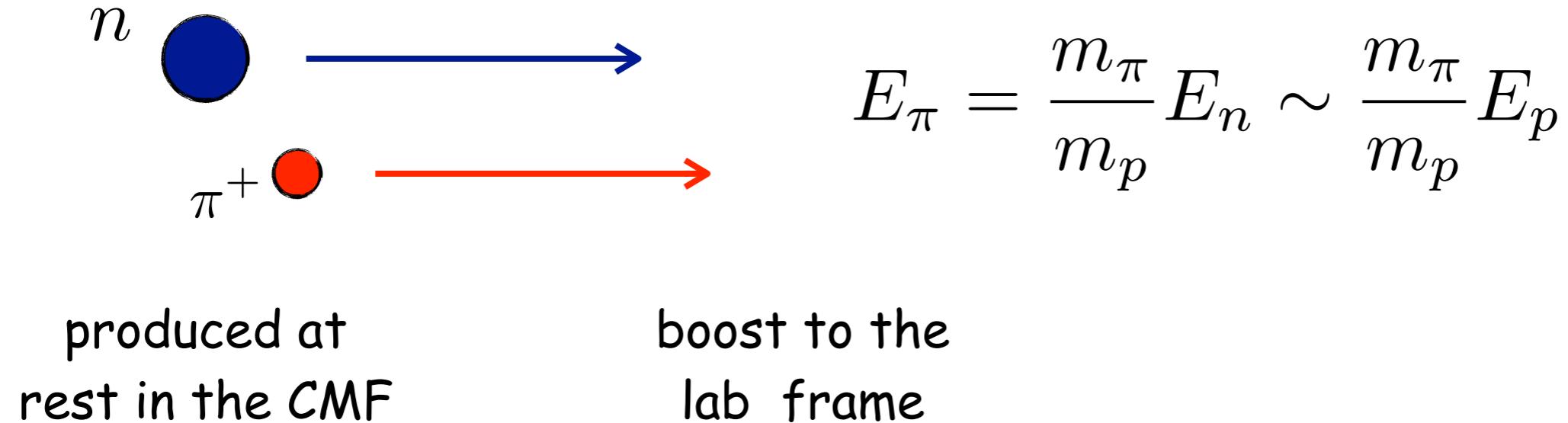


produced at
rest in the CMF

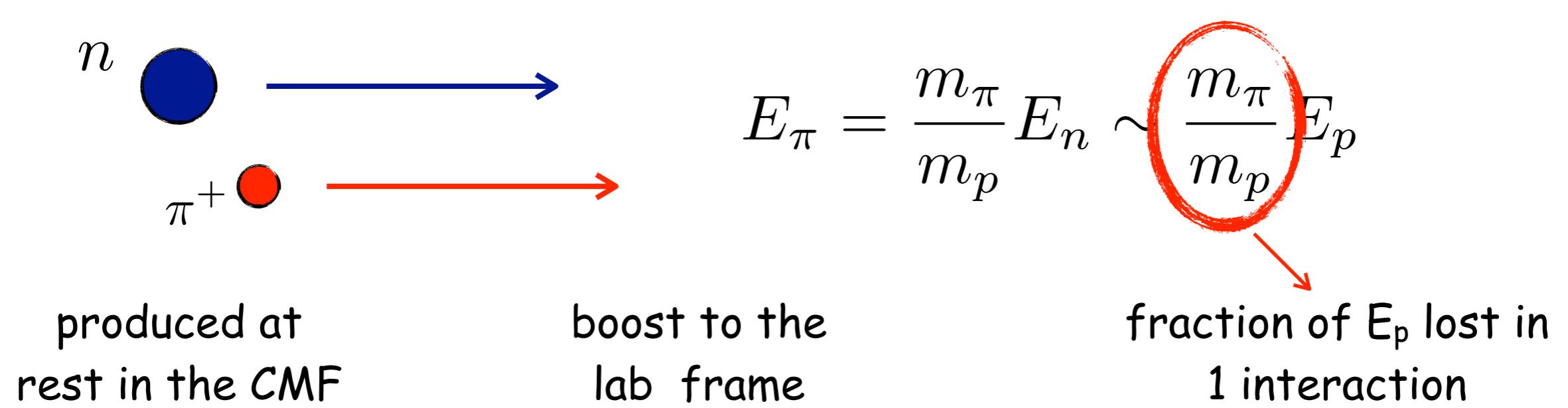
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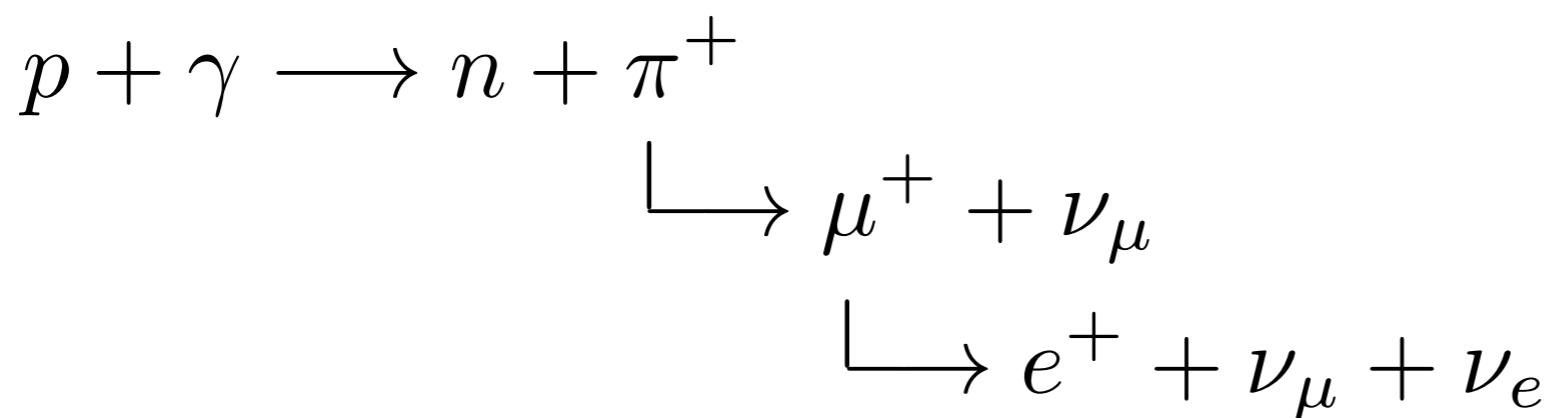
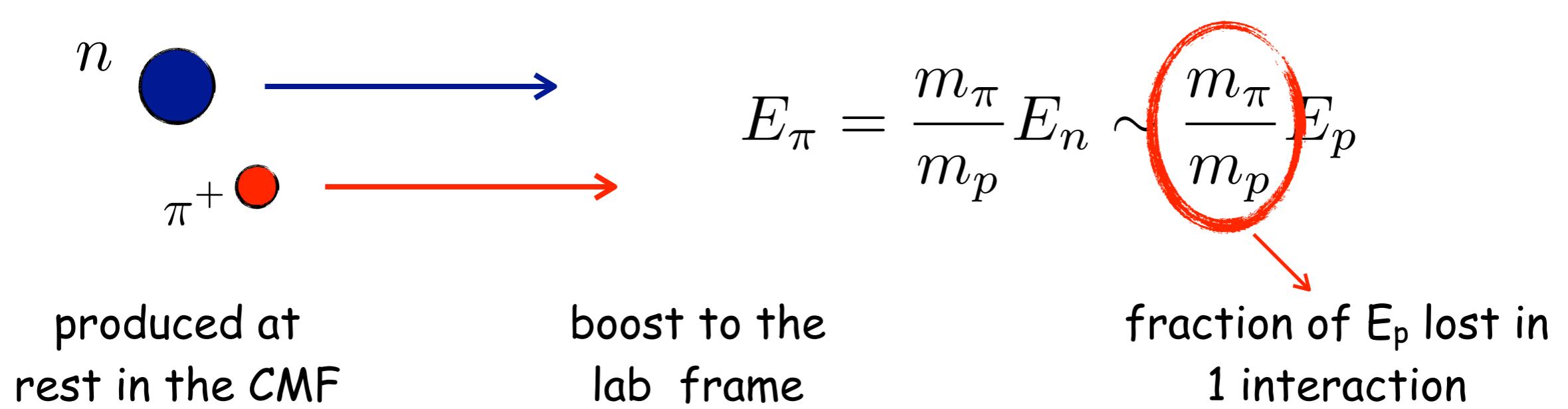
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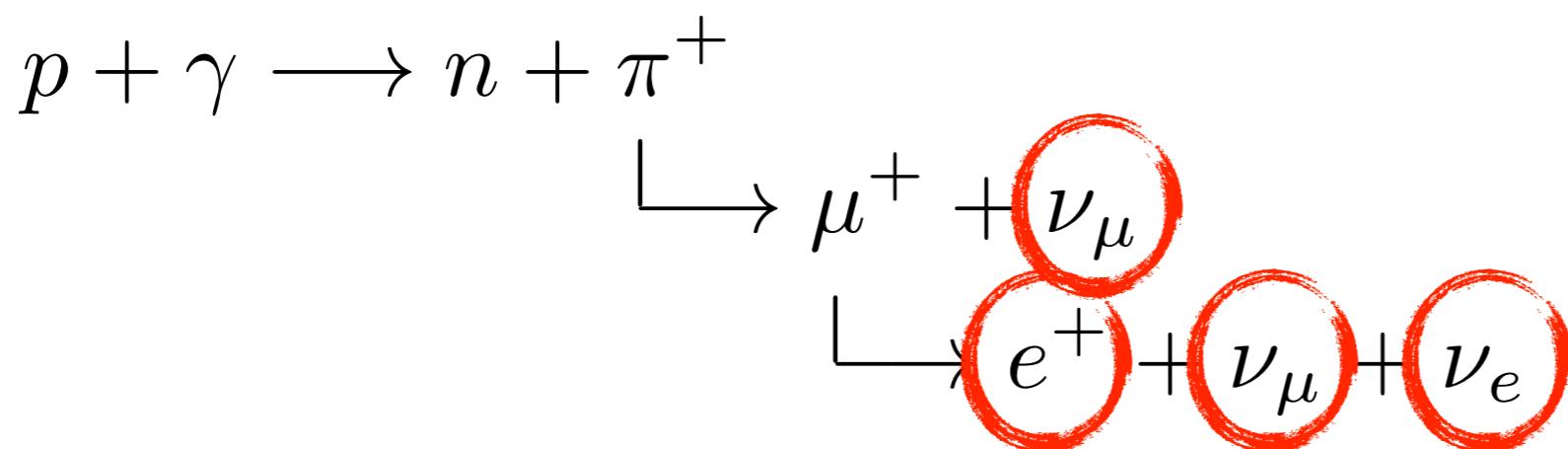
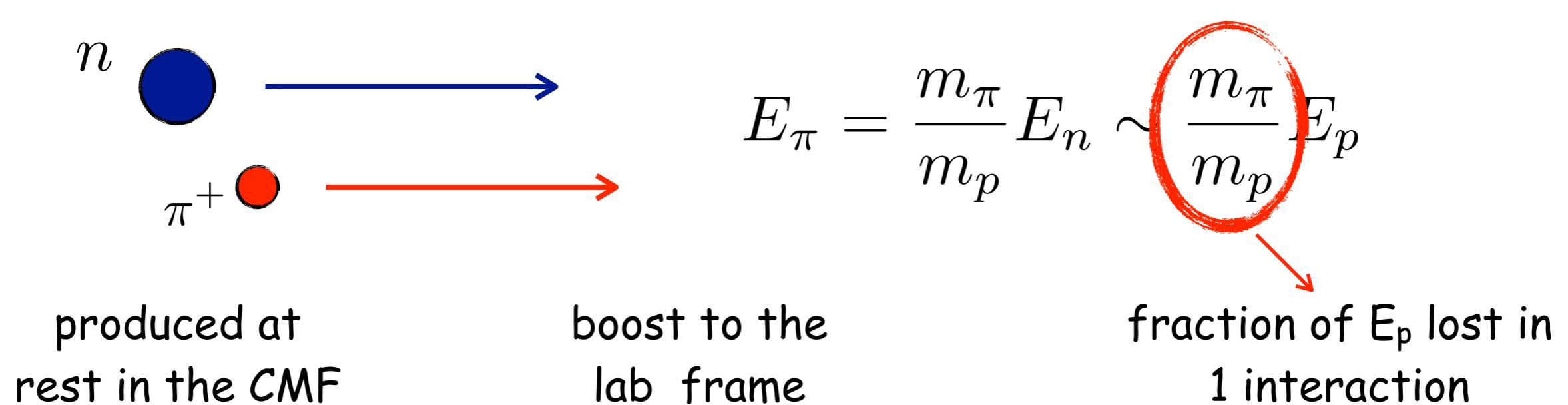
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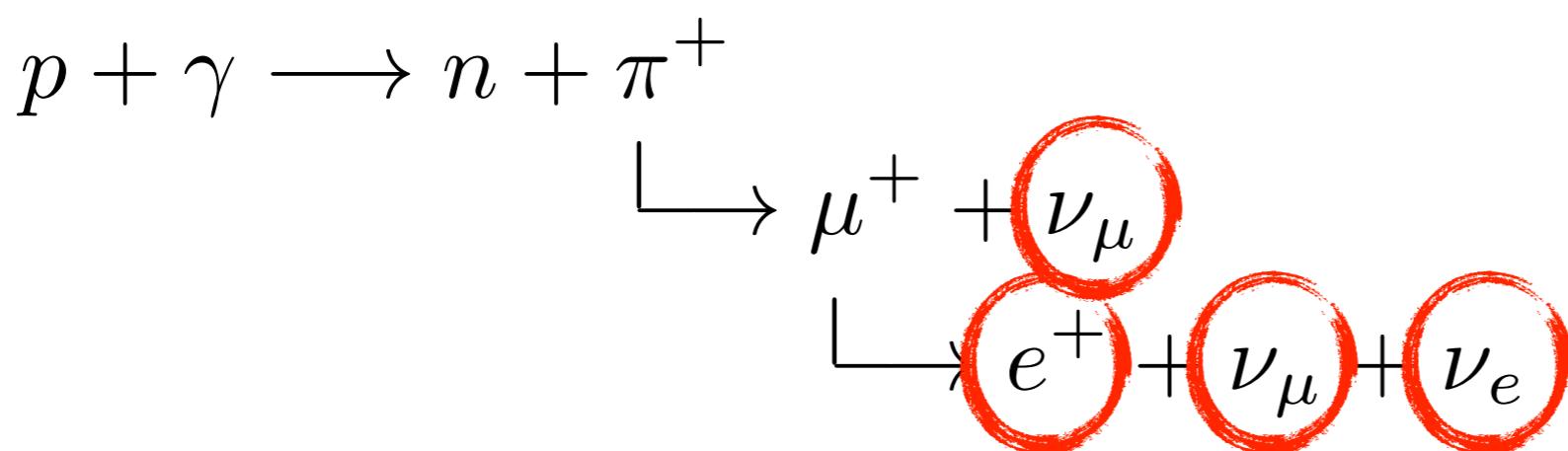
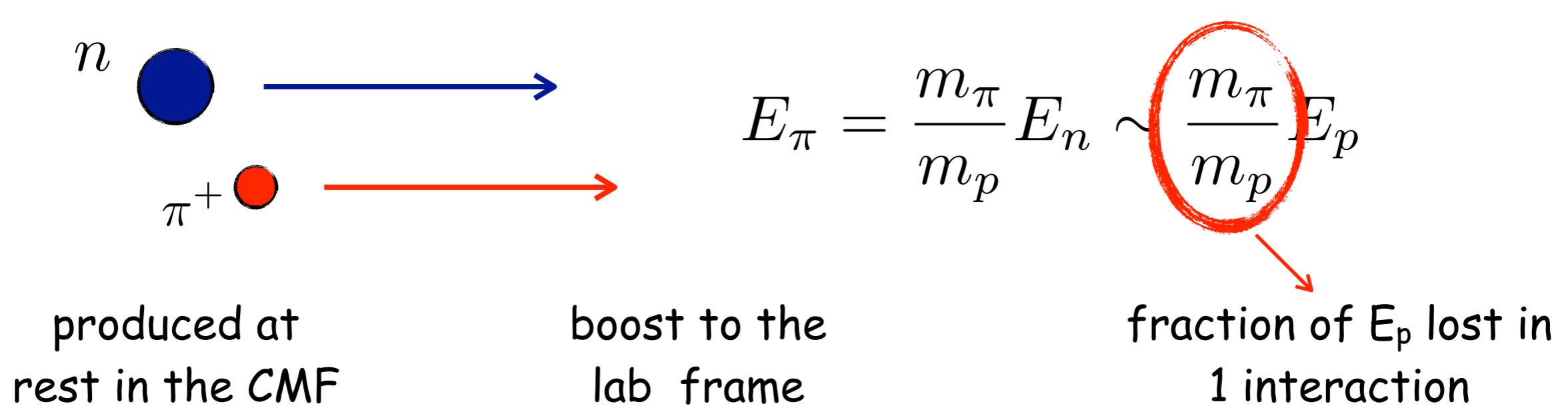


Photomeson production: threshold



4 stable particles carrying ~the same energy

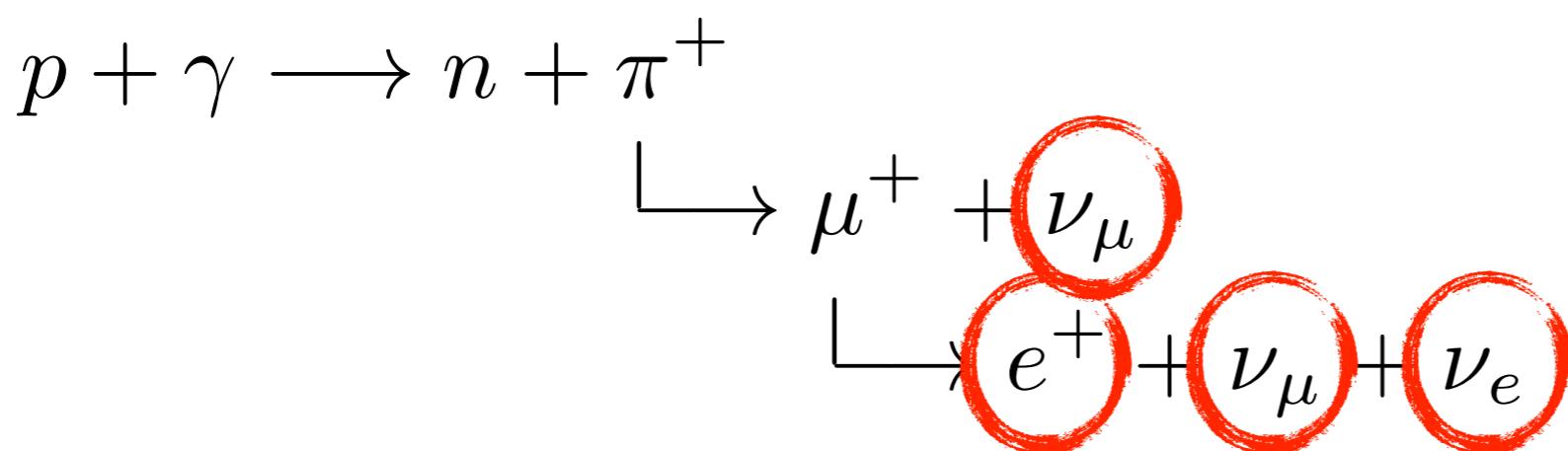
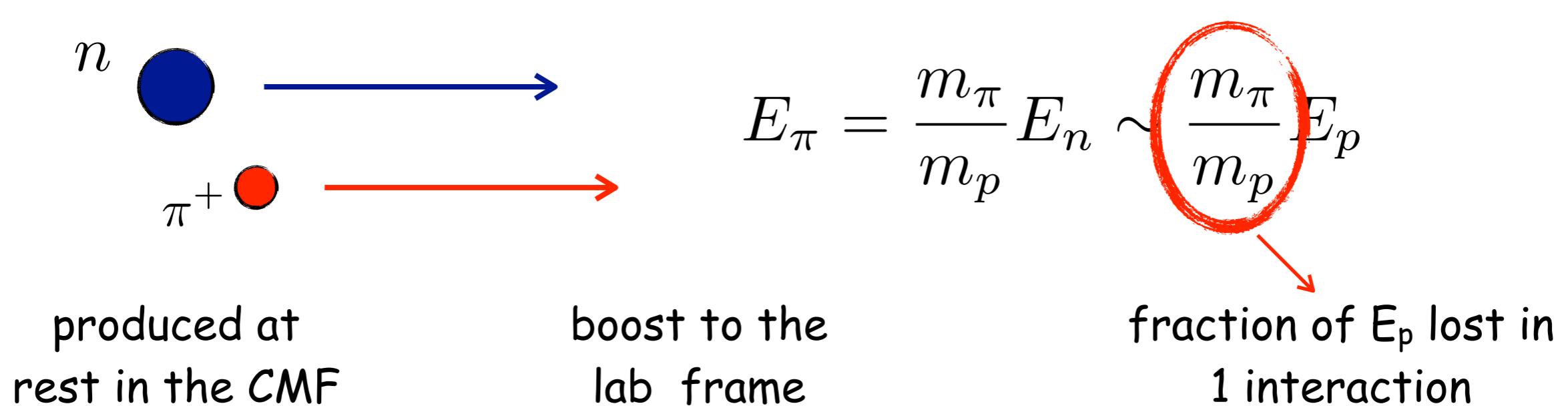
Photomeson production: threshold



4 stable particles carrying
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$$E_\nu \sim \left(\frac{m_\pi}{4m_p} \right) E_p$$

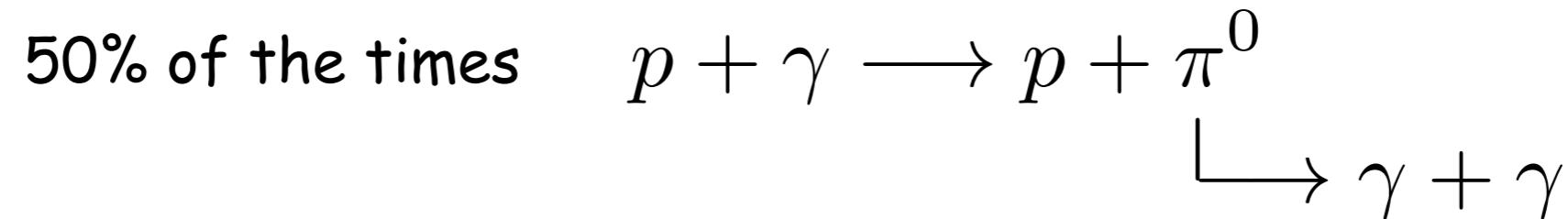
Photomeson production: threshold



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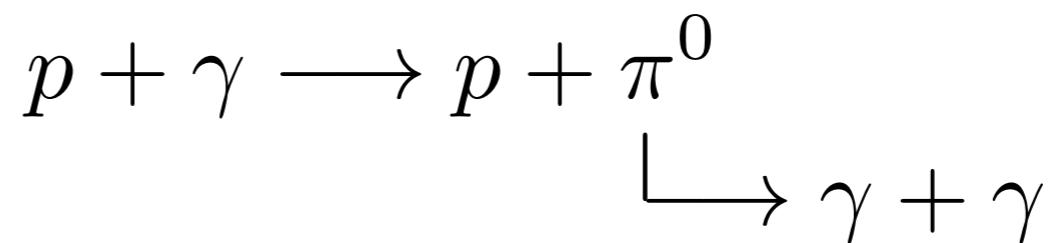
$$E_\nu \sim \left(\frac{m_\pi}{4m_p} \right) E_p \approx 0.04 E_p$$

Absorption of gamma rays



Absorption of gamma rays

50% of the times



- the π^0 carries a fraction of the proton energy equal to $\frac{m_\pi}{m_p}$

Absorption of gamma rays

50% of the times $p + \gamma \longrightarrow p + \pi^0$

$$\downarrow \rightarrow \gamma + \gamma$$

■ the π^0 carries a fraction of the proton energy equal to $\frac{m_\pi}{m_p}$

■ 2 stable particles of average energy

$$E_\gamma \sim \left(\frac{m_\pi}{2m_p} \right) E_p \sim 2 E_\nu$$

Absorption of gamma rays

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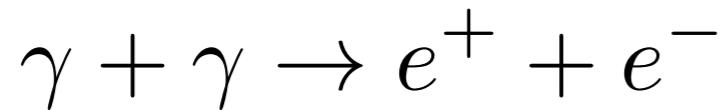
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threshold energy pair production

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$$E_\gamma \epsilon_\gamma (1 - \cos \vartheta) > 2 m_e^2$$

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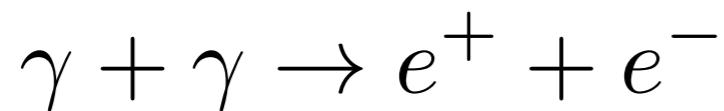
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$$E_\gamma^{min} \approx 0.3 \left(\frac{\epsilon_\gamma}{\text{eV}} \right)^{-1} \text{TeV}$$

Absorption of gamma rays

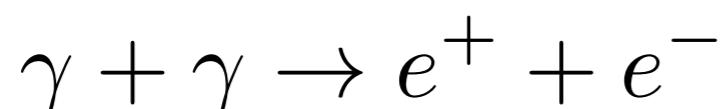
50% of the times $p + \gamma \longrightarrow p + \pi^0$

$$\downarrow \longrightarrow \gamma + \gamma$$

■ the π^0 carries a fraction of the proton energy equal to $\frac{m_\pi}{m_p}$

■ 2 stable particles of average energy

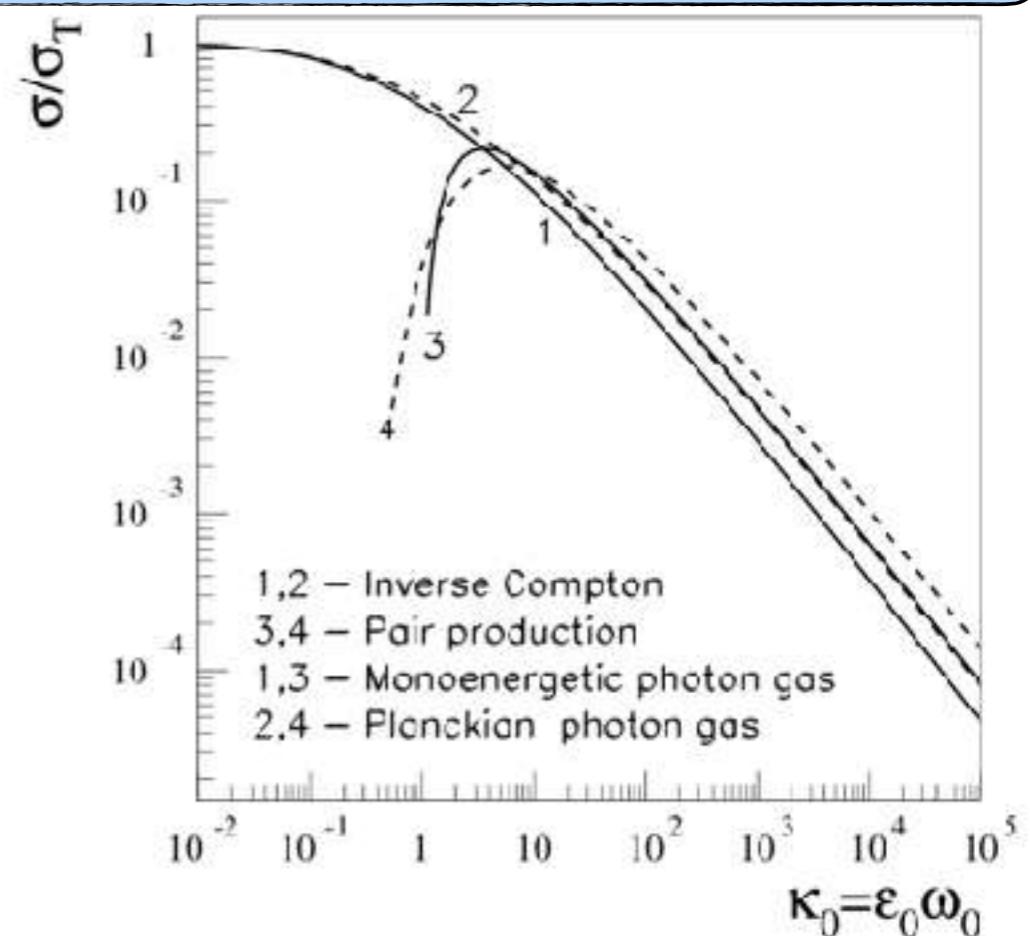
threshold energy pair production



$$E_\gamma \epsilon_\gamma (1 - \cos \vartheta) > 2 m_e^2$$

$$E_\gamma^{\min} \approx 0.3 \left(\frac{\epsilon_\gamma}{\text{eV}} \right)^{-1} \text{TeV}$$

$$E_\gamma \sim \left(\frac{m_\pi}{2m_p} \right) E_p \sim 2 E_\nu$$



Absorption of gamma rays

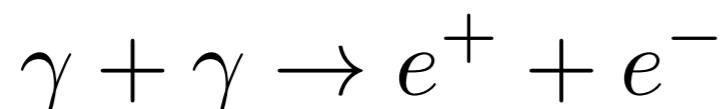
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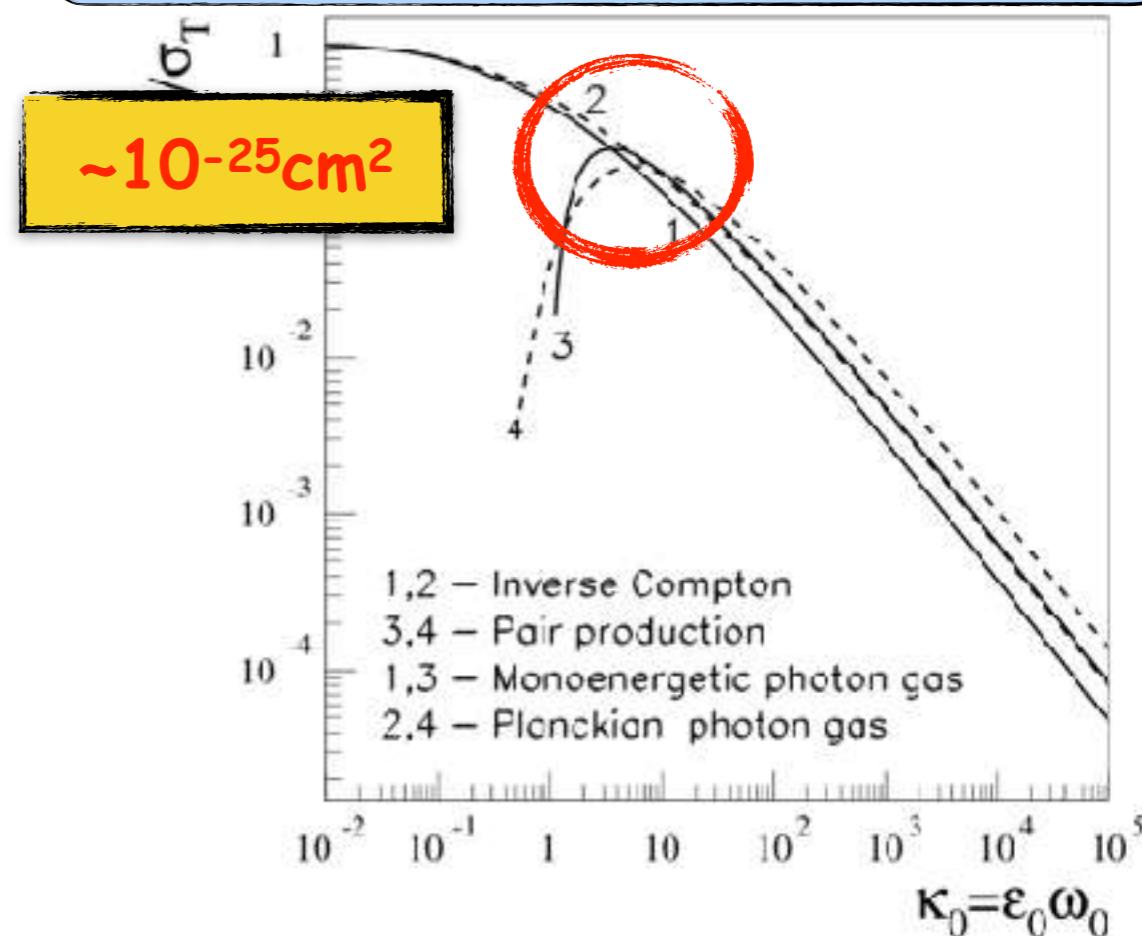
threshold energy pair production



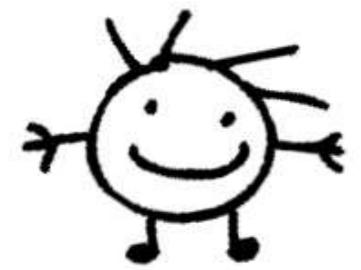
$$E_\gamma \epsilon_\gamma (1 - \cos \vartheta) > 2 m_e^2$$

$$E_\gamma^{\min} \approx 0.3 \left(\frac{\epsilon_\gamma}{\text{eV}} \right)^{-1} \text{TeV}$$

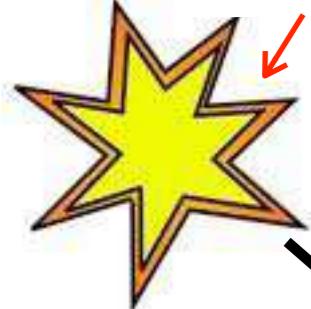
$$E_\gamma \sim \left(\frac{m_\pi}{2m_p} \right) E_p \sim 2 E_\nu$$



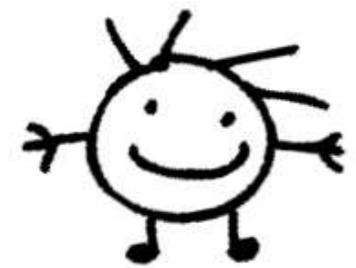
Where do photons go?



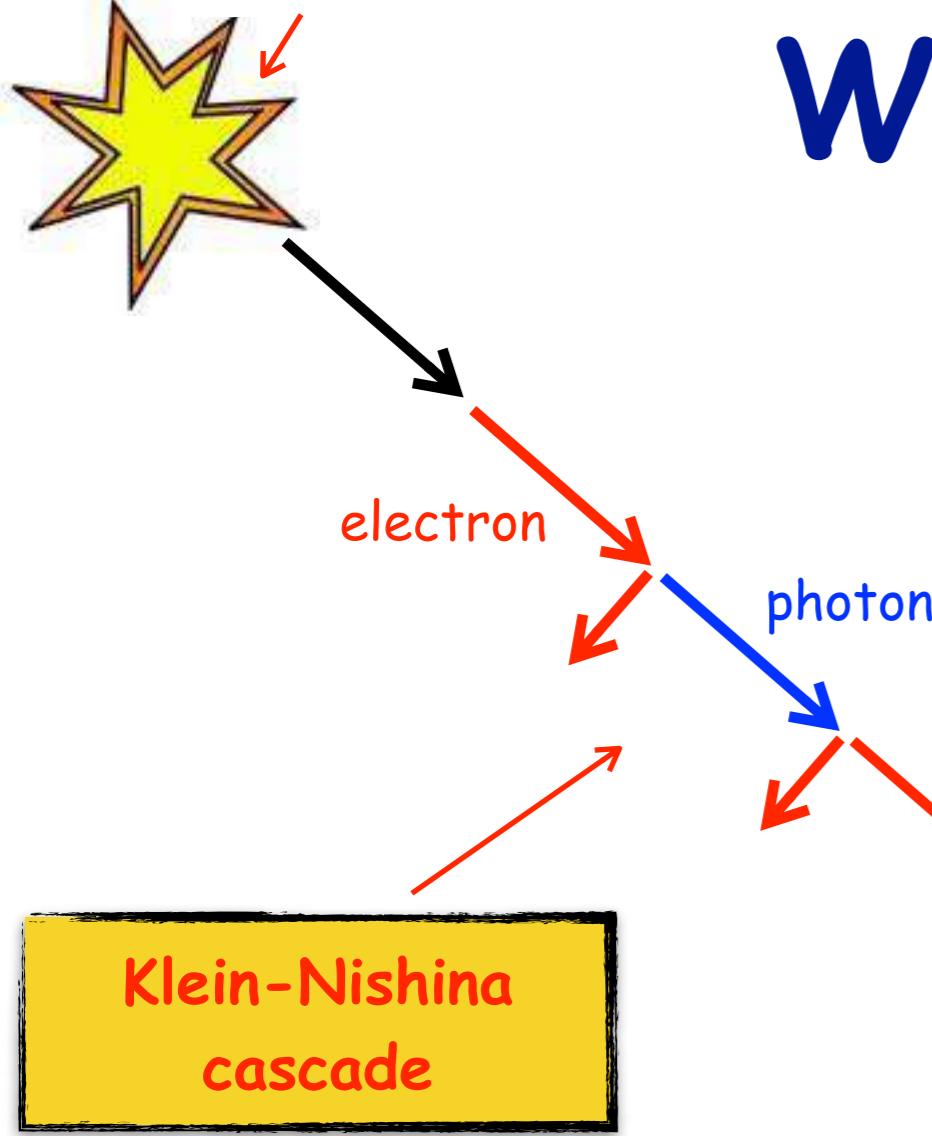
CR source



Where do photons go?



CR source



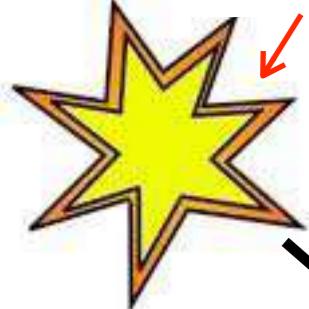
$$E_\gamma \epsilon_\gamma \gg m_e^2$$

inverse Compton + pair production



Where do photons go?

CR source



Where do photons go?

electron

photon

Klein-Nishina
cascade

$E_\gamma \epsilon_\gamma \gg m_e^2$

inverse Compton + pair production

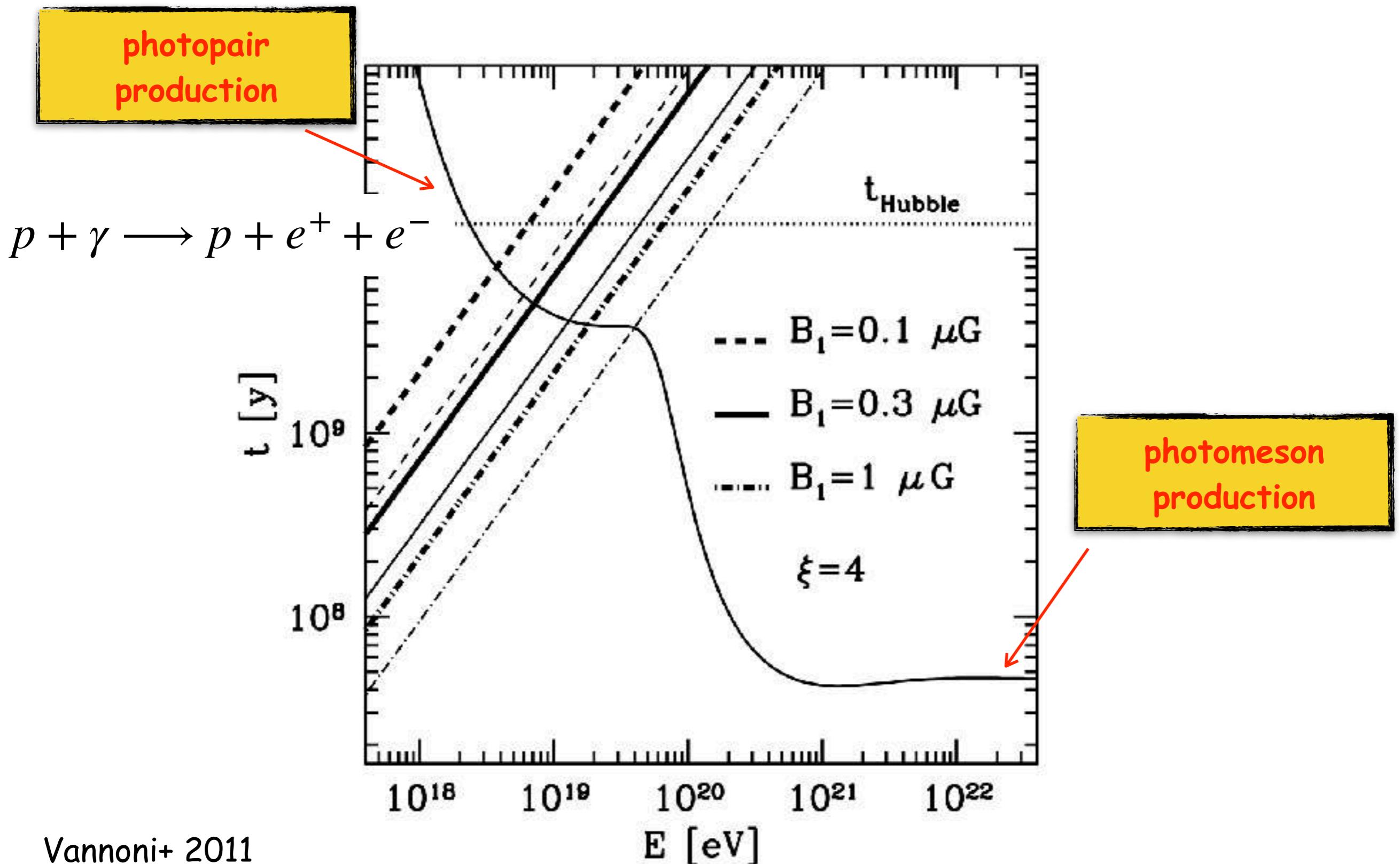
$$E_\gamma \epsilon_\gamma \gtrsim m_e^2$$

Thompson cascade

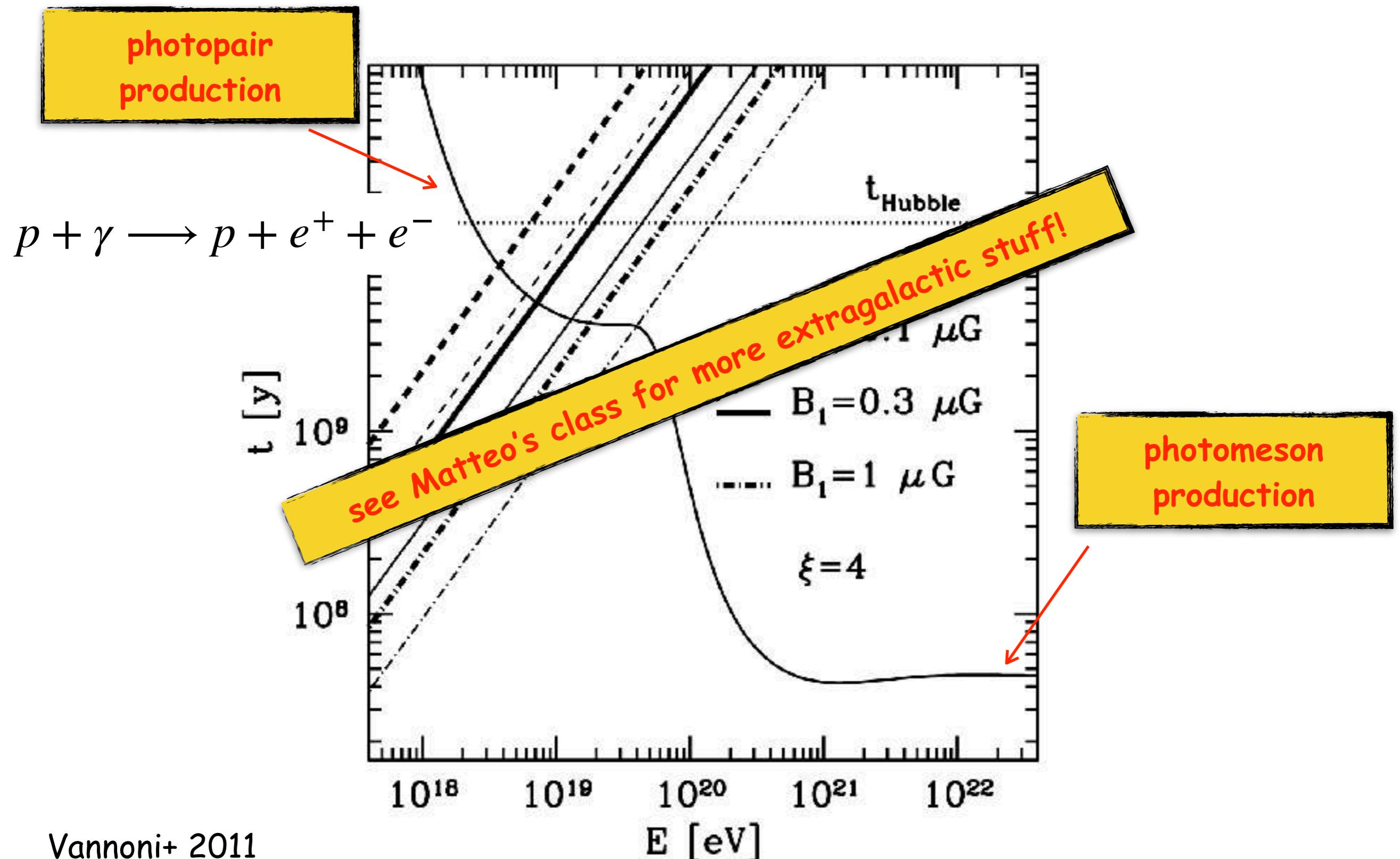
< 100 GeV



Photopair production



Photopair production



next class: apply all this to "Galactic sources"...

