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#### ${\rm FIZ437E\text{-}HW2\text{-}Theoric}$

## 0.1 Part 1

$$\begin{array}{ll} (1.1) & V_t = \alpha \mathbf{V}_{t-1} + \epsilon \mathbf{g}_t \\ (1.2) & \triangle \theta_t = -\mathbf{V}_t \end{array}$$

(1.2) 
$$\triangle \theta_t = -V_t$$

(2.1) 
$$V_t = \beta V_{t-1} + (1 - \beta)g_t$$

$$(2.2) \quad \triangle \theta_t = \delta \mathbf{V}_t$$

$$\Delta \theta_{t-1} = -V_{t-1} \qquad (t \to t-1) \qquad (from 1.2)$$

$$\begin{aligned} \mathbf{V}_t &= -\alpha \triangle \theta_{t-1} + \epsilon \mathbf{g}_t \\ \mathbf{V}_{t-1} &= -\alpha \triangle \theta_{t-2} + \epsilon \mathbf{g}_{t-1} \end{aligned} \qquad (from \ 1.1)$$

(I) 
$$-\triangle \theta_{t-1} = -\alpha \triangle \theta_{t-2} + \epsilon g_{t-1}$$
 (V<sub>t-1</sub>  $\rightarrow$   $-\triangle \theta_{t-1}$ )

$$\triangle \theta_{t-1} = \delta V_{t-1}$$
  $(t \to t-1)$   $(from 2.2)$ 

$$V_{t-1} = \frac{1}{\delta} \triangle \theta_{t-1}$$

$$V_t = \beta \frac{1}{\delta} \triangle \theta_{t-1} + (1 - \beta)g_t \qquad (from 2.1)$$

$$V_t = \frac{\beta}{\delta} \triangle \theta_{t-1} + (1 - \beta)g_t$$

$$V_{t-1} = \frac{\beta}{\delta} \triangle \theta_{t-2} + (1 - \beta)g_{t-1} \quad (t \to t - 1)$$

$$(II)$$
  $\frac{1}{\delta}\triangle\theta_{t-1} = \frac{\beta}{\delta}\triangle\theta_{t-2} + (1-\beta)g_{t-1}$ 

$$I: \triangle \theta_{t-1} = \alpha \triangle \theta_{t-2} - \epsilon g_{t-1}$$

$$II: \frac{1}{\delta} \triangle \theta_{t-1} = \frac{\beta}{\delta} \triangle \theta_{t-2} + (1-\beta)g_{t-1}$$
$$\triangle \theta_{t-1} = \beta \triangle \theta_{t-2} + \delta(1-\beta)g_{t-1}$$

$$\alpha \triangle \theta_{t-2} - \epsilon g_{t-1} = \beta \triangle \theta_{t-2} + \delta (1 - \beta) g_{t-1}$$

$$\alpha = \beta \ and - \epsilon = \delta(1 - \beta) \rightarrow \epsilon = \delta(\beta - 1)$$