Measuring Stellar Elemental Abundances

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Introduction

One way astronomers are able to decipher whether an exoplanet can sustain life is by measuring the spectra of elements from the planet. Computing this directly is very difficult due to the size, distance, and luminosity of exoplanets. However, since planets and their host stars are made of similar materials, we can measure the spectra of stars to understand their satellite planets' compositions better. This is done through the use of Spectroscopy. Every element has its own unique "fingerprint". We can match these "fingerprints" to the spectra of a star in order to determine its composition. In this project, we will look at the Sun's spectra to measure and analyze its elemental abundances using the curve of growth method.

Methods

We will first look at the solar sodium (Na) doublet lines in the Sun's spectrum as well as the curve of growth graph, which is the relationship between the intensity of the absorption line and the number of atoms. The spectral signature of sodium consists of two lines at 5890 and 5896 angstroms.

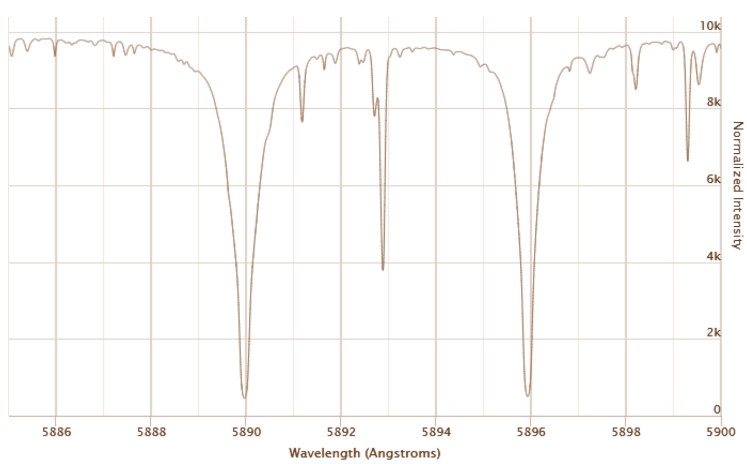

Figure 1: The sodium doublet lines of the Sun's spectra. There are two clear sodium absorption lines at 5890 and 5896 angstroms.

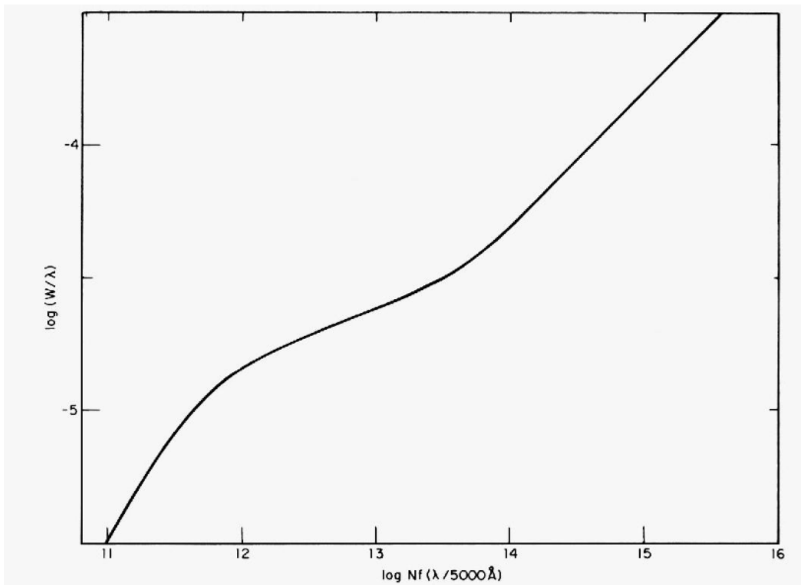

Figure 2: The curve of growth plot.

In order to measure the number density of sodium atoms in the ground state, we must first measure the equivalent width of the sodium absorption lines. Then, using,

$$log\ (W\ /\ \lambda),$$

where *W* is the equivalent width and λ is the wavelength, we can identify the corresponding value on the x-axis of the curve of growth. We will then use this value along with,

$$log\ NF\ (\lambda\ /\ 5000\ \text{Å}),$$

where *F* is the oscillator strength of the transition, to calculate N, the number density of sodium atoms in the ground state.

Next, we want to estimate the number of sodium atoms in the ground state to sodium atoms in the excited state using the Boltzmann Equation,

$$\frac{N_b}{N_a} = (\frac{g_b}{g_a})(e^{-(E_b - E_a)/kT})$$

where N_a is the number density of atoms in the ground state, N_b is the number density of atoms in the excited state, g is the number of separate, individual states that are degenerate in energy, E is the energy at the state, k is the Boltzmann constant, and T is the temperature.

We will then estimate the ratio of neutral sodium atoms to ionized sodium atoms using the Saha equation,

$$\frac{N_{II}}{N_I} = \frac{2Z_{II}}{n_e Z_I}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi/kT}$$

where N_I is the number density of neutral atoms ($N_a + N_b$), N_{II} is the number density of ionized atoms, m_e is electron mass, Z is the partition function, n_e is the number of electrons, h is Plank's constant, and x is the ionization energy.

Results

Using the two ratios above, we can calculate the total number density of sodium atoms where the total number of sodium atoms (N) is $N_I + N_{II}$ and $N_I = N_a + N_b$. We are then able to calculate the sodium abundance relative to hydrogen in the Sun's photosphere. The number density of hydrogen atoms is $6.6 \times 10^{23}\ m^{-2}$. We have expressed this relative abundance in Galactic Astronomer's terms, Physicist's terms, and Stellar Astronomer's terms.

Galactic Astronomer	Physicist: Mole Ratio	Stellar Astronomer
$12 + log_{10}\left(\frac{Na}{H}\right)$	$\frac{N_{Na}}{N_H}$	$\left[\frac{Na}{H}\right] = \log\left(\frac{N_{Na}/N_H}{N_{Na\ sun}/N_{H\ sun}}\right)$
6.51	3.24×10^{-6}	0.21

Table 1: This shows each method of measuring the relative abundance along with the calculated values. *N* is the log abundance and Na and H are the total number densities.

Additionally, we can repeat these calculations to derive the chemical abundance of iron (Fe) relative to hydrogen (H) in the Sun using iron's absorption spectra.

Galactic Astronomer	Physicist: Mole Ratio	Stellar Astronomer
$12 + log_{10}\left(\frac{Fe}{H}\right)$	$\frac{Fe}{H}$	$\left[\frac{Fe}{H}\right] = \log\left(\frac{Fe/H}{Fe_{sun}/H_{sun}}\right)$
7.17	1.50×10^{-5}	-0.31

Table 2: This shows the relative iron (Fe) abundance to hydrogen (H) expressed in Galactic Astronomer's, Physicist's, and Stellar Astronomer's terms.

However, one major source of error in our iron abundance calculations was the constants used in the Saha and Boltzmann equations. These constants, such as the number of separate, individual states that are degenerate in energy depend on the type of atom, and would therefore differ between iron and sodium. However, iron's atomic structure is very complex and per instructions, we just used the sodium constant values. This is probably the source of the discrepancy between our calculated abundance and the observed abundance.

Conclusion

Although we used the Sun's spectra to derive our abundances, we found that our calculated logarithmic abundances are slightly off compared to the reported values. Our calculated logarithmic sodium abundance is *6.51*, while the reported value is *6.30*. We find our logarithmic sodium abundance to be greater than the reported value by *0.21*. Also, our calculated logarithmic iron abundance is *7.17*, while the reported value is *7.48*. We find our logarithmic iron abundance to be less than the reported value by *0.31*. We believe that using the same constants for the iron calculations as we did with sodium caused our discrepancy in the calculated logarithmic iron abundance. As for the discrepancy with sodium, we are not exactly sure where our error lies. It may be due to an incorrect measurement of the equivalent width. We would need to repeat and revise our procedure to better identify the problem.

References

"Atomic Spectra Database." *NIST*, 21 Oct. 2022, https://www.nist.gov/pml/atomic-spectra-database