- 
- 
- 

$I$ $(x, y, t)$ $I(x, y, t)$ $\delta t$ $\delta x$ $\delta y$ $I(x + \delta x, y + \delta y, t + \delta t)$

$$
\begin{aligned}
I(x+\delta x, y+\delta y, t+\delta t) &= I(x,y,t) \\
&\quad + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t + \mathcal{O}(I^2) \\
0 &= \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t \\
0 &= \frac{\partial I}{\partial x}\frac{\delta x}{\delta t} + \frac{\partial I}{\partial y}\frac{\delta y}{\delta t} + \frac{\partial I}{\partial t}\frac{\delta t}{\delta t} \\
0 &= \frac{\partial I}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial I}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial I}{\partial t} \\
0 &= I_x u + I_y v + I_t,
\end{aligned}
$$

$$0 = \nabla I \cdot \vec{V} + I_t.$$

$$\inf\left\{F[u] = \int_\Omega f(x, u(x), \nabla u(x))\,\mathrm{d}x\right\}.$$

$$\frac{\partial f}{\partial u} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial f}{\partial \dot{u}} = 0.$$

$f$

$$F[u] = \iint (I_x u + I_y v + I_t)^2 + \alpha(||\nabla u||^2 + ||\nabla v||^2)\,\mathrm{d}x\mathrm{d}y.$$

$$0 = \nabla I(\nabla I \cdot \vec{V} + I_t) - \alpha^2 \Delta \vec{V}.$$

$n$

$$0 = I_x(p_1)u + I_y(p_1)v + I_t(p_1)$$

$$0 = I_x(p_{mn})u + I_y(p_{mn})v + I_t(p_{mn}),$$

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ \\ I_x(p_{mn}) & I_(p_{mn}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} I_t(p_1) \\ \\ I_t(p_{mn}) \end{bmatrix},$$

$$\nabla \mathbf{I} \cdot \vec{V} = -\mathbf{I}_t.$$

$$
\begin{aligned}
0 &= \nabla \mathbf{I} \cdot \vec{V} + \mathbf{I}_t \\
-\mathbf{I}_t &= \nabla \mathbf{I} \cdot \vec{V} \\
-(\nabla \mathbf{I})^T I_t &= (\nabla \mathbf{I})^T \nabla \mathbf{I} \cdot \vec{V} \\
-\nabla \mathbf{I}^T \mathbf{I}_t &= (\nabla \mathbf{I}^T \nabla \mathbf{I}) \cdot \vec{V} \\
-(\nabla \mathbf{I}^T \nabla \mathbf{I})^{-1} \nabla \mathbf{I}^T \mathbf{I}_t &= (\nabla \mathbf{I}^T \nabla \mathbf{I})^{-1} (\nabla \mathbf{I}^T \nabla \mathbf{I}) \cdot \vec{V} \\
\vec{V} &= -(\nabla \mathbf{I}^T \nabla \mathbf{I})^{-1} \nabla \mathbf{I}^T \mathbf{I}_t.
\end{aligned}
$$

$\nabla \mathbf{I}^T \nabla \mathbf{I}$ $\det(\nabla \mathbf{I}^T \nabla \mathbf{I}) \neq 0$ $\nabla \mathbf{I}^T \nabla \mathbf{I}$ $\lambda_1$ $\lambda_2$ $\lambda_1$ $\lambda_2$

$$x^{(i+1)} \leftarrow \mathcal{F}(x^{(i)}, \Theta), \text{ for } i \in \{1, \dots, n-1\},$$

$n$

$$\min \frac{1}{2}||Ax - y||_{L_2}^2 + ||\lambda x||_{L_1},$$

$x$

$$x^{(k+1)} \leftarrow S_\alpha\Big(x^{(k)} - \frac{1}{L}A^T(Ax^{(k)} - y)\Big),$$

$S_\alpha(\star) \triangleq \operatorname{sign}(\star) \cdot \max\{|\star| - \alpha, 0\}$ $\alpha$ $K$ $K$ $A_t = I - \frac{1}{L}A^T A$ $A_e = \frac{1}{L}A^T$ $I$ $A_t$ $A_e$ $\alpha$ $\ell$

$$\ell(A_t, A_e, \alpha) = \frac{1}{n}\sum_{i=1}^{n} \left|\left|x^{(i)} - \hat{x}^{(i)}(y^{(i)}; A_t, A_e, \alpha)\right|\right|_2^2,$$

$y^1, \dots, y^n$ $x^1, \dots, x^n$

$\varphi(\cdot)$

$$\min_{\vec{V}\in BV(\Omega)} \left|\left|\nabla \mathbf{I}\cdot \hat{\vec{V}} + \mathbf{I}_t\right|\right|_{L_2}^2 + |\varphi(\cdot)|_{L_1}.$$

$\varphi(\cdot)$

$f \notin C_1(\Omega)$

$\text{prox}_{\varphi}(\star) \triangleq \arg\min_z ||\star^{(i-1)} - z||_{L_2}^2 + |\varphi(\cdot)|_{L_1}$

$$
\begin{aligned}
& z^{(i)} \leftarrow \text{prox}_{\varphi}(\vec{V}^{(i-1)}) \\
\Rightarrow\ & z^{(i)} \leftarrow \arg\min_{z} ||\vec{V}^{(i-1)} - z||_{L_2}^2 + |\varphi(\cdot)|_{L_1} \\
& \vec{V}^{(i-1)} \leftarrow z^{(i)} + \eta^{(i)} \nabla \mathbf{I}^T (\mathbf{I}_t - \nabla \mathbf{I} z^{(i)}).
\end{aligned}
$$

$\eta_i$

$\arg\min_z ||\vec{V}_{i-1} - z||_{L_2}^2$

$$\ell = \frac{1}{n}\sum_{i=1}^{n} \left|V_i - \hat{V}_i\right|_{L^2}$$

$$\ell = \frac{1}{n}\sum_{i=1}^{n} \left|V_i - \hat{V}_i\right|_{L_2}^2 + \frac{1}{N}\sum_{i=1}^{N} \left|\nabla I \cdot \hat{V}_i + I_t\right|_{L^2}^2$$

$\dots$

$\dots$