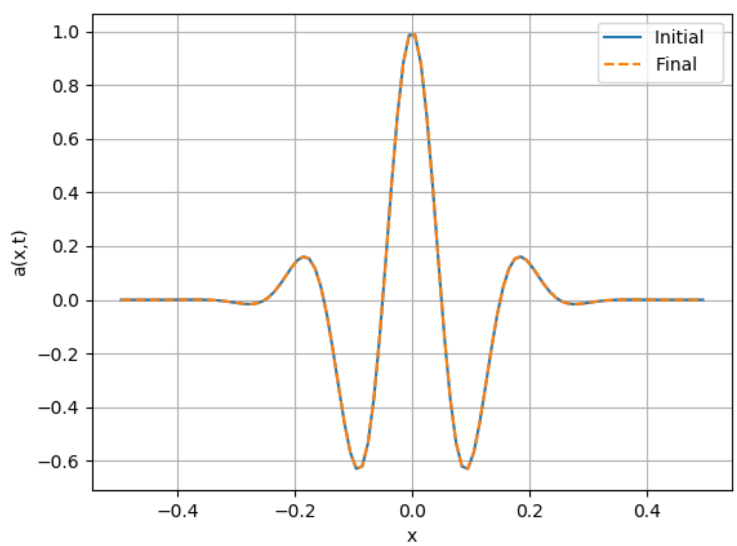
Notes and Results for Chapter 9

Benjamin Klimko

April 20, 2018

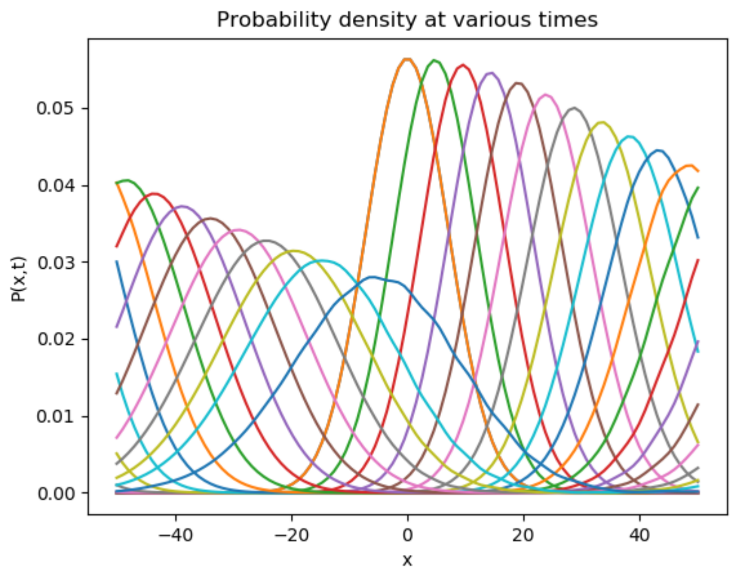
Problem A

See below for the output using the Crank-Nicolson method. The method can introduce dispersion into a system but in this case the output matches the original Gaussian almost exactly.

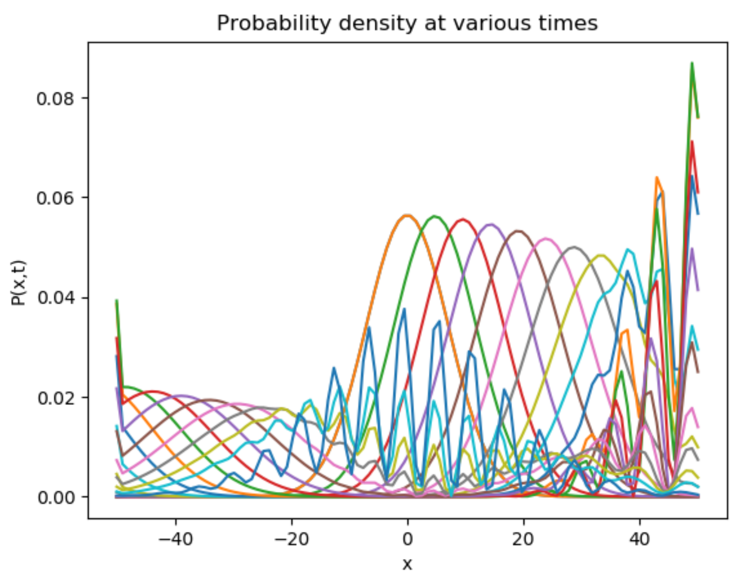


Problem 15

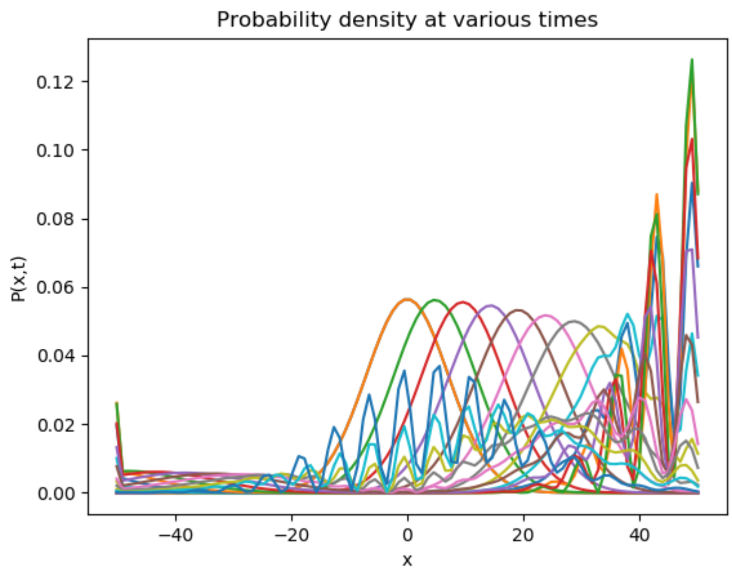
Below are runs of the system with a potential $V(x) = U\delta(x - L/2)$, where top to bottom U is 0.1, 1, and 2 times the energy of the particle. Note the increased probabilities near $L/2$ are increased due to reflections from the potential barrier for $U \geq E$.



Probability density at various times
P(x,t)
0.00
0.01
0.02
0.03
0.04
0.05
−40
−20
0
20
40
x

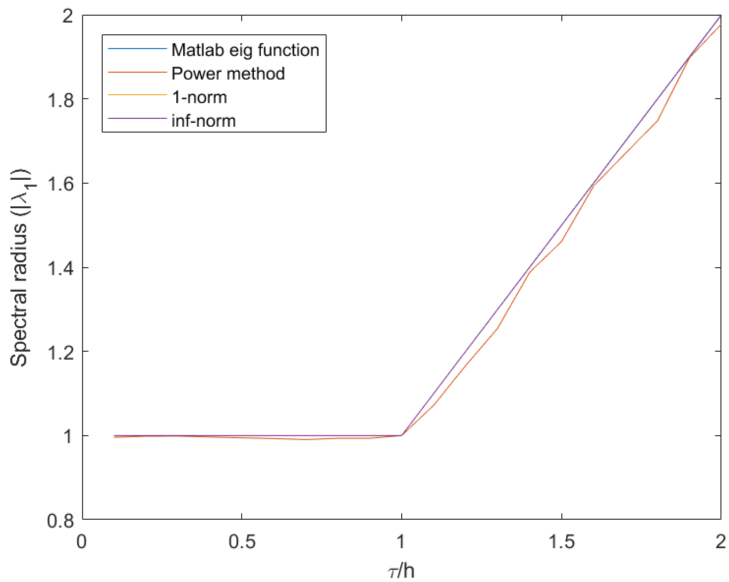


Probability density at various times
P(x,t)
0.00
0.02
0.04
0.06
0.08
−40
−20
0
20
40
x



Problem 9

According to the CFL condition, the Lax scheme is stable when $\tau < \frac{h}{c}$ or, given that $c = 1$, $\frac{\tau}{h} < 1$. The graph below shows that when τ/h grows larger than one the spectral radius becomes larger than one, showing that the scheme is unstable beyond that threshold. Note that the power method appears to slightly undershoot the values given by MATLAB's **eig** function, as well as the 1-norm and infinity-norm.



Matlab eig function
Power method
1-norm
inf-norm
Spectral radius (|λ_1|)
τ/h