Notes and Results for Chapter 3b Problems

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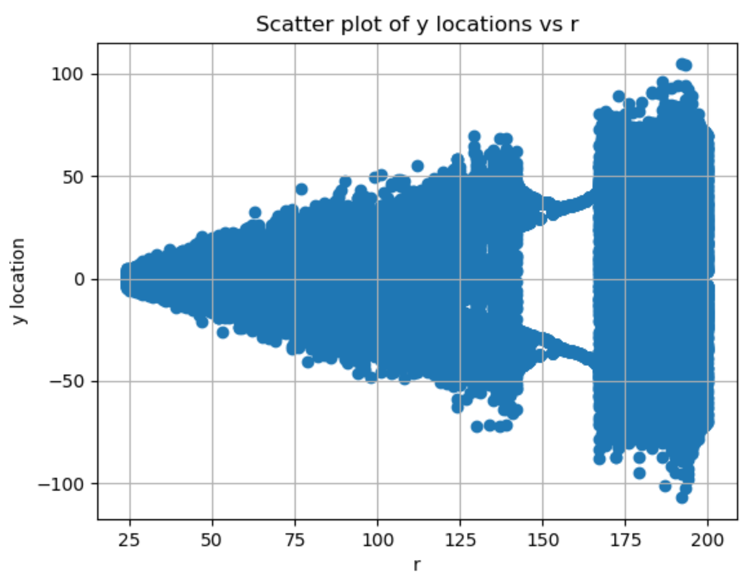
1 Lorenz Model

Bifurcation diagram, created by scatter plot of y locations when the trajectory crosses the x=0 plane versus parameter r:



2 Problem 22

2.1 a

The various parameters in the Lotka-Volterra model have physical significance: a represents the reproduction and mortality rate of prey and predators, respectively; b represents the carrying capacity of the environment for the prey (e.g. availability of food); and c represents predation rate.

To obtain the steady state solutions for the rabbit and fox populations, we set each equation equal to 0 and solve for r and f:

$$0 = a(1 - \frac{r}{b})r - crf$$
$$0 = -af + crf$$

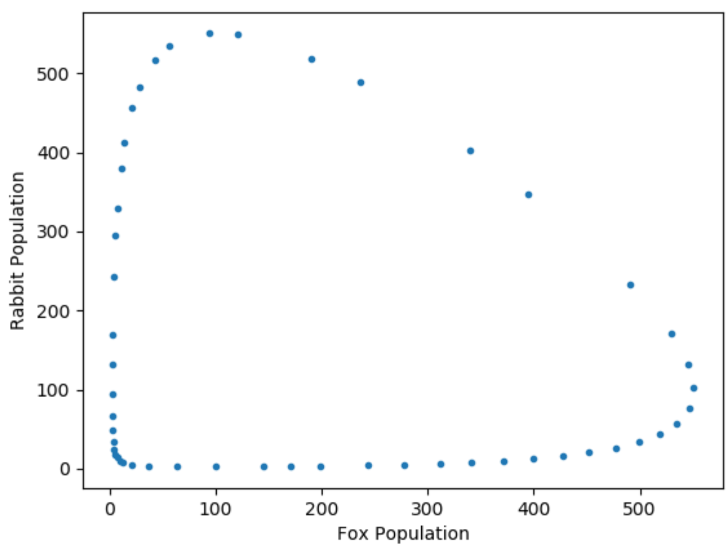
giving

$$r = \frac{a}{c}$$
$$f = \frac{a}{c} - \frac{a^2}{bc^2}$$

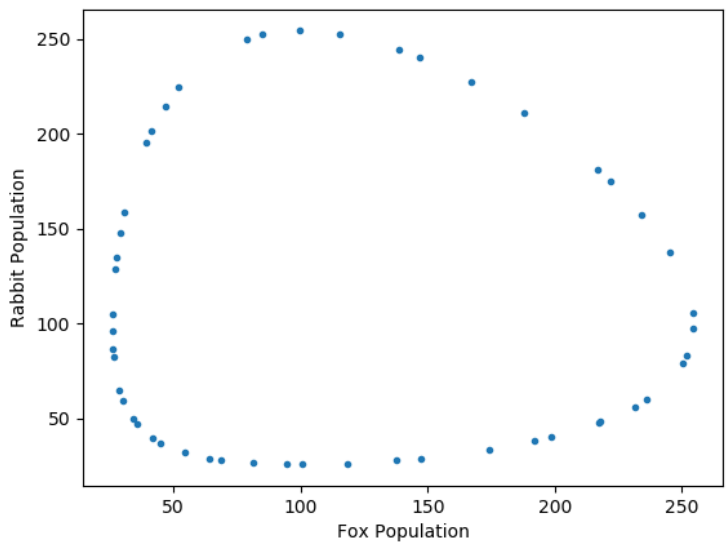
which for our parameters of $a = 10$, $b =, 10^6$, $c = 0.1$ yields $r = 100$ and $f = 100$.

2.2 b

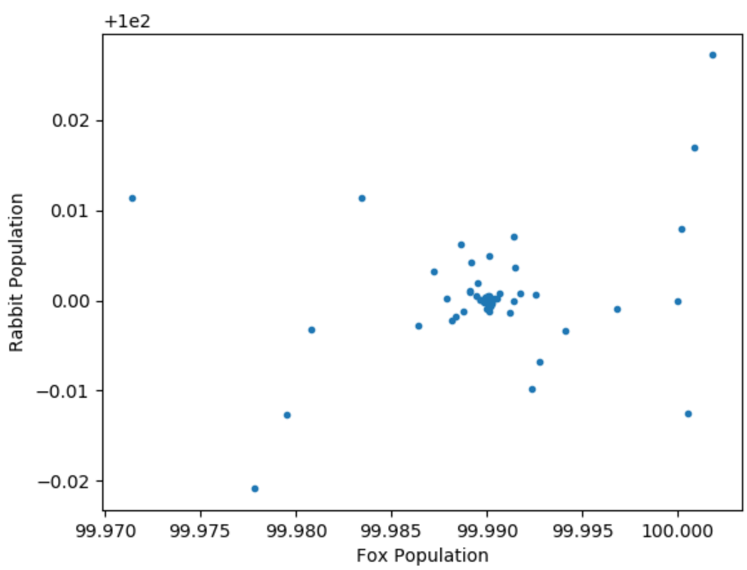
Population trajectories for various initial conditions:
$r(0) = 10,\ f(0) = 10$



$r(0) = 50,\ f(0) = 34$

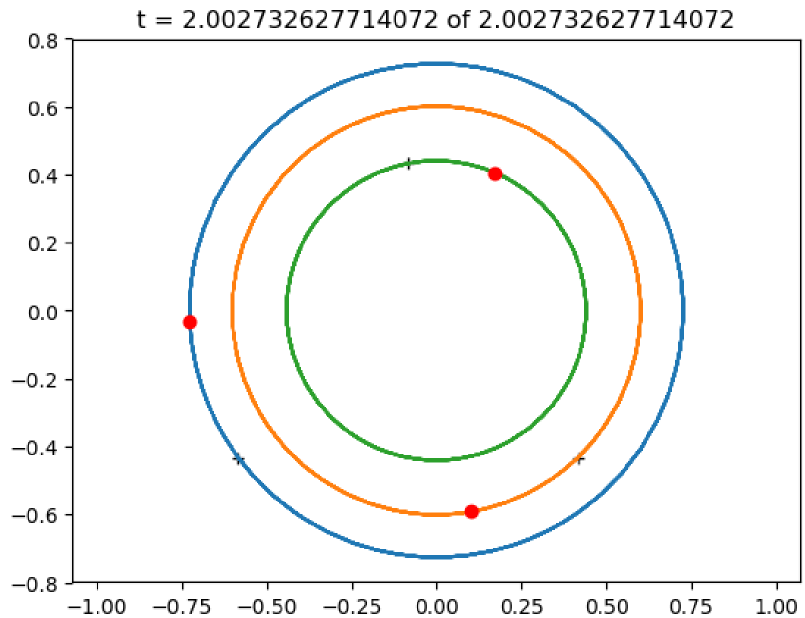


When you start the program at the steady state solution ($r = 100$ and $f = 100$) the solution "bounces" around within a very small range:

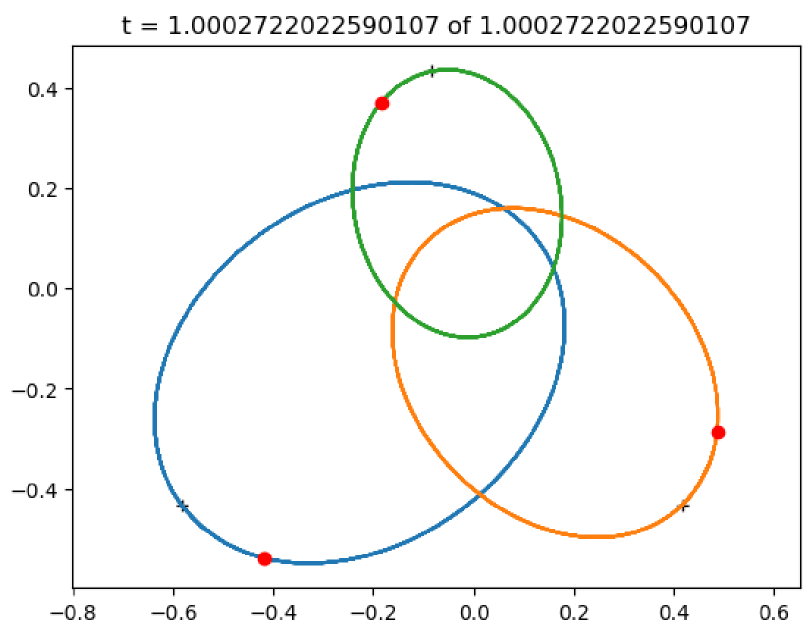


3 Lagrange Problem

Plot of 3 object system with circular orbits:



Plot of 3 object system with elliptical orbits:



4 Problem 19

4.1 a

To obtain equations of motion from the Lagrangian we solve

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \frac{\partial L}{\partial \theta_1}$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = \frac{\partial L}{\partial \theta_2}$$

For the first equation we have

$$(m_1 + m_2)L_1^2\ddot{\theta}_1 + m_2L_1L_2\ddot{\theta}_2\cos(\theta_1 - \theta_2) - m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) + m_2L_1L_2\dot{\theta}_2^{\,2}\sin(\theta_1 - \theta_2) = \\ - m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) - (m_1 + m_2)gL_1\sin\theta_1$$

which leads to the equation of motion

$$(m_1+m_2)L_1\ddot{\theta}_1+m_2L_2\ddot{\theta}_2\cos(\theta_1-\theta_2)+m_2L_2\dot{\theta}_2^{\,2}\sin(\theta_1-\theta_2)+(m_1+m_2)g\sin\theta_1 = 0$$

Similarly for the second equation we have

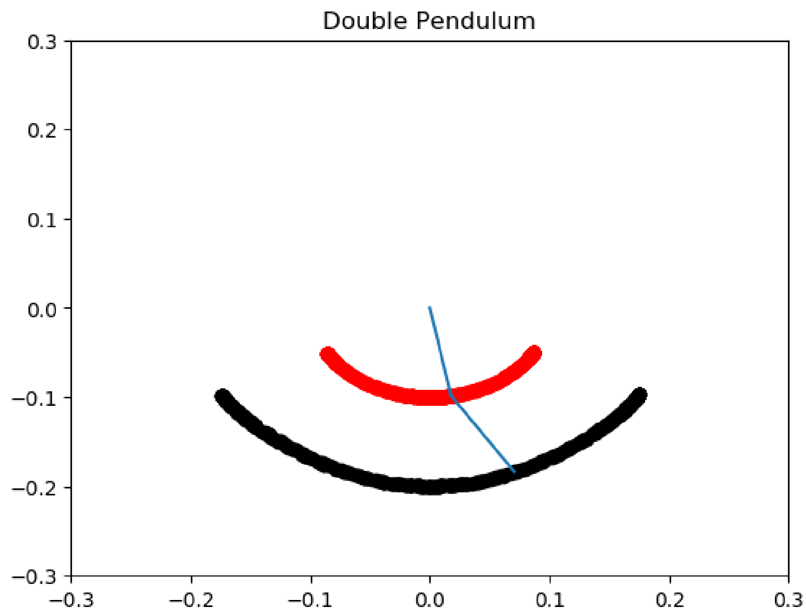
$$m_2L_2^2\ddot{\theta}_2 + m_2L_1L_2\ddot{\theta}_1\cos(\theta_1 - \theta_2) - m_2L_1L_2\dot{\theta}_1^{\,2}\sin(\theta_1 - \theta_2) + m_2L_1L_2\dot{\theta}_2\dot{\theta}_1\sin(\theta_1 - \theta_2) = \\ m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\sin(\theta_1 - \theta_2) - m_2gL_2\sin\theta_2$$

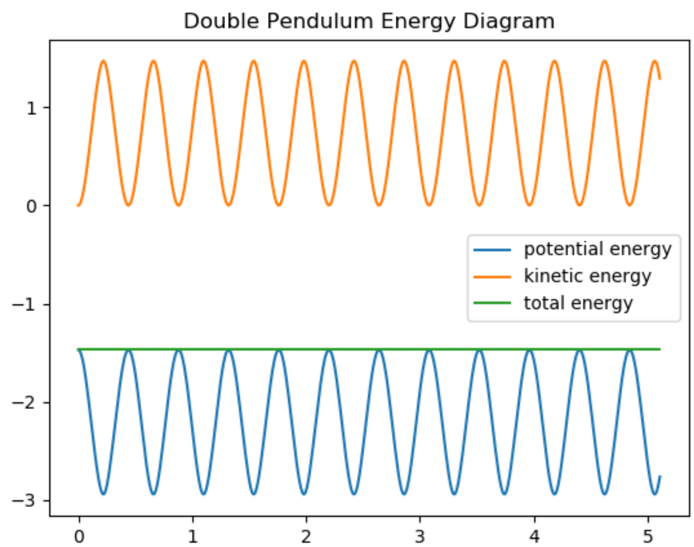
which leads to the equation of motion

$$m_2L_2\ddot{\theta}_2 + m_2L_1\ddot{\theta}_1\cos(\theta_1 - \theta_2) - m_2L_1\dot{\theta}_1^{\,2}\sin(\theta_1 - \theta_2) + m_2g\sin\theta_2 = 0$$

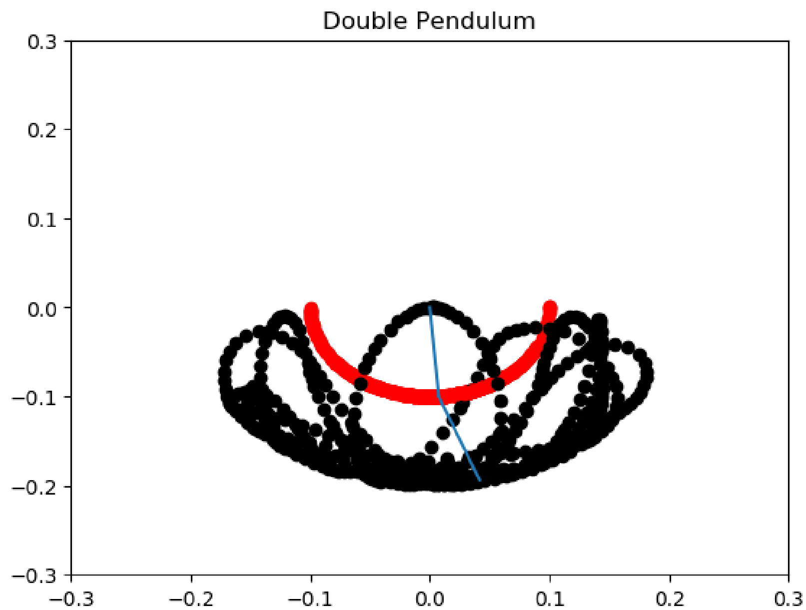
4.2 b

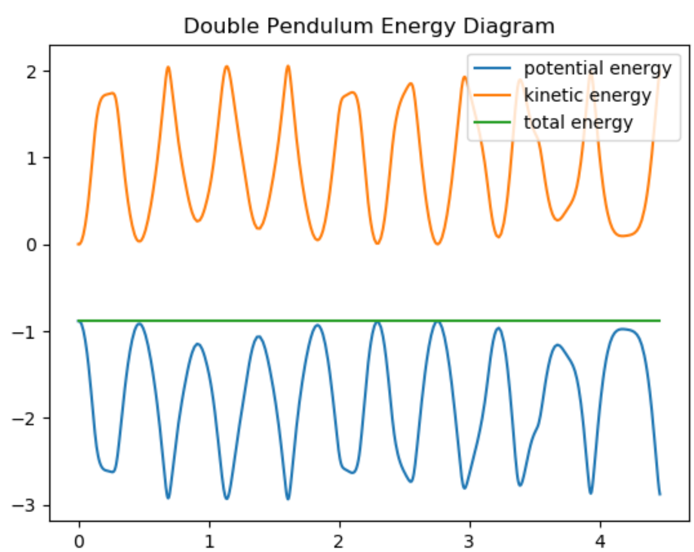
Below are several plots of the motion of a double pendulum for stated initial conditions.
$\theta_1 = \frac{\pi}{3}$, $\theta_2 = \frac{\pi}{3}$, $\dot{\theta}_1 = 0$, $\dot{\theta}_2 = 0$:



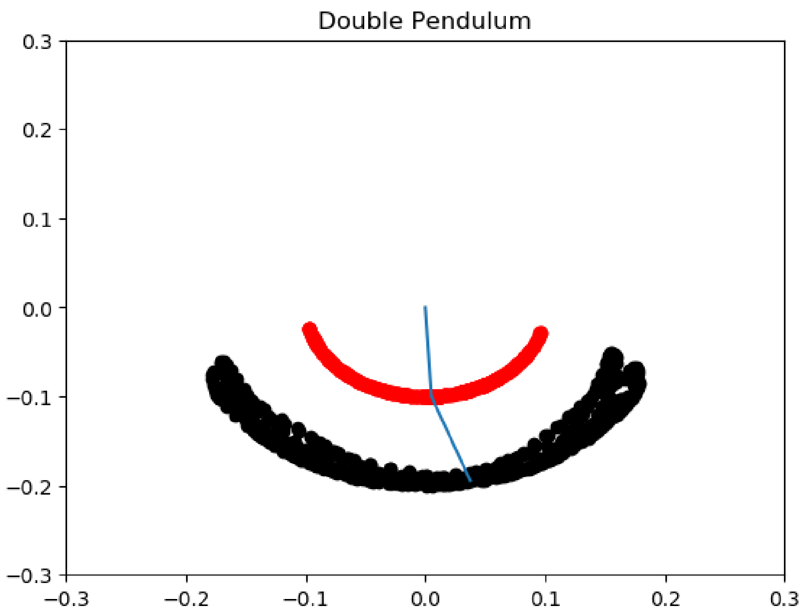


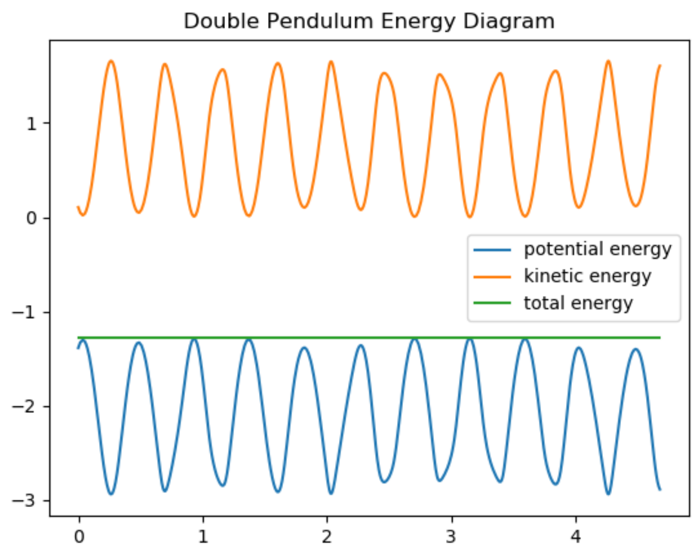
$\theta_1 = \frac{\pi}{2}$, $\theta_2 = \frac{\pi}{7}$, $\dot{\theta_1} = \frac{\pi}{6}$, $\dot{\theta_2} = 0$:





$\theta_1 = \frac{\pi}{4}$, $\theta_2 = \frac{\pi}{2}$, $\dot{\theta_1} = \pi$, $\dot{\theta_2} = \frac{\pi}{12}$:


Double Pendulum
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3


Double Pendulum Energy Diagram
1
0
−1
−2
−3
0
1
2
3
4
potential energy
kinetic energy
total energy