Notes and Results for Chapter 4 Problems

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1 Problem 13

(a)

To create a matrix that has a unique solution (i.e. is not singular) I set the location of the first mass, x_0, to 0. This led to the force equations

$$\begin{aligned}
F_1 &= -k_1(x_1 - 1) + k_1(x_2 - x_1 - 1) + k_2(x_3 - x_1 - 1) \\
F_2 &= -k_1(x_2 - x_1 - 1) + k_1(x_3 - x_2 - 1) - k_2(x_2 - 1) \\
F_3 &= -k_1(x_3 - x_2 - 1) - k_2(x_3 - x_1 - 1)
\end{aligned}$$

where 1 is the rest length of all springs. From here I created the matrix equation

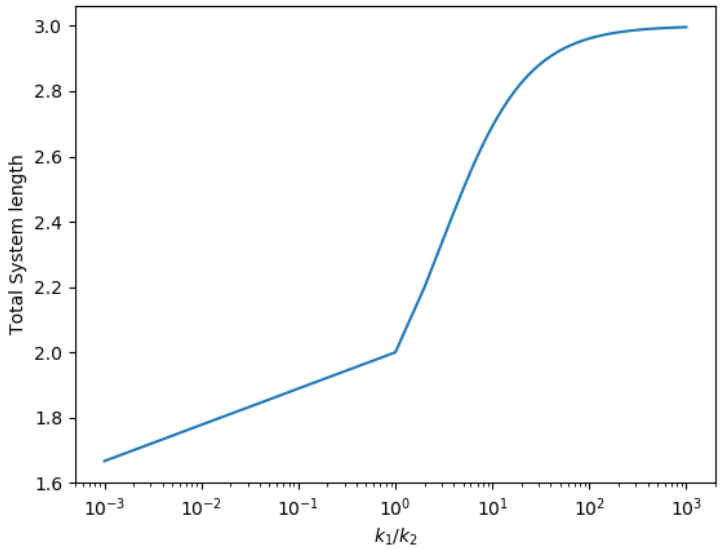
$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} -2k_1 - k_2 & k_1 & k_2 \\ k_1 & -2k_1 - k_2 & k_1 \\ k_2 & k_1 & -k_1 - k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} k_2 \\ -k_2 \\ -k_1 - k_2 \end{bmatrix}$$

To find the equilibrium positions of the blocks we set the force vector to $\vec{0}$, which gives

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2k_1 - k_2 & k_1 & k_2 \\ k_1 & -2k_1 - k_2 & k_1 \\ k_2 & k_1 & -k_1 - k_2 \end{bmatrix}^{-1} \begin{bmatrix} k_2 \\ -k_2 \\ -k_1 - k_2 \end{bmatrix}$$

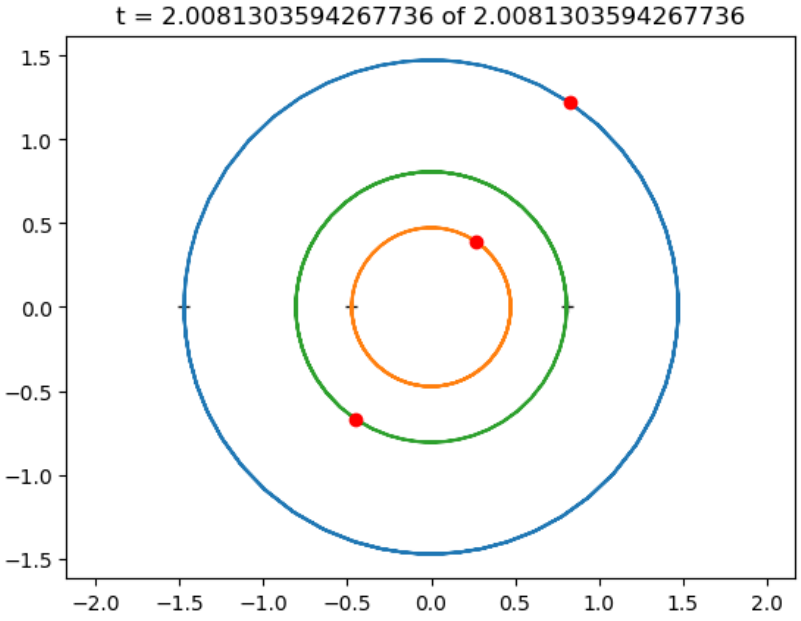
(b)

Plot of system length vs the ratio of parameters $\frac{k_1}{k_2}$:

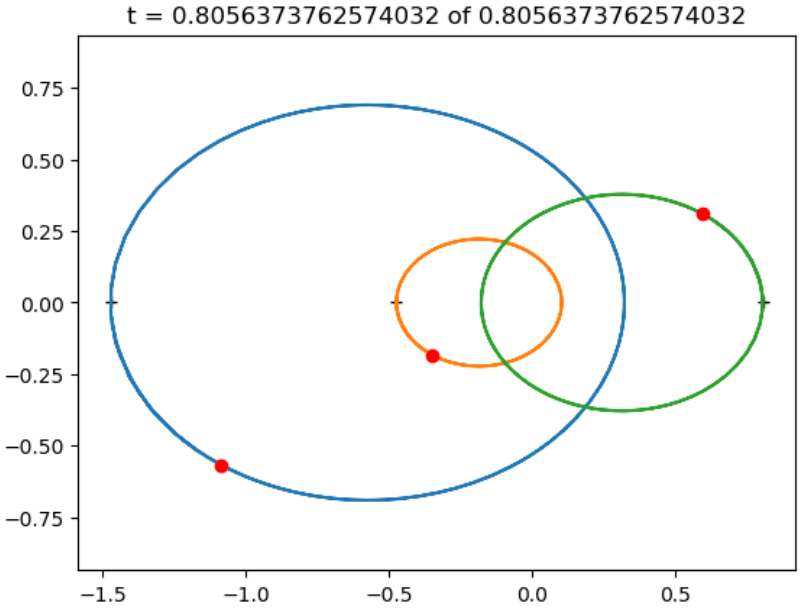


2 Problem A

Plot of elliptical orbits:



Plot of circular orbits:



3 Problem 24

Plot of the equilibrium position of the mass-spring system: