NAME:

This is an open notes and open-book exam. Write all your answers on these pages. Do all the problems within 55 minutes.

105 points total

1 Sum of Products Minimization (16 points)

Consider the following specification:

$$F(w,x,y,z) = \Sigma(7,8,9,13,15)$$
$$D(w,x,y,z) = \Sigma(1,2,3,5,6,12,14)$$

A. Draw a 4-variable Karnaugh map and fill it in with 0s, 1s, and don't cares corresponding to the specification **(8 points)**.

B. Using your Karnaugh map, write down a *minimal* sum-of-products expression for F(w,x,y,z) **(4 points)**.

$$F(w,x,y,z) =$$

C. Using **only** nand gates and inverters, draw a schematic implementing F(w,x,y,z) **(4 points)**.

2 Product of Sums Minimization (16 points)

Consider the following specification:

$$F(w,x,y,z) = \Pi(0,3,5,7,8,13)$$
$$D(w,x,y,z) = \Pi(2,6,10,14)$$

A. Draw a 4-variable Karnaugh map and fill it in with 0s, 1s, and don't cares corresponding to the specification **(8 points)**.

B. Using your Karnaugh map, write down a *minimal* product-of-sums expression for F(w,x,y,z) **(4 points)**.

$$F(w,x,y,z) =$$

C. Using **only** nor gates and inverters, draw a schematic implementing F(w,x,y,z) **(4 points)**.

3 7-Segment Display Controller and Decoder (31 points)

Inputs		Outputs			
x	y	D_0	D_1	D_2	D_3
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

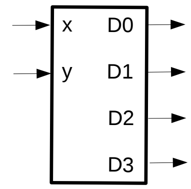


Figure 1: 2–4 Decoder Specification and Block Diagram

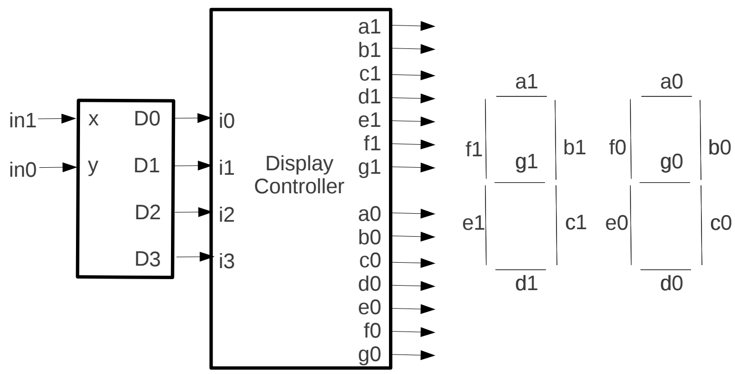


Figure 2: Two 7-Segment Displays

Consider the two-to-four decoder specified below and its block diagram in Figure 1. Your task is to design part of the *Display Controller* shown in Figure 2.

A. (**3 Points**) The two 7-segment displays are used to display the *base 10* value of a 2-bit **2's-complement** number $i_1 i_0$, where i_1 is the **most significant bit**.

Each segment in the two 7-segment displays is controlled by the corresponding control signals a_0 through g_1. In this part, for each value of $i_1 i_0$, specify the base 10 value to be displayed when $i_1 i_0$ is interpreted as a 2's-complement number. As an example, the case for 11_2 is shown below. Fill in the remainder of the table.

in_1	in_0	Display
0	0	
0	1	
1	0	
1	1	-1

B. (**14 Points**) The four input variables to the display controller are i_0 through i_3 corresponding to each of the four input cases. Fill in the table below specifying the value of each display segment. **Note:** (1) for negative numbers, use the leftmost 7-segment display for the negative sign, and (2) use b_0 and c_0 for the numeral "1".

i_0	i_1	i_2	i_3	a_1	b_1	c_1	d_1	e_1	f_1	g_1	a_0	b_0	c_0	d_0	e_0	f_0	g_0
1	0	0	0														
0	1	0	0														
0	0	1	0														
0	0	0	1														

C. (**14 points**) Based on the table above, write **minimal** logical formulas in terms of the inputs i_0, i_1, i_2, i_3 for the following display segments:

$b_1 =$

$g_1 =$

$b_0 =$

$c_0 =$

$d_0 =$

$f_0 =$

$g_0 =$

4 Design with Multiplexers (20 points)

Consider the following function:

$$F(w,x,y,z) = (x+y)\cdot(w'+y)\cdot(y+z)\cdot(w+x'+y'+z')\cdot(w'+x+y'+z')$$

A. Fill in the truth table **(4 points)**

w	x	y	z	F(w,x,y,z)
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

B. We can write F(w,x,y,z) using Shannon's expansion as:

$$\begin{aligned} F(w,x,y,z) = &(w' \cdot x' \cdot y') \cdot F(0,0,0,z) + (w' \cdot x' \cdot y) \cdot F(0,0,1,z) + \\ &(w' \cdot x \cdot y') \cdot F(0,1,0,z) + (w' \cdot x \cdot y) \cdot F(0,1,1,z) + \\ &(w \cdot x' \cdot y') \cdot F(1,0,0,z) + (w \cdot x' \cdot y) \cdot F(1,0,1,z) + \\ &(w \cdot x \cdot y') \cdot F(1,1,0,z) + (w \cdot x \cdot y) \cdot F(1,1,1,z) \end{aligned}$$

Write the formulas for each function below **(8 points)**.

$F(0,0,0,z) =$

$F(0,0,1,z) =$

$F(0,1,0,z) =$

$F(0,1,1,z) =$

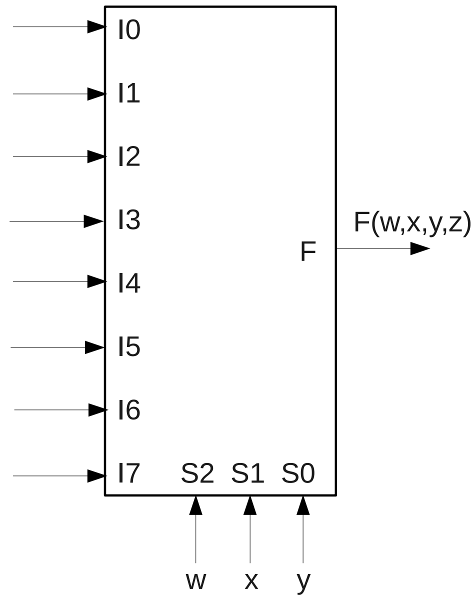
$F(1,0,0,z) =$

$F(1,0,1,z) =$

$F(1,1,0,z) =$

$F(1,1,1,z) =$

C. What are the inputs to the multiplexer shown below that implements F(w,x,y,z)? **(8 points)**



5 Full Subtractor Design (22 points)

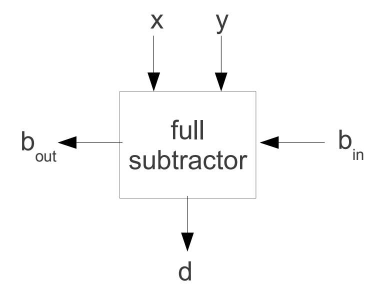


Figure 3: Full Subtractor

A full subtractor has three inputs x, y, and b_{in}, and two outputs b_{out} and d. The behavior of the full subtractor is described as follows:

$$x_{10} - y_{10} - b_{in10} = -2 \times b_{out} + d.$$

In other words, the value of $x - y - b_{in}$ is represented as a 2-bit 2's-complement number $b_{out}d$.

A. Given the informal behavioral description of a full subtractor, fill in the following table **(14 points)**.

x	y	b_{in}	Base 10 value	b_{out}	d
0	0	0			
0	0	1			
0	1	0			
0	1	1	-2	1	0
1	0	0			
1	0	1			
1	1	0			
1	1	1			

B. Implement the full subtractor using two 4:1 multiplexers as shown below (**8 Points:**).