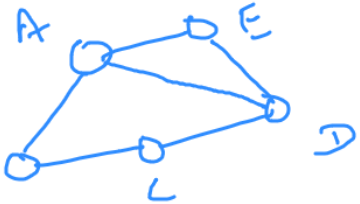
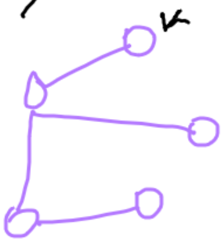
Graph : → collection of nodes/ vertices connected by edges.

G (V, E)

cyclic graph.

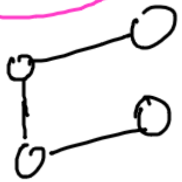


(1) cyclic Graph & Acyclic Graph ✓



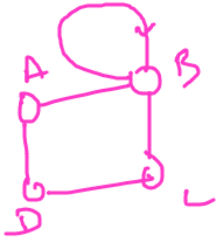
Acyclic graph

(2) Simple & multigraph



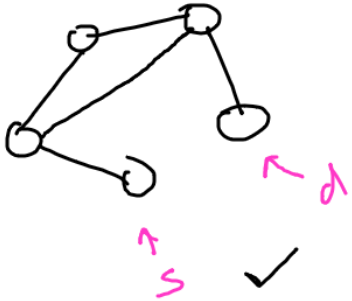
No loops →

Loop →

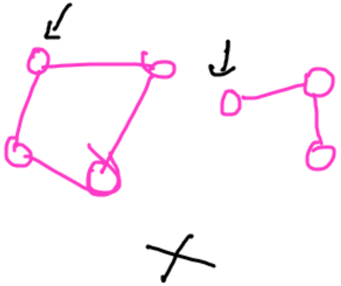


Multigraph

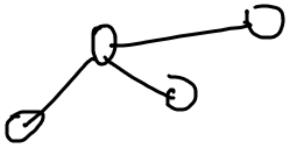
Connected graph



$\boxed{\text{Simple connected graph}}$



Simple connected Acyclic graph is a Tree

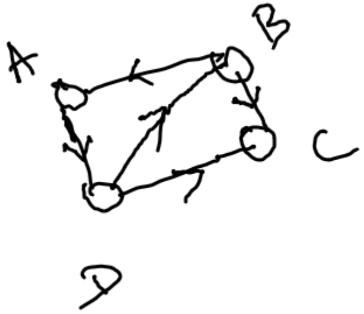


Note:-> All trees are graphs but not vice-versa

Digraph

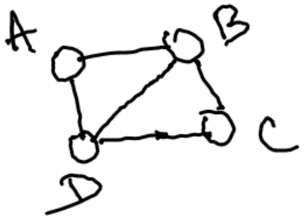
↑

Directed graph



A ← B

Undirected graph

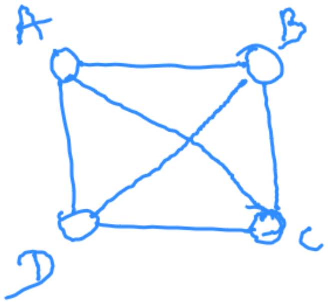


A B

Complete Graph (K_n)

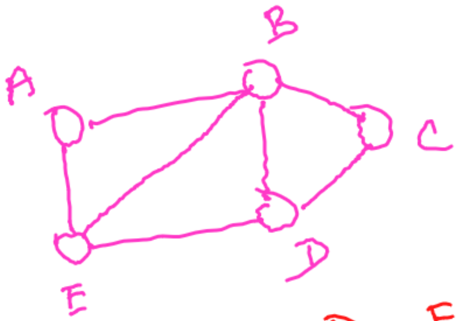
↓

n → no of nodes



Graph Representation

(1) Adjacency Matrix ✓
(2) Adjacency List ✓
(3) Edge List ✓
(4) Path matrix



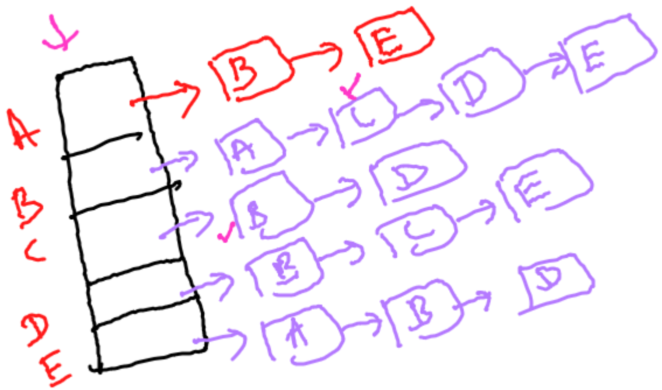
Adjacency matrix ⟹

$$
\begin{array}{c|ccccc}
 & A & B & C & D & E \\
\hline
A & 0 & 1 & 0 & 0 & 1 \\
B & 1 & 0 & 1 & 1 & 1 \\
C & 0 & 1 & 0 & 1 & 0 \\
D & 0 & 1 & 1 & 0 & 1 \\
E & 1 & 1 & 0 & 1 & 0
\end{array}
$$

$S(n) = O(n^2)$

(Dense graph)

$O(n + 2E)$

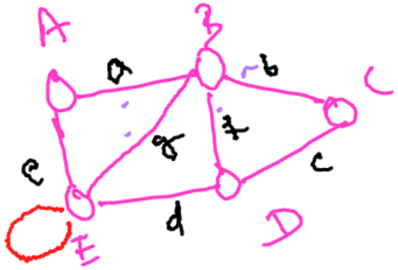


⇐ Adjacency List

(sparse graph)

Edge List

a	A	B
b	B	C
c	C	D
d	D	E
e	E	A
f	B	D
g	B	E
e	E	E



Degree

'No. of edges connected to a node.'

Degree (A) = 2 , Degree (B) = 4

Path matrix

$$P = A^1 + A^2 + A^3 + A^4 + \dots A^n$$

A → Adjacency matrix

$$A^2 = [A] \times [A]$$

$$A^3 = [A^2] \times [A]$$

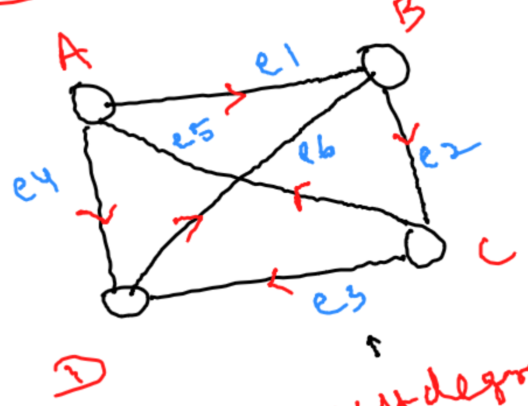
$A^2 \to$ path of length 2
matrix

$\begin{bmatrix} - - 3 \end{bmatrix}$ There are 3 paths of length 2.

<- path=2

$P = \begin{bmatrix} & 3 \end{bmatrix}$ There are 3 paths of length n or less than n.

Directed Graph



Indegree & **outdegree**

Indegree ⇒ no. of incoming edges

$\boxed{A = 1}$

outdegree ⇒ no. of outgoing edges.

$\boxed{A = 2}$

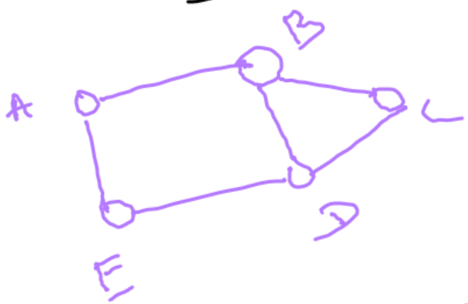
Incidence Matrix → Represents a directed graph.

$$
\begin{array}{c|cccc}
 & A & B & C & D \\
\hline
e1 & 1 & -1 & 0 & 0 \\
e2 & 0 & 1 & -1 & 0 \\
e3 & 0 & 0 & 1 & -1 \\
e4 & 1 & 0 & 0 & -1 \\
e5 & -1 & 0 & 1 & 0 \\
e6 & 0 & -1 & 0 & 1
\end{array}
$$

incoming ⇒ -1

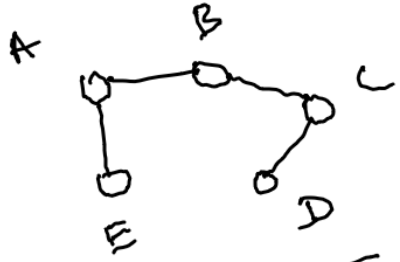
outgoing ⇒ $+1$

Spanning Tree



$$\boxed{\text{No. of edges} = n-1}$$

$n \rightarrow$ no. of nodes.



Spanning Tree 1

spanning Tree 2

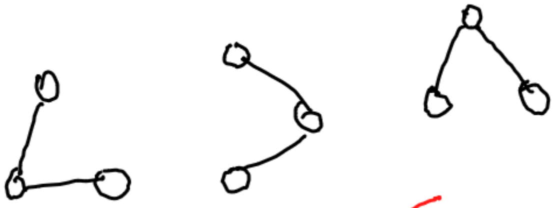
$\Rightarrow$ multiple spanning trees are possible for a graph.

For a complete graph K_n, $n \to$ no of nodes.

$\Rightarrow$ $\boxed{\text{no. of spanning trees} = n^{n-2}}$

$n = 3$

$n^{n-2} = 3^{3-2} = 3$



3 S.T.s.

$\boxed{\text{Kirchoff's Theorem} \rightarrow \text{for non complete graph}}$