are modified only in that the sums of errors (or fractional errors) are replaced by quadratic sums. Further, the old expressions (3.4) and (3.8) will be proven to be upper bounds that always hold whether or not the uncertainties are independent and random. Thus, the final versions of our two main rules are as follows:

Uncertainty in Sums and Differences

Suppose that $x, \ldots, w$ are measured with uncertainties $\delta x, \ldots, \delta w$ and the measured values used to compute

$$q = x + \cdots + z - (u + \cdots + w).$$

If the uncertainties in $x, \ldots, w$ are known to be *independent and random,* then the uncertainty in q is the quadratic sum

$$\delta q = \sqrt{(\delta x)^2 + \cdots + (\delta z)^2 + (\delta u)^2 + \cdots + (\delta w)^2} \tag{3.16}$$

of the original uncertainties. In any case, δq is never larger than their ordinary sum,

$$\delta q \leqslant \delta x + \cdots + \delta z + \delta u + \cdots + \delta w. \tag{3.17}$$

and

Uncertainties in Products and Quotients

Suppose that $x, \ldots, w$ are measured with uncertainties $\delta x, \ldots, \delta w$, and the measured values are used to compute

$$q = \frac{x \times \cdots \times z}{u \times \cdots \times w}.$$

If the uncertainties in $x, \ldots, w$ are *independent and random,* then the fractional uncertainty in q is the sum in quadrature of the original fractional uncertainties,

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2}. \tag{3.18}$$

In any case, it is never larger than their ordinary sum,

$$\frac{\delta q}{|q|} \leqslant \frac{\delta x}{|x|} + \cdots + \frac{\delta z}{|z|} + \frac{\delta u}{|u|} + \cdots + \frac{\delta w}{|w|}. \tag{3.19}$$

Notice that I have not yet justified the use of addition in quadrature for independent random uncertainties. I have argued only that when the various uncertainties

$$f = \left(\frac{R_p}{R_*}\right)^2$$

$$K = v_* \sin i = \left(\frac{M_p}{M_*}\right)\sqrt{\frac{GM_*}{a}} \sin i$$

$$R_p = \sqrt{f} \cdot R_*$$

$f = 0.0247 \pm .000554$

$R_* = 0.768 \pm .025\ R_{sun}$

$$R_p = \left((|f|)^{1/2} \pm \tfrac{1}{2}(\delta f)\right)\left(|R_*| \pm \partial R_*\right)$$

$= 0.121 \pm 0.00394\ R_{sun}$

$= 13.198 \pm 0.4298\ R_\oplus$

$$M_p = K\sqrt{\frac{aM_*}{G}} \cdot \frac{1}{\sin i}$$

$K = 200.7999 \pm 6.63\ m/s$

$a = 0.03115 \pm .000493\ AU$

$M_* = 0.8187 \pm 0.0385\ M_{sun}$

$i = 85.743$

$au = 1.496 \times 10^{11}\ m$

$M_{sun} = 1.989 \times 10^{30}\ kg$

$G = 6.67 \times 10^{-11}\ m^3 kg^{-1} s^{-2} = 3.962 \times 10^{-14}\ au^3 M_{sun}^{-1} s^{-2}$

$$aM^* = 0.0255 \pm .00127$$

$$\frac{aM^*}{G} = 6.436 \times 10^{11} \pm 3.205 \times 10^{10}$$

$$\left(\frac{aM^*}{G}\right)^{1/2} = 802255.78 \pm 79910$$

$$K\left(\frac{aM_*}{G}\right)^{1/2} = 0.00108 \pm .0001076$$

$$\frac{\text{"} \quad \text{"}}{\sin i} = 1.08 \times 10^{-3} \pm 1.079 \times 10^{-4} \; M_{sun}$$

$$= 359.672 \pm 35.934 \; M_\oplus$$

$$D = \frac{M}{\frac{4}{3}\pi R^3}$$

$$R^3 = 2298.923 \pm 224.597$$

$$\frac{4}{3}\pi R^3 = 9629.7 \pm 940.79$$

$$\frac{M}{\frac{4}{3}\pi R^3} = 0.051 \pm .00713 \quad M_\oplus / R_\oplus^3$$