Boltzmann

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-\frac{E_2 - E_1}{KT}} = \frac{4}{2} e^{-\left(\frac{(-3.04)-(-5.14)\,eV}{8.62\times10^{-5}\frac{eV}{K}\cdot 5780\,K}\right)}$$

$$= 0.03$$

$$g = 2J + 1$$

$$g_2 = 2\left(\tfrac{3}{2}\right) + 1 = 4 \qquad E_2 = -3.04\ eV$$

$$g_1 = 2\left(\tfrac{1}{2}\right) + 1 = 2 \qquad E_1 = -5.14\ eV$$

Saha

$$\frac{N_{aII}}{N_{aI}} = \frac{2\cdot KT}{1\,N\cdot m^{-2}} \cdot \frac{1}{2.4} \cdot \left(\frac{2\pi\, m_e\, KT}{h^2}\right)^{\frac{3}{2}} \cdot e^{\frac{-5.1}{KT}} =$$

$$\frac{2(1.38\times10^{-23}\,J\cdot K^{-1})(5780\,K)}{1\,N\cdot m^{-2}} \cdot \frac{1}{2.4} \cdot \left(\frac{2\pi(9.11\times10^{-31}\,kg)(1.38\times10^{-23}\,J\cdot K^{-1})}{(6.63\times10^{-34}\,kg\cdot m^2\cdot s^{-1})^2}\right)^{\frac{3}{2}} \cdot e^{\frac{-5.1\,eV}{(8.62\times10^{-5}\,eV\cdot K^{-1})(5780\,K)}}$$

$$= 2522.6410$$

Unit analysis

$$\frac{m^2\cdot kg\cdot s^{-2}}{kg\cdot m\cdot s^{-2}\cdot m^{-2}} \cdot \left(\frac{kg\cdot m^2\cdot kg\cdot s^{-2}}{kg^2\cdot m^4\cdot s^{-2}}\right)^{\frac{3}{2}} \Rightarrow m^3\cdot\left(m^{-2}\right)^{\frac{3}{2}} = 1$$

$$[KT] = m^2\cdot kg\cdot s^{-2}$$

$$[N] = kg\cdot m\cdot s^{-2}$$

$$[m_e] = kg$$

$$[h]^2 = kg^2\cdot m^4\cdot s^{-2}$$

$$N = N_{aI} + N_{aII}$$

$$N_1 = 8.24 \times 10^{14} \ cm^{-2}$$

$$N_2 = 0.03 \cdot N_1 = 0.03(8.24 \times 10^{14} \ cm^{-2}) = 2.47 \times 10^{13} \ cm^{-2}$$

$$N_{aI} = N_1 + N_2 = 8.49 \times 10^{14} \ cm^{-2}$$

$$N_{aII} = 2522.6410 \cdot N_{aI} = 2.142 \times 10^{18} \ cm^{-2}$$

$$N_{Total} = N_{aI} + N_{aII} = 2.143 \times 10^{18} \ cm^{-2}$$

Galactic astronomer version

Solar sodium log abundance : 6.30

Compare to $12 + \log_{10}\left(\frac{N(Na)}{N(H)}\right) = ?$

N(H) given : $6.6 \times 10^{23} \ cm^{-2}$

$$12 + \log_{10}\left(\frac{2.143 \times 10^{18} \ cm^{-2}}{6.6 \times 10^{23} \ cm^{-2}}\right) = 6.51$$

Physicist mole ratio version

$$N_{Na}/N_H = 10^{6.51-12} = 3.24 \times 10^{-6}$$

Stellar astronomer version

Solar sodium log abundance : 6.30

$$N_{Na}/N_H = 10^{(6.51-12)}$$

$$(N_{Na}/N_H)_\odot = 10^{(6.3-12)}$$

$$\log\left(\frac{N_{Na}/N_H}{(N_{Na}/N_H)_\odot}\right) = 0.21$$