Notes and Results for Chapter 2 Problems

Benjamin Klimko

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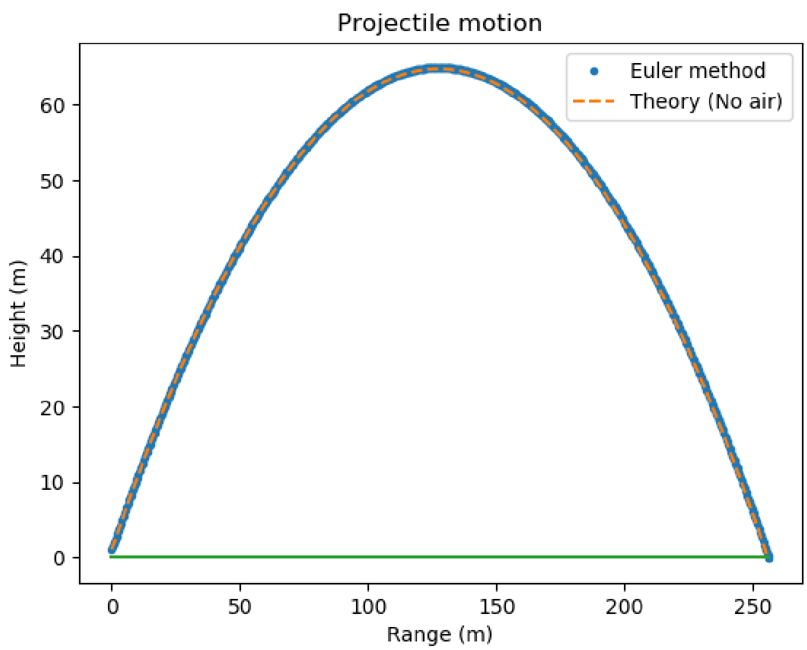
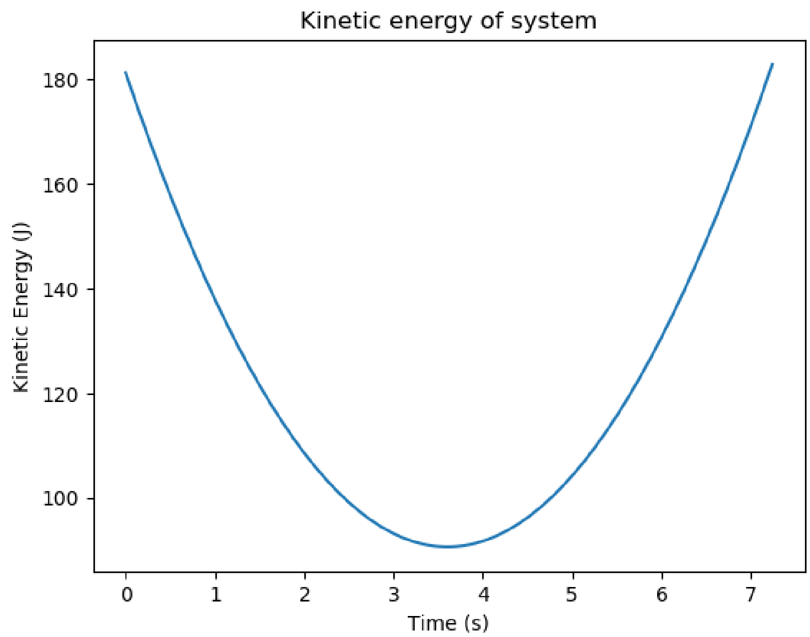
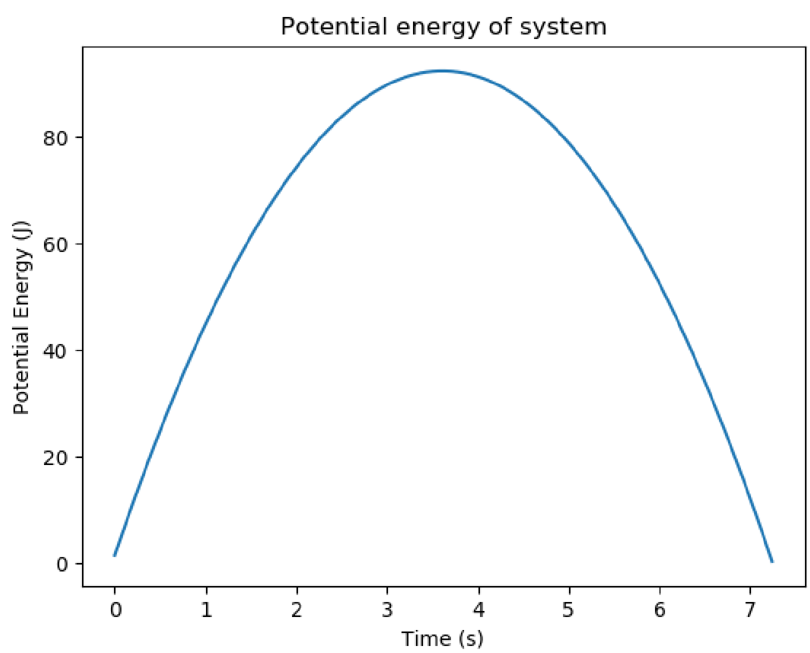
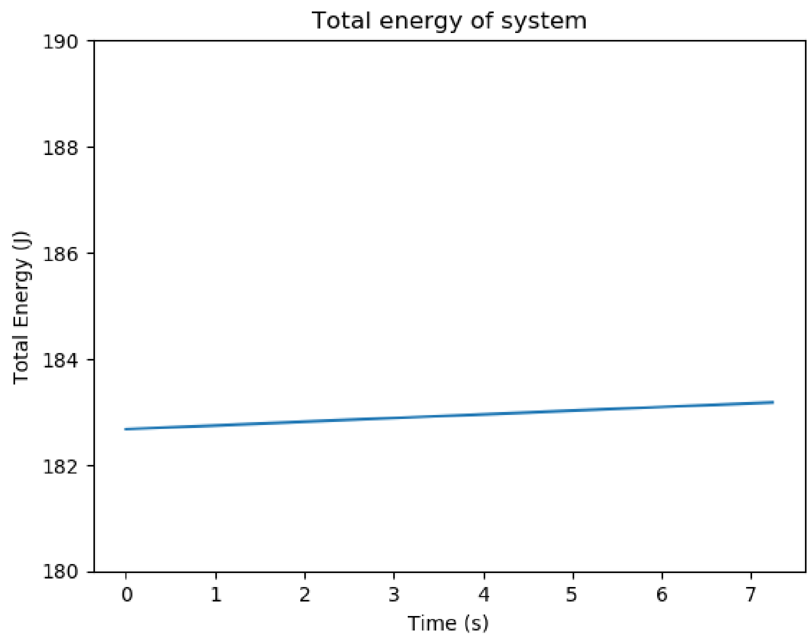
1 Problem 3

After trying several values of the time step τ I determined that 0.01 was optimal for both its speed and accuracy– the program executes very quickly while providing results for maximum range within .14% of the analytic solution and time of flight within .0006%. Below are graphs of the plotted projectile motion, kinetic energy, potential energy, and total energy of the system along with a sample console output.




Kinetic energy of system
Kinetic Energy (J)
180
160
140
120
100
0
1
2
3
4
5
6
7
Time (s)


Potential energy of system
Potential Energy (J)
80
60
40
20
0
0
1
2
3
4
5
6
7
Time (s)

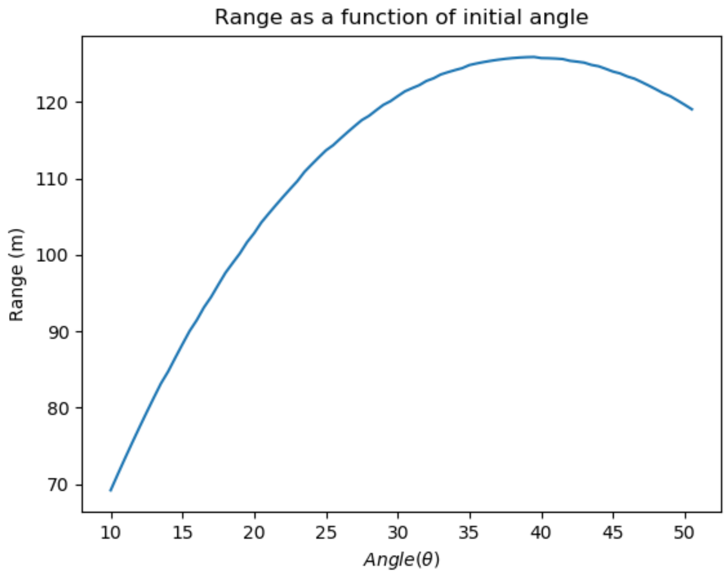



```
Enter initial height (meters): 1
Enter initial speed (m/s): 50
Enter initial angle (degrees): 45
Air resistance? (Yes:1, No:0):0
Enter timestep, tau (sec): .01
Maximum range is  256.19028901671834  meters
Time of flight is  7.236155625514474  seconds

Process finished with exit code 0
```

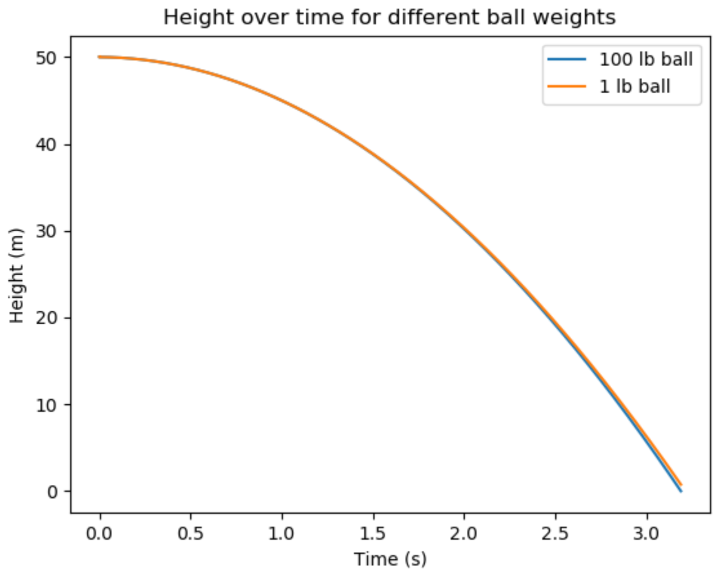
2 Problem 5

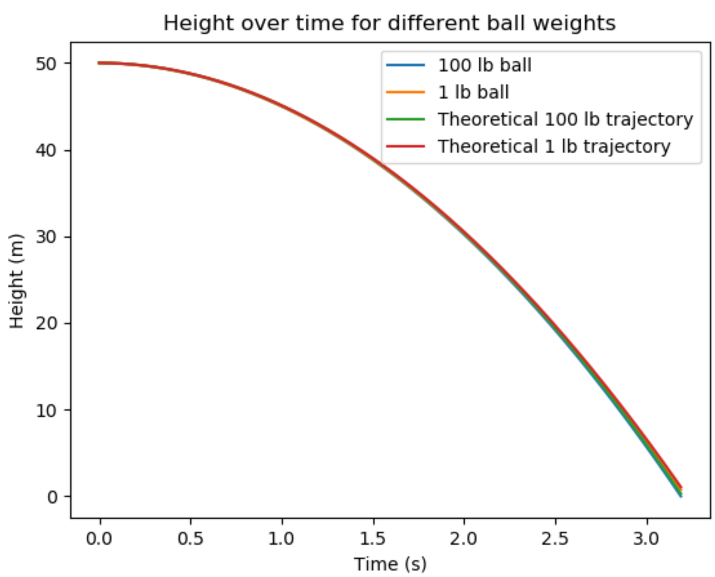
The maximum range is achieved at 39.5°.



3 Problem 6

Quite sadly for Galileo, when the larger ball hits the ground the lighter ball is more than two inches behind it (according to the program's output, it is nearly 2.5 feet behind). Below are plots of height vs time and the same with theoretically computed trajectories.

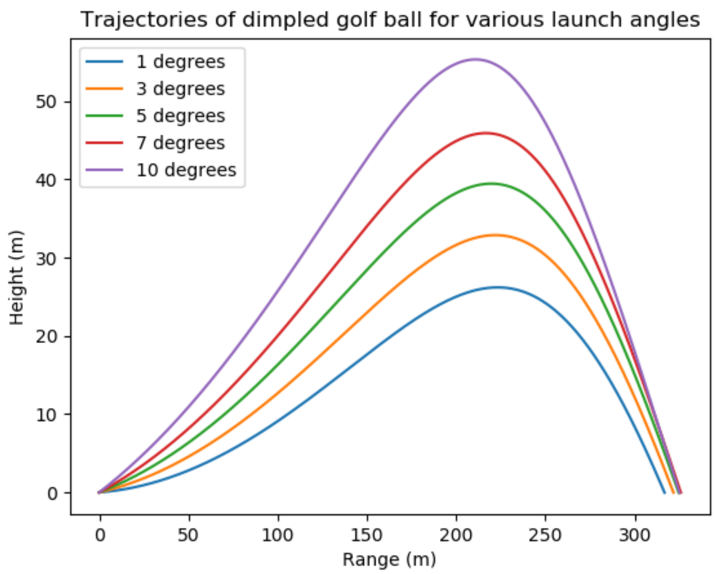


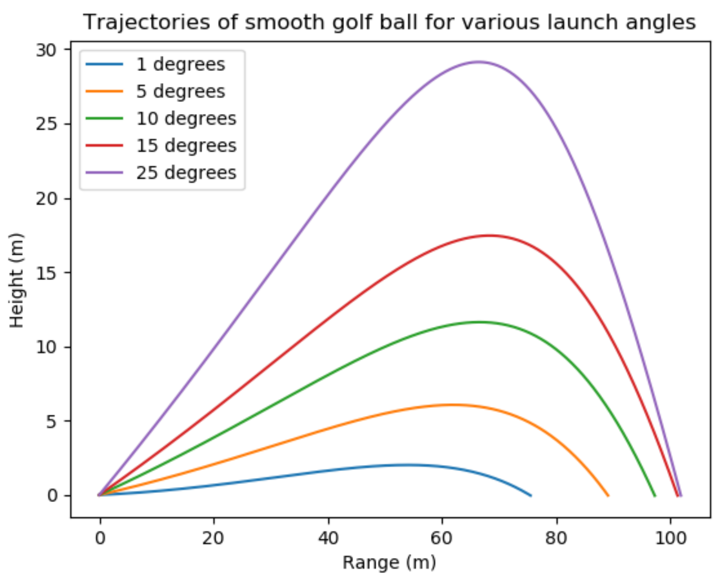


As can be seen in the above figure, the theoretically computed trajectories overlay the numerically calculated height vs time plot, verifying the accuracy of the solution.
C_d would need to be roughly 0.03 for Galileo to be correct in his assertion.

4 Problem A

The optimal angle for achieving maximum range for a dimpled ball is 7.35° while for a smooth ball it is 20.2°.





5 Problem 15

The following is showing that the energy E of a pendulum (in the small angle approximation $E = \frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 - mgL$)increases monotonically with time when the Euler method is used to compute the motion.

$$\theta_{n+1} = \theta_n + \tau\omega_n$$

$$\omega_{n+1} = \omega_n - \frac{g}{L}\tau \sin\theta_n \text{ but for small angles } \sin\theta \approx \theta \text{ so } \omega_{n+1} \approx \omega_n - \frac{g}{L}\tau\theta_n$$

$$E_{n+1} = \frac{1}{2}mL^2\omega_{n+1}^2 + \frac{1}{2}mgL\theta_{n+1}^2 - mgL$$

$$= \frac{1}{2}mL^2(\omega_n - \frac{g}{L}\tau\theta_n)^2 + \frac{1}{2}mgL(\theta_n + \tau\omega_n)^2 - mgL$$

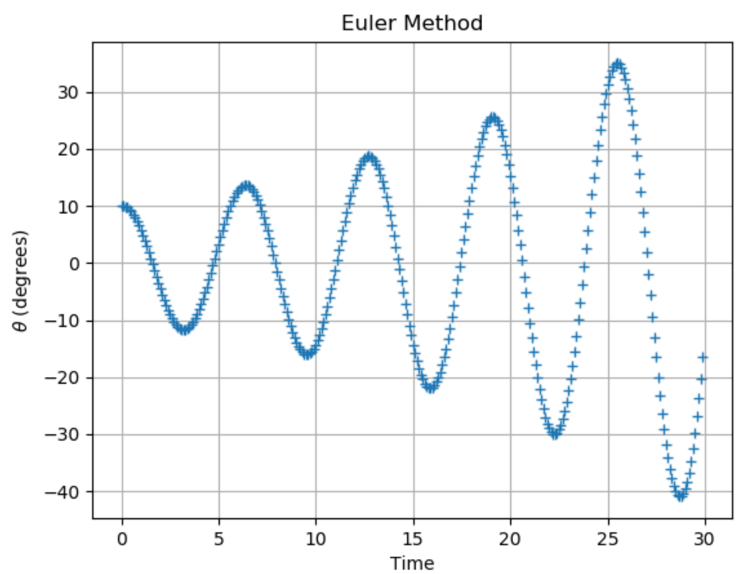
$$= \frac{mL^2\omega_n^2}{2} - mgL\tau\theta_n\omega_n + \frac{mg^2\tau^2\theta_n}{2} + \frac{mgL\theta_n^2}{2} + mgL\tau\theta_n\omega_n + \frac{mgL\tau\omega_n^2}{2} - mgL$$

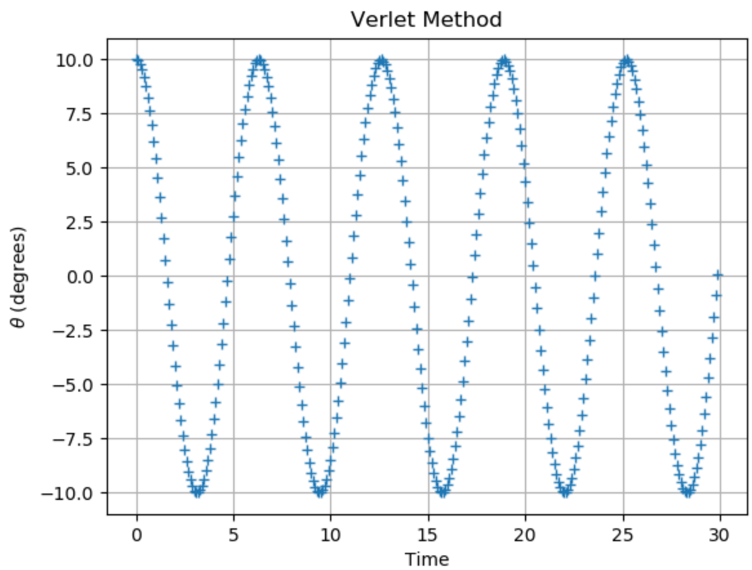
$$= E_n + \frac{mg\tau^2}{2}\left(g\theta_n + L\omega_n^2\right)$$

The second term $\frac{mg\tau^2}{2}\left(g\theta_n + L\omega_n^2\right)$ is always positive so the energy will increase monotonically with time.

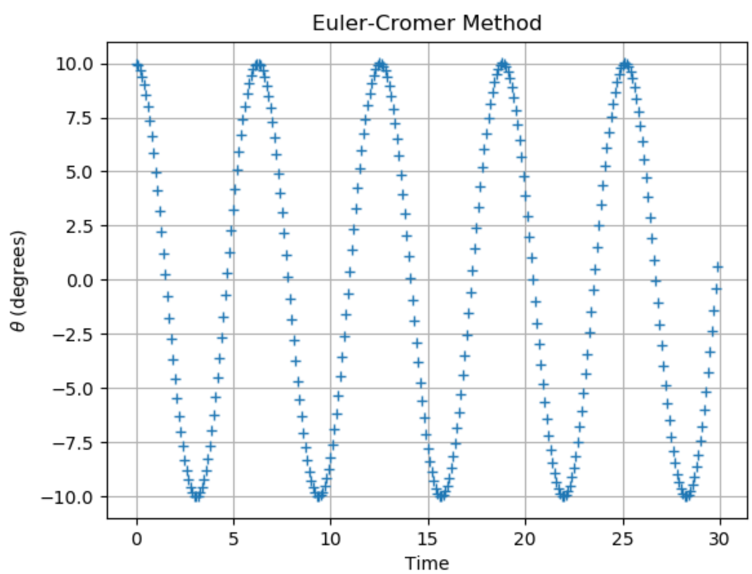
6 Problem 16

Sample plots for $\theta_0 = 10°$, $\tau = 0.1$, and 300 steps:

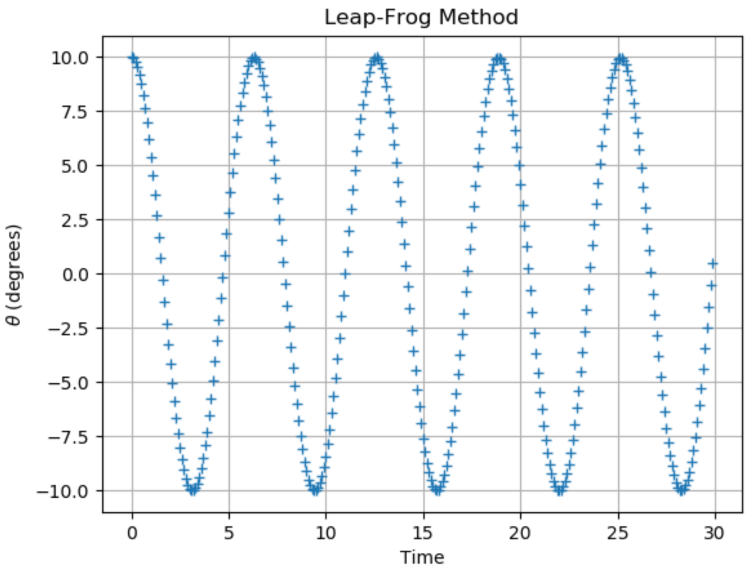


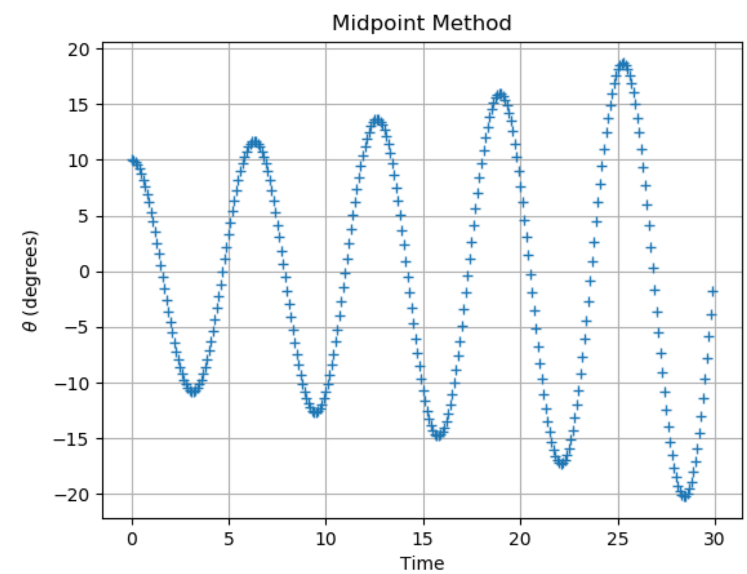


Verlet Method
θ (degrees)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
5
10
15
20
25
30
Time

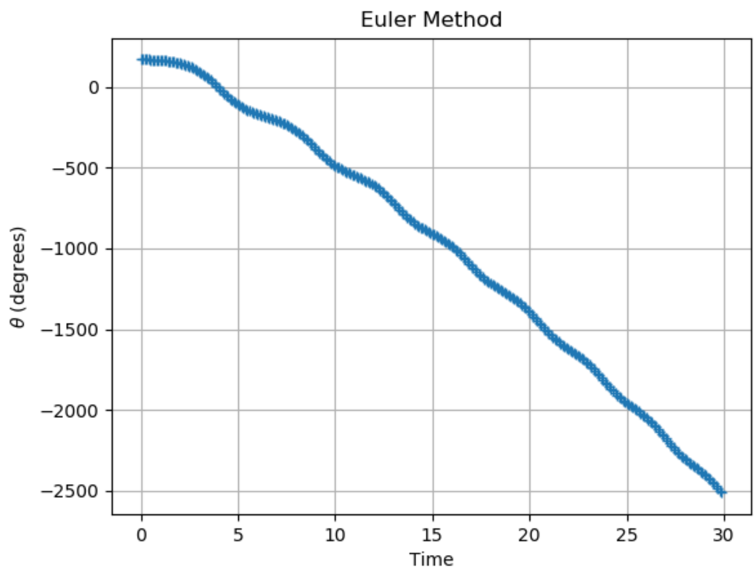


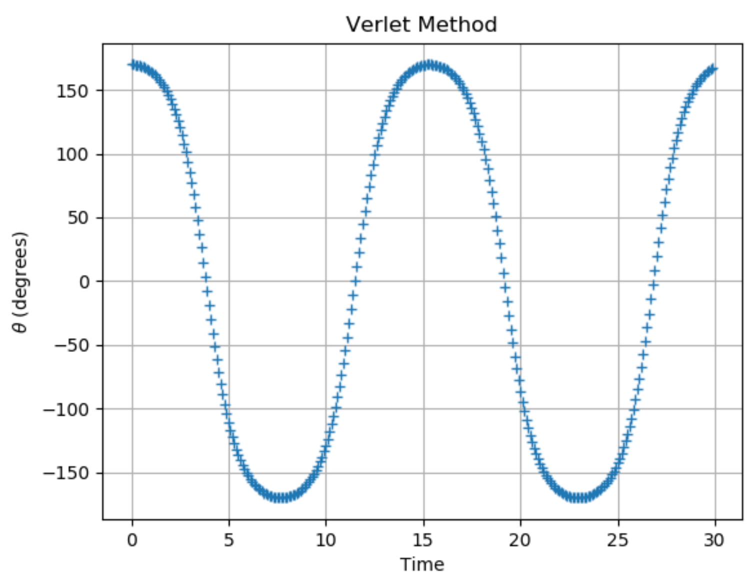
Euler-Cromer Method
θ (degrees)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
0
5
10
15
20
25
30
Time

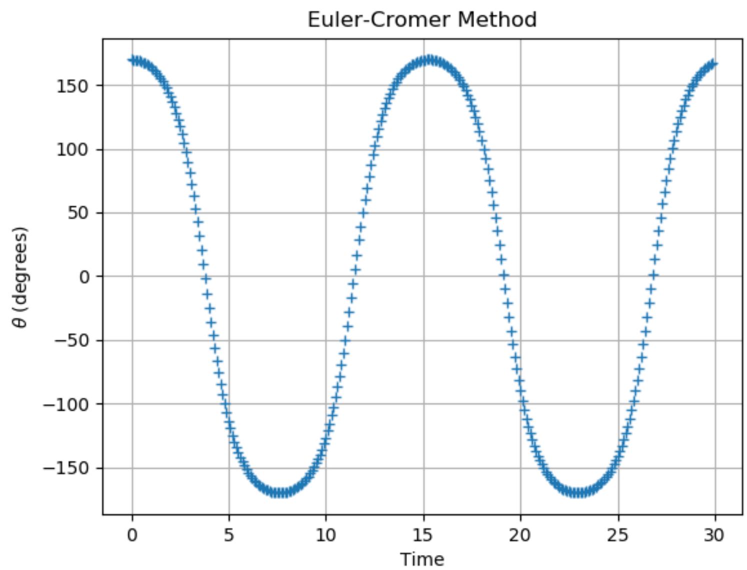


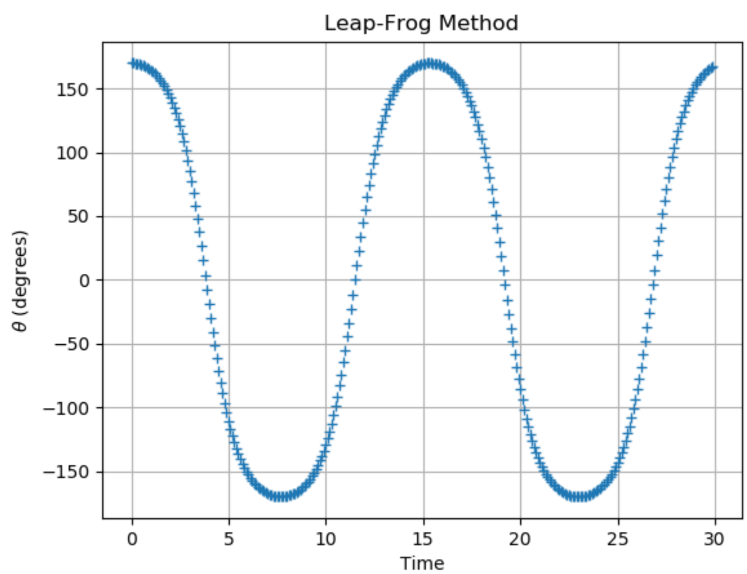


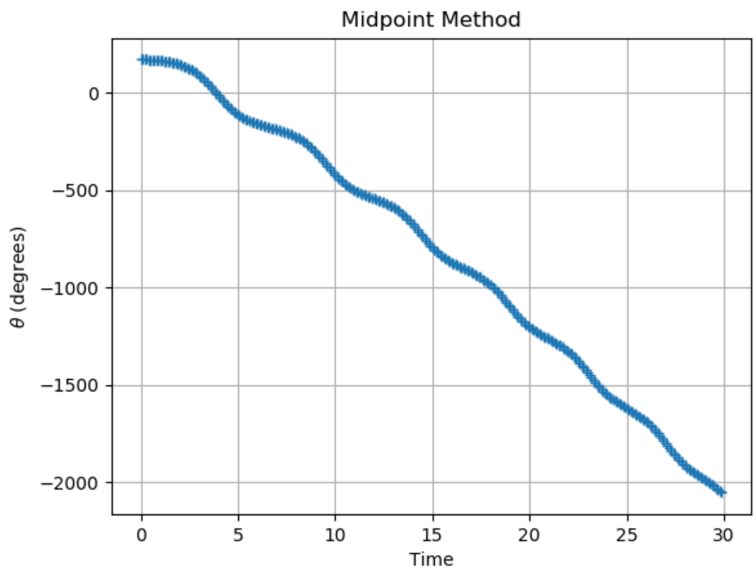
Sample plots for $\theta_0 = 170^\circ$, $\tau = 0.1$, and 300 steps:


Euler Method
0
−500
−1000
−1500
−2000
−2500
θ (degrees)
0
5
10
15
20
25
30
Time


Verlet Method
150
100
50
0
−50
−100
−150
θ (degrees)
0
5
10
15
20
25
30
Time


Euler-Cromer Method
θ (degrees)
Time


Leap-Frog Method
θ (degrees)
Time



Notice that for both cases the midpoint method fails in a manner similar to the Euler method and the resulting graphs do not show conservation of energy.