

ECE 4813 - Electromagnetics II - Laboratory 2 Experiment Report

Burak Koryan | ID : 3505874 | bkoryan@unb.ca

Experiment : Capacitance by Time Constant

TA : Alex Colpitts Instructor : Dr.Erik Scheme

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Objective:

The aim of this experiment is to determine capacitance of a circuit by using time constants (TC) and known equipment parameters.

Equipment:

- Signal generator capable of square waves at 100 kHz
- Oscilloscope probes (1:1)
- Reference capacitor of 1000 pF
- Two 7.62 (cm) by 15.24 (cm) plates
- Two 15.24 (cm) by 15.24 (cm) plates
- Five 7.62 (cm) by 15.24 (cm) sheets of paper
- Five 15.24 (cm) by 15.24 (cm) sheets of paper
- Five 7.62 (cm) by 15.24 (cm) sheets of acetate
- Five 15.24 (cm) by 15.24 (cm) sheets of acetate
- Wires
- Brick as a weight
- Tape measure

Procedure & Results:

1) The oscilloscope probes have been set to 1-to-1 scale and calibrated using the oscilloscopes square wave output.

2) The numbers on the copper plates as well as thicknesses of acetate and paper sheets have been measured and noted in the table below.

Parameter	Value
Big Plate Number	5
Small Plate Number	5

Table 1 : Copper plate numbers and paper/acetate sheet thickness

3) Paper and acetate sheets have been measured for their thicknesses and the measurements have been noted below in Table 2.

Parameter	Value
Big Plate Number	5
Small Plate Number	5
Average Paper Sheet Thickness	0.2mm
Average Acetate Sheet Thickness	0.1mm

Table 2 : Paper and Acetate sheets thicknesses

4) Dimensions of the copper plates used for making parallel plate capacitor have been measured with a tape measure and their areas have been calculated as shown in Table 3.

$$RectArea = length * width \quad (1)$$

$$SqArea = (side)^2 \quad (2)$$

Measured length of (small) rectangle copper plate : **15.2 cm**

Measured width of (large) square copper plate : **7.60 cm**

- Area of a rectangle can be found using Eq.1 above:

$$RectArea = 15.2cm * 7.60cm = 0.011552m^2$$

$$\text{Small Plate area} = 11.55 \times 10^{-3} m^2$$

Measured side dimension of the (large) square copper plate : **15.2 cm**

- Area of square can be found using Eq.2 above

$$SqArea = (15.2cm)^2 = 0.023104m^2$$

$$\text{Large Plate area} = 23.10 \times 10^{-3} m^2$$

5) A Rigol DG1022 function generator was connected to a Rigol DS1052E oscilloscope with an oscilloscope probe provided by the university. A 100 kHz square wave was applied from the function generator to the oscilloscope and the square wave was observed.

6)

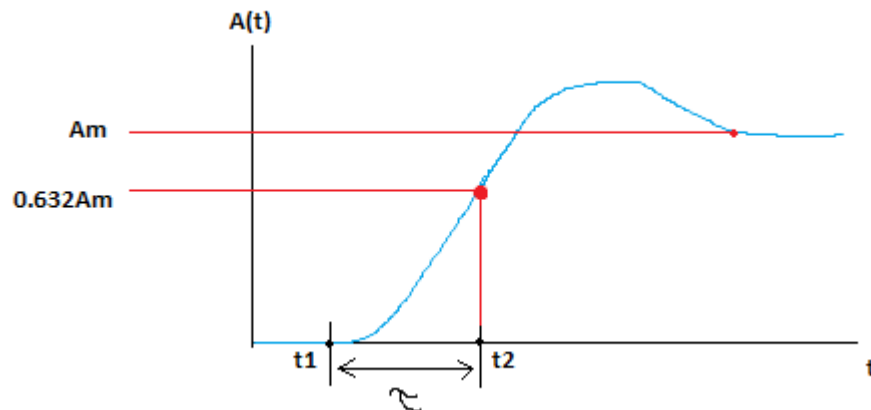


Figure 1 : Measuring time constant (TC) of function generator (Plot not to scale)

Note: The volt-div of the oscilloscope was at 10V/sq

$$\begin{aligned} \text{Decay point} &= 0.632 \cdot A_m \\ \text{Max. measured Amplitude} &= 50\text{Vdc} \end{aligned} \quad (3)$$

Using Eq.3:

$$\text{Decay point} = 0.632 \cdot 50\text{Vdc} = \mathbf{31.6\text{Vdc}}$$

$$t_1 = 20 \text{ ns (measured from oscilloscope)}$$

$$t_2 = 50 \text{ ns (measured from oscilloscope)}$$

$$\tau = t_2 - t_1 \quad (4)$$

Using Eq.4:

$$\tau = 50 \text{ ns} - 20 \text{ ns} = \mathbf{30 \text{ ns}}$$

or if the time-div of the oscilloscope is at 20 ns and if t_1 is 1.5 squares away horizontally from t_2 , we can say that $1.5 \times 20 \text{ ns (time-div)} = 30 \text{ ns}$

7) The stray capacitances ($C_s + C_o$) is calculated, when the source impedance is 50Ω , and shown below. (C_s : Source capacitance, C_o : oscilloscope capacitance)

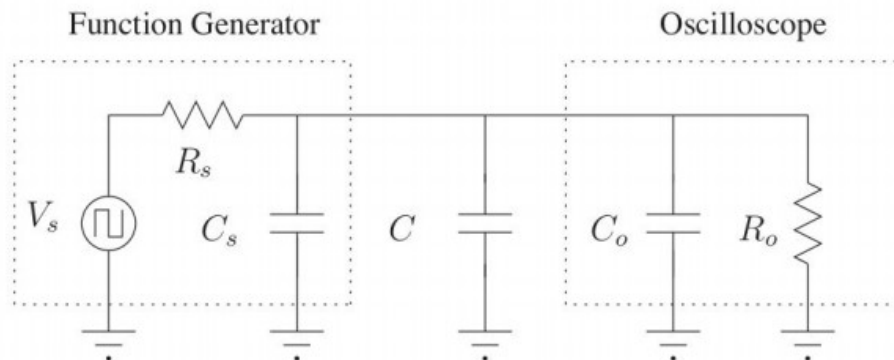


Figure 2 : Circuit diagram of parallel connection of function generator and oscilloscope

$$\tau = R \cdot C \quad (5)$$

where $R = 50\Omega$ (given)

if measured $\tau = 30 \text{ ns}$ then using Eq.5:

$$30 \text{ ns} = 50\Omega \cdot C$$

$$C = C_s + C_o = \mathbf{600 \text{ pF}}$$

8)

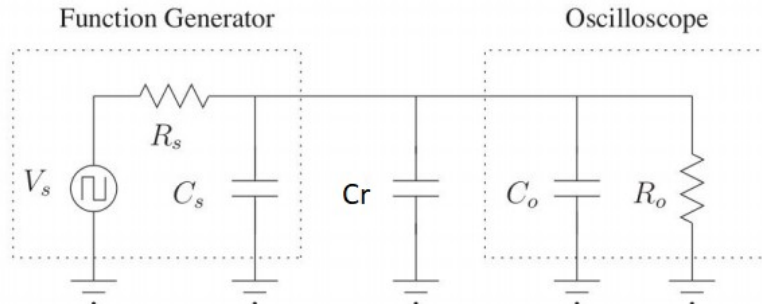


Figure 3 : Circuit diagram of parallel connection of function generator, oscilloscope, and C_r

$$C_{\text{total}} = C_r + C_s + C_o \quad (6)$$

Provided reference capacitor (C_r) in the laboratory experiment : **1000 pF**

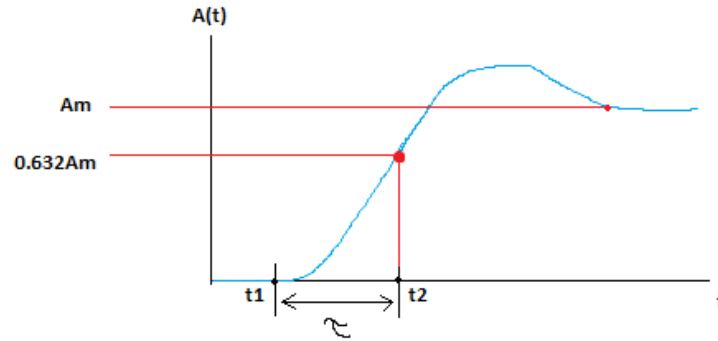


Figure 4 : TC measurement when C_r is connected in parallel with oscilloscope and function generator

Max. measured Amplitude(A_m) = 50Vdc

Using Eq.3 we can find the decay point:

$$\text{Decay point} = 0.632A_m = 0.632(50\text{Vdc}) = 31.6\text{Vdc}$$

Time-div of the oscilloscope is at 50 ns. The distance between t_1 and t_2 is 1.5 squares horizontally. Then we can say that $\tau_{(\text{measured})} = 1.5\text{sq} * 50 \text{ ns}(\text{time-div}) = \mathbf{75 \text{ ns}}$

Using Eq.5:

$$\tau_{(\text{measured})} = 75 \text{ ns} = R * C$$

$$75 \text{ ns} = 50\Omega * C$$

$$C_{\text{total}} = \mathbf{1.5 \text{ nF}}$$

In step 7 of the experiment, $C_s + C_o$ was found to be 600 pF. If Eq.6 says:

$$C_{\text{total}} = C_r + C_s + C_o$$

$$C_{r(\text{measured})} = C_{\text{total}} - (C_s + C_o) \quad (7)$$

If $C_s + C_o = 600 \text{ pF}$ and $C_{\text{total}} = 1.5 \text{ nF}$ then measured C_r can be found using Eq.7:

$$C_{r(\text{measured})} = 1.5 \text{ nF} - 600 \text{ pF} = \mathbf{900 \text{ pF}}$$

The nominal value of the provided C_r is 1000 pF and the measured value, found using Eq.7 is 900 pF. The difference is **10%** and within acceptable range.

9) For each size of plate and paper/acetate sheet, the TC has been measured on the oscilloscope and noted in Table 3.

Number of Sheets	TC small paper sheets(ns)	TC large paper sheets (ns)	TC small acetate sheets(ns)	TC large acetate sheets(ns)
1	248	314	92	346
2	123	258	84	276
3	109	236	82	224
4	103	222	72	236
5	98	160	68	220

Table 3 : Table of Time Constant measurements

10) Using data in Table 3, the varying capacitance has been calculated, in MATLAB (code shown in Appendix A), using TC eq.5. It can be clearly seen that as the number of sheets, either paper or acetate, increases in between the parallel plates, the τ decreases and in return the capacitance of the parallel plate capacitor decreases too. Therefore, the log-log plot of the capacitance and the increasing distance d , due to increasing number of sheets in between the plates, has a negative slope.

Number of Sheets	C with small paper sheets(nF)	C with large paper sheets (nF)	C with small acetate sheets(nF)	C with large acetate sheets(nF)
1	4.96	6.28	1.84	6.9
2	2.46	5.16	1.68	5.5
3	2.18	4.72	1.64	4.48
4	2.06	4.44	1.44	4.72
5	1.96	3.20	1.36	4.40

Table 4 : Table of varying capacitance with number of dielectric sheets and parallel plate size

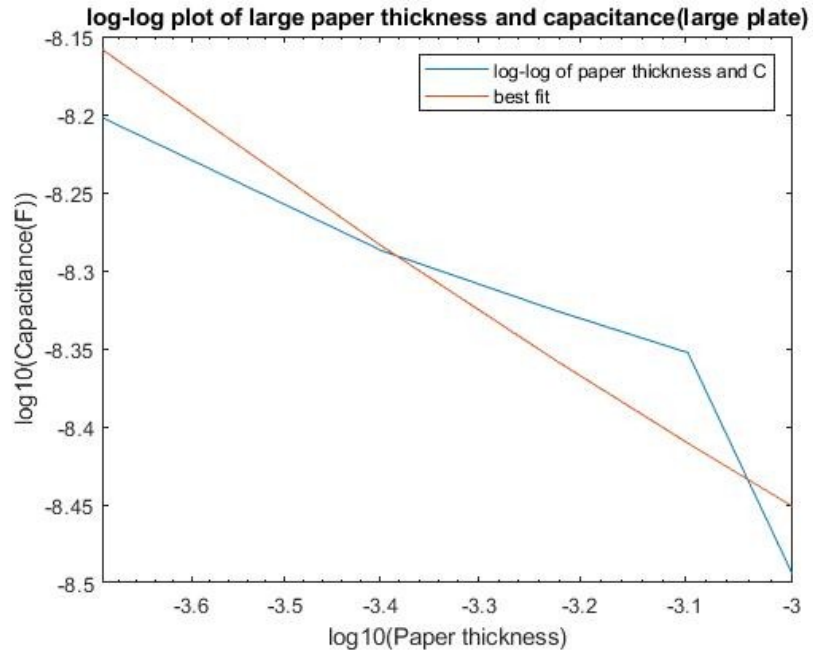


Figure 5: log-log plot of capacitance and distance d between large plates when the dielectric material is paper

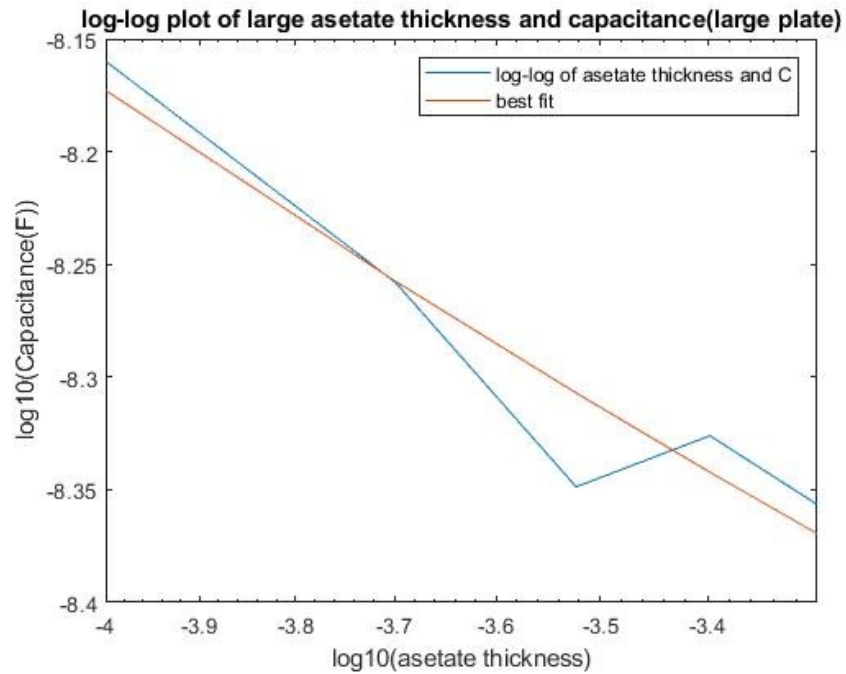


Figure 6: log-log plot of capacitance and distance d between large plates when the dielectric material is acetate sheet

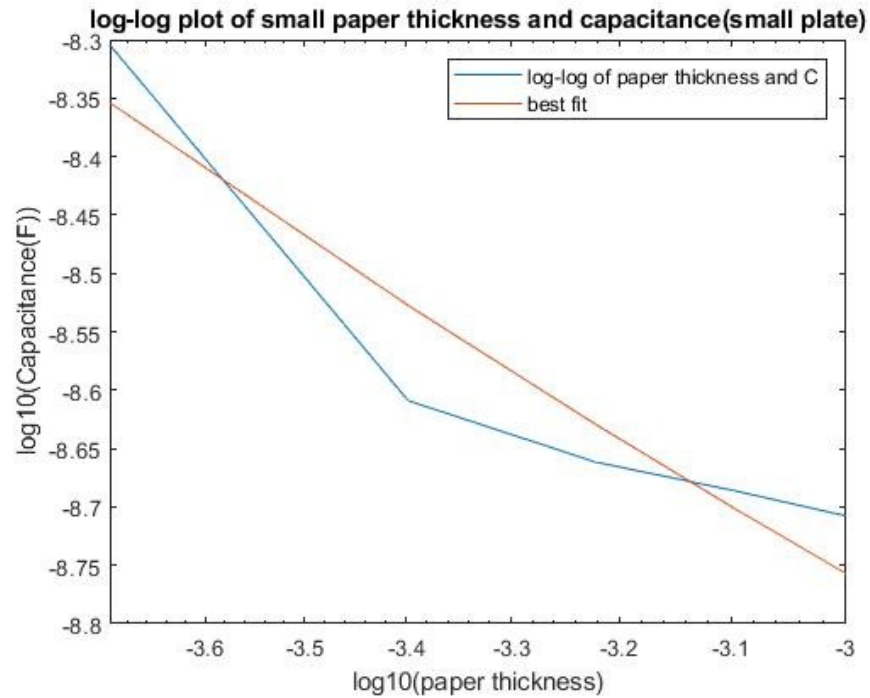


Figure 7: log-log plot of capacitance versus distance d with small parallel plates and paper as a dielectric material

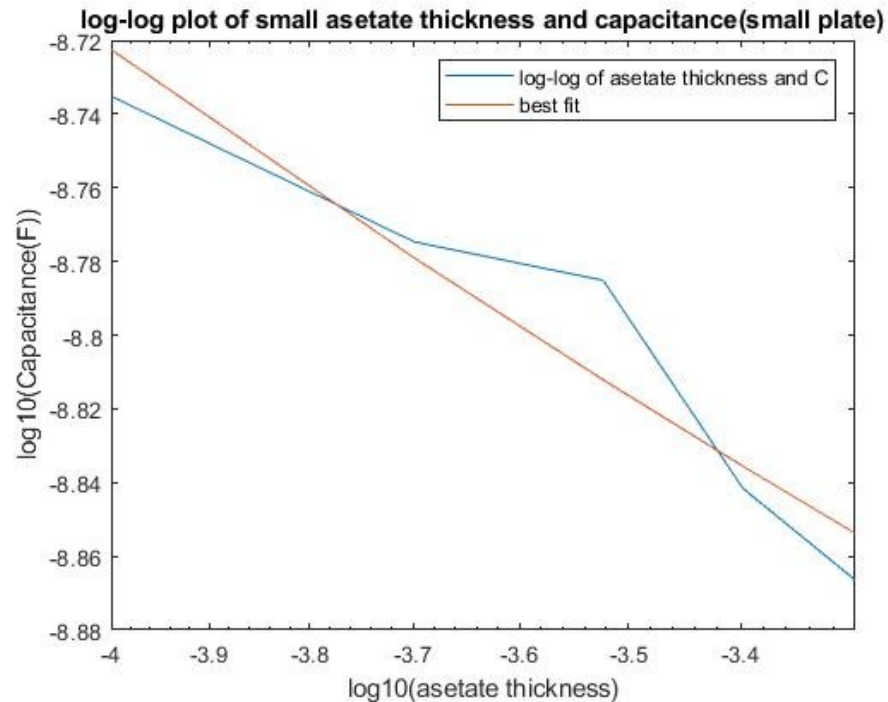


Figure 8: log-log plot of capacitance versus distance d between plates when using small plates and acetate sheets as dielectric material

calculated	small paper sheets	large paper sheets	small acetate sheets	large acetate sheets
m	-0.5769	-0.4189	-0.1878	-0.2813
b	-10.4878	-9.7079	-9.4738	-9.2983

Table 5 : Calculated slope(m) and y-intercept(b) of best fit lines

$$C = (\epsilon_r \times \epsilon_0 \times A) / d \quad (8)$$

where $\epsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$

Using Eq.8 ϵ_r , as shown in Table 6, can be found as:

$$\epsilon_r = (C \times d) / (\epsilon_0 \times A) \quad (9)$$

where A : area of the (small/large) plates
d : # of sheets x thickness of each sheet

Number of Sheets	small paper sheets(F/m)	large paper sheets (F/m)	small acetate sheets(F/m)	large acetate sheets(F/m)
1	9.70	4.85	1.80	3.38
2	9.62	4.81	3.29	5.40
3	12.8	6.40	4.81	6.57
4	16.1	8.06	5.63	9.23
5	19.2	9.58	6.65	10.80

Table 6: Estimated relative permittivity ϵ_r (F/m)

Discussions:

In the beginning of this experiment, we started with measuring the thicknesses of acetate sheets and paper sheets. It was found that each paper sheet has 0.2mm and each acetate sheet has 0.1mm thicknesses. This means that a paper sheet is thicker than an acetate sheet. Then the following task was calculating the effective area of the provided copper plates. Two copper plates were given in different dimensions. One like a square (large plate) with **15.2cm** by **15.2cm** dimension and one like a rectangle (small plate) with **15.2cm** and **7.60cm** dimension. The effective area for the large plate is calculated to be **$23.10 \times 10^{-3} \text{ m}^2$** and for the small plate **$11.55 \times 10^{-3} \text{ m}^2$** . This means that the large plate should give higher capacitance assuming that when the dielectric material is the same as well as the distance d between the plates.

In step 6 of the experiment, the decay point is found using eq.3. and the maximum amplitude of the signal is measured to be 50VDC. As shown in Figure 1, the decay point is at **31.6V**. τ was measured to be **30 ns** using time-div of the oscilloscope.

In step 7, the stray capacitances ($C_s + C_o$) is calculated from the τ in step 6 using eq.5.

since the total resistance in the schematic shown in Figure 1 is assumed to be 50Ω . Then stray capacitance is found **600 pF** ($C_s + C_o$).

In step 8, there is a reference capacitor C_r is connected in parallel to the oscilloscope and the function generator. The ceramic reference capacitor (with 102 code on it) is **1000 pF**. This additionally connected reference capacitor **increases** the total capacitance of the connection between the oscilloscope and the function generator. The measured τ is **75 ns** which translates into, using eq.5, total capacitance (C_{total}) of **1.5 nF**. Earlier, the stray capacitance calculated to be **600 pF** which means, using eq.7, the measured reference capacitance is **900 pF**. From this, it can be concluded that there is **10%** difference between the actual and measured value of the reference capacitor, which is an acceptable tolerance.

In step 9, the τ was measured with varying parallel plate sizes and the number of (paper/acetate) sheets between the parallel plates. There is a clear direct relationship between the number of sheets inbetween the plates and the τ measured. As the number of sheets **increases** inbetween the plates, τ **decreases** regardless the dielectric material.

In step 10, the τ measured is translated into capacitance using eq.5. The direct relationship between the number of sheets and the τ can also be seen here. As the distance d **increases** between the plates, the capacitance created by the parallel plates **decreases**. Multiple log-log plots are created to prove this relationship as shown from Figures 5 to 8. The relative permittivity (ϵ_r) is approximated using eq.9 and calculated capacitances earlier as shown in Table 6. As the number of (paper/acetate) sheets increases, the relative permittivity increases. A small paper sheet has approximately a relative permittivity of 9.70(F/m) whereas a large paper sheet has a relative permittivity of 4.85(F/m). Inversely, a small acetate sheet has a relative permittivity of 1.80(F/m) but a large acetate sheet has a relative permittivity of 3.38 (F/m). The relative permittivity (ϵ_r) is affected by the effective area of the plates as well as the capacitance and the distance d between the plates. As the effective area of the plates decreases, the capacitance created increases. Conversely, as the capacitance created increases and the distance d between the plates, the relative permittivity (ϵ_r) increases.

Conclusion:

In conclusion, the capacitance of a parallel plate capacitance only depends on the geometry of the plates and the dielectric material inbetween the plates. It is proven that as the number of either paper or acetate sheets increases, the capacitance of the parallel plate capacitor decreases.

The large conductive plate gives higher capacitance comparing to the small one regardless the number of (paper/acetate) sheets inbetween them. The nominal value for the reference capacitor is 1000 pF however the measured value is calculated to be 10% less of it (900pF) which means there is a 10% difference between them.

The log-log plots of capacitance versus the distance d between plates have negative slopes which means as the distance d increases, the capacitance the parallel plates create decreases. In the end of the experiment, it can be concluded that the large plates with acetate sheets give the highest capacitance.

Appendix A : Matlab code for plots and best-fit calculation

```
% Burak Koryan | bkoryan@unb.ca | Feb 3 2019
% ECE4813 - Lab 2 - Matlab code for plots and best-fit calculation

clc;

% Constants:
e0 = 8.854e-12;
er = 1;
LPlate = 15.24e-2;
SPlateL = 15.24e-2;
SPlateW = 7.6e-2;

dDefault = 0;
PaperThick = 0.2e-3;
AseThick = 0.1e-3;
Resist = 50;

% convert data from table to array from the workspace. The collected TC data has been
% copied/pasted into the workspace on matlab.

TableArr = table2array(data)*1e-9;
TableC = TableArr/Resist;

% Thickness of paper and asetate sheets from count 1 to 5
dPaper = [0.2e-3;0.4e-3;0.6e-3;0.8e-3;1e-3];
dAse = [0.1e-3;0.2e-3;0.3e-3;0.4e-3;0.5e-3];

%%
% Large plate with large paper sheet
logC = log10(TableC(:,2));
change_C = zeros(1,3);
logC_change = [logC(2)-logC(1);logC(3)-logC(2);logC(4)-logC(3);logC(5)-logC(4)];
log_change_paper = [(log10(dPaper(2))-log10(dPaper(1))),(log10(dPaper(3))-log10(dPaper(2))),(log10(dPaper(4))-log10(dPaper(3))),(log10(dPaper(5))-log10(dPaper(4)))];

m = mean(logC_change) / mean(log_change_paper); % m = change in capacitance /
change in paper thickness % b = y+m*x
b = (sum(log10(TableC(:,2)))-m*(sum(log10(dPaper))))/5; % y = mx+b
y = m*log10(dPaper)+b; % Plot log-log of paper thickness
semilogx(log10(dPaper),log10(TableC(:,2))) % Plot best fit
and C
hold on;
plot(log10(dPaper),y)
hold off;
xlabel('log10(Paper thickness)')
ylabel('log10(Capacitance(F))')
legend('log-log of paper thickness and C','best fit')
title('log-log plot of large paper thickness and capacitance(large plate)')
%%
% Large plate with large asetate sheet
logC = log10(TableC(:,4));
change_C = zeros(1,3);
logC_change = [logC(2)-logC(1);logC(3)-logC(2);logC(4)-logC(3);logC(5)-logC(4)];
log_change_ase = [(log10(dAse(2))-log10(dAse(1))),(log10(dAse(3))-log10(dAse(2))),(log10(dAse(4))-log10(dAse(3))),(log10(dAse(5))-log10(dAse(4)))];
```

```

m = mean(logC_change) / mean(log_change_ase);
b = (sum(log10(TableC(:,4)))-m*(sum(log10(dAse))))/5;
y = m*log10(dAse)+b;
semilogx(log10(dAse), log10(TableC(:,4)));
hold on;
semilogx(log10(dAse),y);
hold off;
xlabel('log10(asetate thickness)')
ylabel('log10(Capacitance(F))')
legend('log-log of asetate thickness and C','best fit')
title('log-log plot of large asetate thickness and capacitance(large plate)')

%%
% Small plate with small paper sheet
logC = log10(TableC(:,1));
change_C = zeros(1,3);
logC_change = [logC(2)-logC(1);logC(3)-logC(2);logC(4)-logC(3);logC(5)-logC(4)];
log_change_ase = [(log10(dPaper(2))-log10(dPaper(1)))+(log10(dPaper(3))-log10(dPaper(2)))+(log10(dPaper(4))-log10(dPaper(3)))+(log10(dPaper(5))-log10(dPaper(4)))];

m = mean(logC_change) / mean(log_change_ase);
b = (sum(log10(TableC(:,1)))-m*(sum(log10(dPaper))))/5;
y = m*log10(dPaper)+b;
semilogx(log10(dPaper), log10(TableC(:,1)));
hold on;
semilogx(log10(dPaper),y);
hold off;
xlabel('log10(paper thickness)')
ylabel('log10(Capacitance(F))')
legend('log-log of paper thickness and C','best fit')
title('log-log plot of small paper thickness and capacitance(small plate)')

%%
% Small plate with small asetate sheet
logC = log10(TableC(:,3));
change_C = zeros(1,3);
logC_change = [logC(2)-logC(1);logC(3)-logC(2);logC(4)-logC(3);logC(5)-logC(4)];
log_change_ase = [(log10(dAse(2))-log10(dAse(1)))+(log10(dAse(3))-log10(dAse(2)))+(log10(dAse(4))-log10(dAse(3)))+(log10(dAse(5))-log10(dAse(4)))];

m = mean(logC_change) / mean(log_change_ase);
b = (sum(log10(TableC(:,3)))-m*(sum(log10(dAse))))/5;
y = m*log10(dAse)+b;
semilogx(log10(dAse), log10(TableC(:,3)));
hold on;
semilogx(log10(dAse),y);
hold off;
xlabel('log10(asetate thickness)')
ylabel('log10(Capacitance(F))')
legend('log-log of asetate thickness and C','best fit')
title('log-log plot of small asetate thickness and capacitance(small plate)')

```