Notes and Results for Chapter 8 Problems

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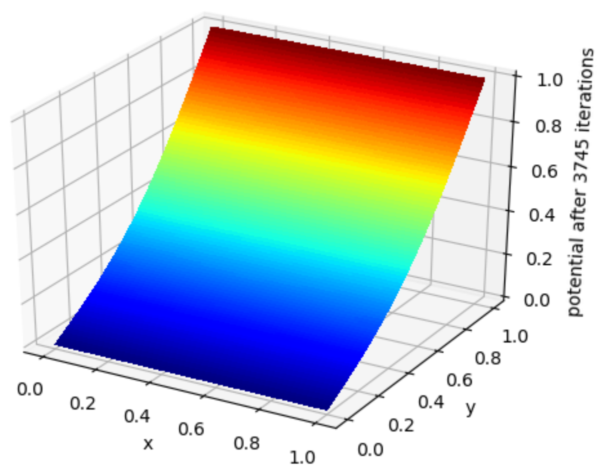
Problem A

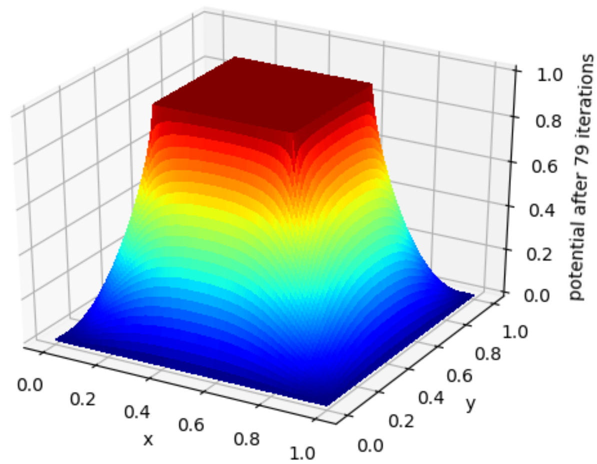
When Neumann boundary conditions are imposed $\left(\frac{\partial \phi}{\partial x} = 0 \,\text{at}\, x = 0 \,\text{and}\, x = 1\right)$ the following graph results.



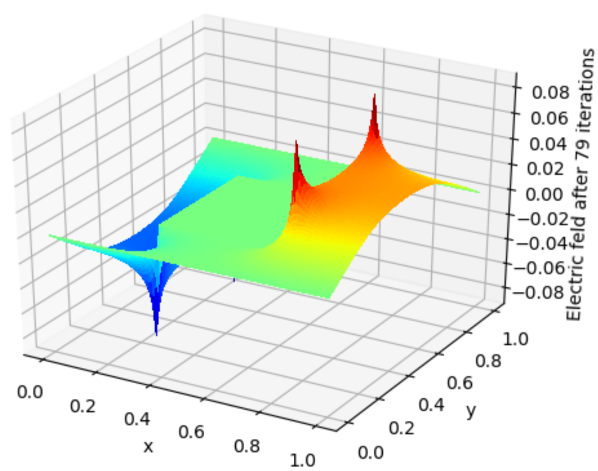
Problem B

a

The resulting potential is seen below:

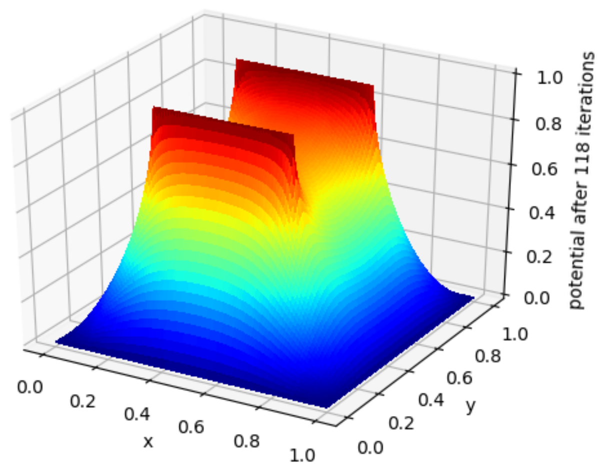


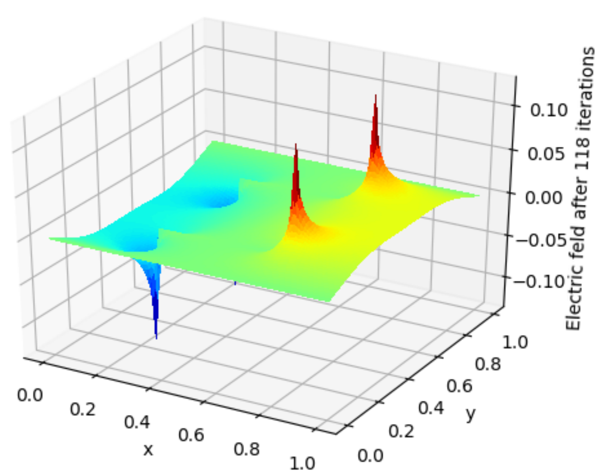
For a given potential V, the electric field is calculated by $\vec{E} = -\nabla V$, which can be seen below. This shows the greatest magnitude of the electric field occurs at the corners of the interior box.



b

Similarly to the last part, the electric field is calculated by $\vec{E} = -\nabla V$ and it can be seen that the largest magnitude of the electric field occurs at the end points of both capacitor plates.





Problem C

For the Laplacian in cylindrical coordinates

$$\nabla^2 \Phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2}$$

I created the discretization

$$\phi_{r,\varphi} = \frac{\frac{1}{r}\frac{\phi_{r+1,\varphi}-\phi_{r-1,\varphi}}{2h_r} + \frac{\phi_{r+1,\varphi}+\phi_{r-1,\varphi}}{h_r^2} + \frac{1}{r^2}\frac{\phi_{r,\varphi+1}+\phi_{r,\varphi-1}}{h_\varphi^2}}{\frac{2}{h_r^2}+\frac{2}{r^2 h_\varphi^2}}$$

where h_r and h_φ are the radius and angle spacing, respectively, r is the radius, and
$phi_{r,\varphi}$ is the potential at radius r and angle φ.
Unfortunately in the given time I was unable to make my program able to properly solve Laplace's equation due to working on the project and multiple tests in other courses within a few days. I did plot the analytic solution, which is seen below in Cartesian coordinates.