Stevens Institute of Technology

Financial Engineering

Implied Volatility and Corporate Earnings

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Abstract

The W-shaped implied volatility smile has emerged in various options markets in recent years, resulting from traders' anticipation of major market events, such as corporate earnings announcements. There are two extreme outcomes possible, being an up or down stock movement. The W-shape smile appears most prominently in short-dated options immediately before the announcement takes place. We proposed to model the W-shape implied volatility smile in the absence of arbitrage. We accomplished this by using market option data for Amazon, Inc., and using parametric models, optimization and smoothing techniques to replicate the implied volatility smiles. The probabilities of the two main outcomes occurring, being under or over performance, will be obtained by constructing bimodal stock price risk neutral density distribution.
Keywords: options, volatility, earnings

1 Introduction & Motivation

The standard volatility curve follows a smile or smirking shape; however, shortly before a quarterly earnings announcement, the curve resembles a mustache, or W-shape smile. Investigating this phenomenon can give insight into how stocks move prior to expected positive or negative earnings news. If these W-shaped volatility smiles can be modeled, they can be used to predict the risk-neutral probability density distribution of stock prices at the time of earnings announcements.

With this knowledge, investors can develop trading strategies based on the risk-neutral probability density of the stock distribution that is implied from the W-shaped volatility curve, shortly before corporate earnings announcements. This valuable information will help investors buy into or sell out of the respective stocks ahead of time, thus increasing the chances of profiting, or decreasing the chances of losses to their portfolio.

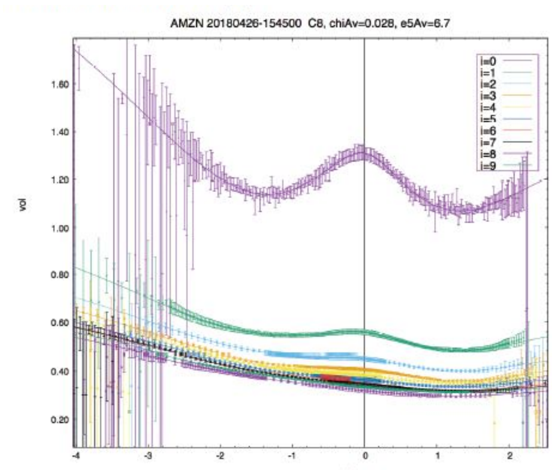

Figure 1: The implied volatility of AMZN options with maturity 27-Apr-2019, fifteen minutes before earnings announcements.

As an illustration we show in Figure 1 the implied volatility for AMZN options with maturity 27-Apr-2018 fifteen minutes prior to earnings announcements. Similar shapes are observed in several situations of market stress, such as the options on USD/GBP before the Brexit vote, and during the recent COVID-19 crisis in S&P500 index option markets.

2 Literature Review

2.1 SVI Literature

Gatheral and Jacquier (2014) present their methodology for calibrating the stochastic volatility inspired (SVI) parameterization of the implied volatility smile. Originally developed at Merrill Lynch in 1999, SVI quickly gained popularity due to two main characteristics that facilitated the parameterization process for fitting implied volatility. The first being that for a fixed time to expiry t, the implied Black-Scholes variance $\sigma^2_{BS}(k,t)$ is linear in the log-strike k as $k \to \infty$ consistent with Roger Lee's moment formula. The second is that it is relatively straightforward to fit SVI to listed option prices while ensuring the absence of calendar spread arbitrage, which guarantees that the total variance is an increasing function of the time to maturity. The greatest contribution of Gatheral and Jacquier (2014) is their exploration into a class of SVI models having a simple closed form solution and absence of static arbitrage. Static arbitrage assures that there exists a non-negative martingale on a filtered probability space so that a European call option call can be calculated as the risk neutral expected value of its final payoffs. Additionally, the necessary conditions are derived to ensure the absence of butterfly arbitrage, which corresponds to a non-negative implied density from the SVI parameterization and therefore presents call prices as a decreasing and convex function of strike prices. Since its introduction in 1999, SVI has been manipulated and implemented in hopes of eliminating static arbitrage and finding the simple closed form solution proposed by Gatheral and Jacquier (2014). Proving a volatility surface is free of both calendar spread and butterfly arbitrage for each time slice of the surface is sufficient to ensure that it is free of static arbitrage. This also guarantees the monotonicity of option prices with respect to maturity in addition to the existence of a risk neutral martingale measure for option pricing.

Gatheral and Jacquier (2014) go on to introduce three variations of SVI and their respective conditions for guaranteeing absence of static arbitrage, accompanied by the necessary proofs to exhibit these characteristics. The first to be mentioned is the raw SVI parameterization consisting of five parameters, $X_R = \{a, b, p, m, \sigma\}$, such that total variance is defined by the following equation:

$$w(k; X_R) = a + b\{p(k-m) + \sqrt{(k-m)^2 + \sigma^2}\}$$

where

$$\sigma^2_{BS}(k,t) = \frac{w(k,t)}{t}$$

This requires the following conditions in order to achieve freedom of static arbitrage and a non-negative minimum for $w(k; X_R)$:

$$a \in R, b \geq 0, p < 1, m \in R, \sigma > 0, a + b\sigma\sqrt{1-p^2} \geq 0$$

From this parameterization we can observe the effect of each parameter on the resulting volatility smile. Increasing a corresponds to an increase in the general total variance for all strikes or a vertical transformation of the implied volatility curve. Increasing the value of b translates to an increase in the slope of both the put and call wings of the curve, effectively tightening the smile. An increase in p will decrease the slope of the left wing while increasing the slope of the right wing resulting in a counterclockwise rotation of the curve. Moreover, an increase in m will translate the smile to the right and increasing reduces the at-the-money (ATM) curvature of the smile. The second parameterization that is introduced, natural SVI, is only a slight variation on raw SVI and the third, SVI Jump-Wings, adds complexity to the calculations in order to provide more intuitive and meaningful parameters. The SVI Jump-Wings parameterization is intended for traders so that they are able to understand the meaning behind each portfolio and exactly how each parameter affects their positions. This model is given by the parameters $X_J = \{\nu_t, \psi_t, p_t, c_t, \overline{\nu_t}\}$ such that ν_t represents the ATM variance, ψ_t gives the ATM skew, p_t the slope of the put (left) wing, c_t the slope of the call (right) wing, and $\overline{\nu_t}$ the minimum implied variance. This benefit, however, is not as relevant to our purpose, and therefore, does not warrant the added complexity of the SVI Jump-Wings parameterization. Moreover, we selected the raw SVI implementation as our SVI model and moving forward any reference of SVI will be in reference to this model.

2.2 SSVI Literature

Klassen (2016) literature defines the simple stochastic volatility inspired (SSVI/S3) parameterization of the implied volatility smile. SSVI is the simplest curve utilizing three parameters to describe the at-the-money behavior of implied volatilities for a given term, while also having a sensible functional form in the call and put wings. Three parameters at minimum are required to describe realistic volatility curves, where the parameters are the ATM (at-the-money) volatility, slope and curvature. By considering dimensionless parameters in normalized strike space, a simple picture of the no-arbitrage region emerges. The no-arbitrage region is surprisingly large for most realistic volatilities. SSVI is thought of as the "null hypothesis" of implied volatility curve fitting, as the defining characteristics of SSVI/S3 are motivated by simplicity and avoiding arbitrage, which should hold for all underliers. However, volatility curves for options on more liquid underliers, at least in the equity domain, can have significantly more structure and require more degrees of freedom to be consistently fit within bid-ask spreads. For example, the curvature c_2 can become significantly negative for maturities right after important events (earnings for technology names, FOMC for the SPY ETF, etc). This is a qualitative feature not allowed by the SSVI/S3 or SVI curves.

Klassen (2016) guides us in the implementation of the SSVI model for modeling the W-shape implied volatility smile. The three parameters in SSVI are ATF volatility 0, slope s_2 and curvature c_2. Specifically, for the SSVI/S3 curve we write the square of the implied volatility at a given term T as

$$\sigma^2(z) = \sigma_0^2[\frac{1}{2}(1 + s_2 z) + \sqrt{\frac{1}{4}(1 + s_2 z)^2 + \frac{1}{2}c_2 z^2}]$$

$$where\ z := \frac{LN(K/F)}{\hat{\sigma_0}} = \frac{y}{\hat{\sigma_0}}$$

is defined as the normalized strike. Normalized strike (NS) provides a convenient and intuitive way of thinking about moneyness, as it makes moneyness (more) comparable across maturities and underliers. It is defined in terms of the normalized arbitrage free (ATF) volatility

$$\hat{\sigma_0} := \sigma_0\sqrt{T}$$

To ensure ATF, the priori range of parameters is $\sigma_0 \geq 0,\ -\infty < s_2 < \infty, c_2 \geq 0$.

Most recently, VolaDynamics has begun modeling these W-shaped volatility smiles (as well as many different volatility surfaces) VolaDynamics (2020). They use a proprietary smoothing technique, but have also provided some motivation for this project.

2.3 PDF Calculation

After obtaining the best parametric fit for the implied volatility W-shape curve, the Risk-neutral density from the implied volatility Breeden Litzenberger formula can be used to determine the risk-neutral probability distribution $f(S)$ of stock prices.

$$f(s) = \frac{d^2C(K,T)}{dK^2}|_{K=S} = \frac{d^2P(K,T)}{dK^2}|_{K=S}$$

where $C(K,T)$ and $P(K,T)$ are the call and put option prices. They are expressed in terms of the implied volatility $\sigma_{BS}(K;T)$ using the Black-Scholes formula

$$C(K,T) = e^{rT}\ [F(T)N(d_1 - KN(d_2)]$$

$$P(K,T) = e^{rT}[KN(-d_2 - F(T)N(-d_1)]$$

$$with\ d_{1,2} = \frac{LN(\frac{F(T)}{K} \pm \frac{1}{2}\sigma_{BS}^2(K,T)T}{\sigma_{BS}\sqrt{T}}$$

The derivatives can be computed by discretization on a uniform grid of strikes K_i with step $\triangle K$ as finite differences

$$\frac{d^2C(K,T)}{dK^2} \simeq \frac{C(K_{i+1},T) - 2C(K_i,T) + C(K_{i-1},T)}{\triangle K^2}$$

3 Main Objectives

- To explain the dynamics of the implied volatility smile close to earnings announcements
- To model the W-shaped implied volatility smile and interpolate the strike and maturity of an option that ensures the absence of arbitrage
- To construct a risk neutral probability density stock distribution

4 Data

The Amazon option data was provided by Voladynamics. The data was taken on April 27, 2018, just a few seconds prior to the Amazon corporate earnings announcement.

We were provided with 16 Amazon option maturity data sets with the final maturity occurring on January 17, 2020. All maturities are shown in Figure 2.

Maturity	
2018-04-27	2018-08-17
2018-05-04	2018-09-21
2018-05-11	2018-10-19
2018-05-18	2018-11-16
2018-05-25	2019-01-18
2018-06-01	2019-02-15
2018-06-15	2019-06-21
2018-07-20	2020-01-17

Figure 2: All AMZN option maturities included in the data.

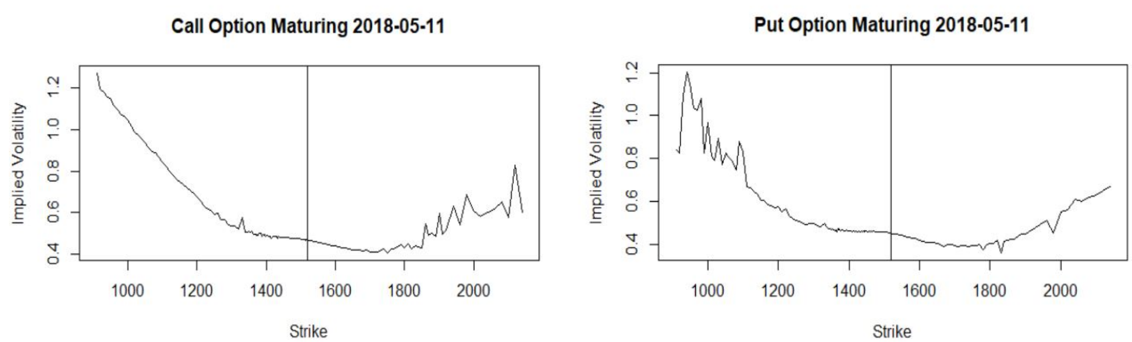

Figure 3: The call and put implied volatility curves for the May 11, 2018 maturity.

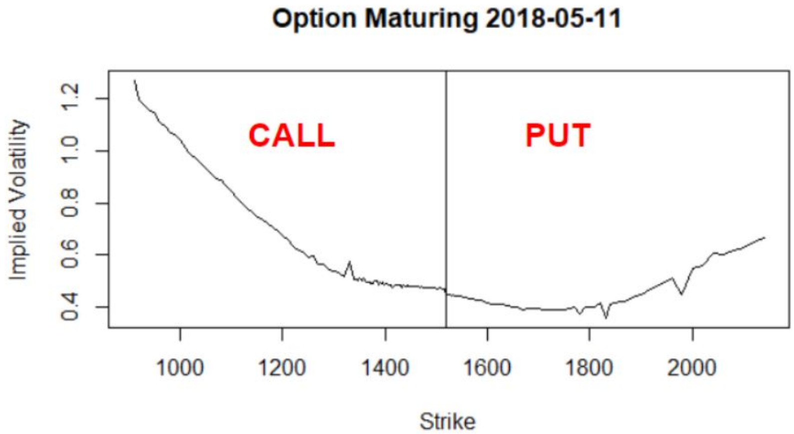

Figure 4: The combined implied volatility curve for the May 11, 2018 maturity.

Before proceeding with the models to determine the risk-neutral probability density, the implied volatility curve needed to be calculated for each maturity. This was done by applying the Bisection Method to the Black-Scholes equation. This was the most efficient way to find the implied volatility because all other inputs for the Black-Scholes equation were given in the market data. The Bisection Method looped through different possible values for implied volatility and returned the value that allowed the Black-Scholes price to match the given market price from the data. As demonstrated in Figure 3, there tended to be a lot of excess noise on the right side of the call implied volatility curves and the left side of the put implied volatility curves across all maturities. In order to reduce this noise, the two implied volatility curves were combined into one single implied volatility curve with call volatility on the left and put volatility on the right as shown in Figure 4. This is a valid method because it has no impact on the actual values of the implied volatility. It helps make the curve smoother so that the implied volatility models used later will handle the data better.

The dynamics of the implied volatility curve can also be observed. For short-dated options, the W-shaped curve is very pronounced, showing a sharp uptick in volatility for options at-the-money. As the maturity of the Amazon option data increases, the W-shaped curve becomes less pronounced. Rather, the shape looks more like a smirk or smile for options maturing.

5 Methodology & Results

In this section we will highlight 3 implied volatility models we determined to best recreate the W-shape curve for all maturities of the Amazon data. These models are Cubic Spline Double SVI Fit, SVI Wing Cubic Spline Fit, and SSVI Wing Cubic Spline Fit.

The first section describes the Cubic Spline Double SVI fit, and begins by explaining some of the initial implementations of SVI. The second section describes the SVI Wing Cubic Spline Fit. The third and fourth section describe the Single SSVI fit and Double

SSVI fit implementation, which resulted in the fifth section, SSVI Wing Cubic Spline Fit.

5.1 Cubic Spline Double SVI Fit

This model addressed the problems we had faced when implementing SVI to interpolate the entire W-shaped implied volatility curve. This parameterization failed to capture the inflection points and changes in convexity when we attempted to fit a single SVI curve to the smile as shown in the figure below and its accompanied risk neutral probability density function where we applied this method to the earliest maturity in the data set, April 27, 2018.

Figure 5: Single SVI Fit and PDF

While the fit tracks the wings closely, it fails to capture the W-shape, and as a result yields a probability density function that is more indicative of a volatility smile in a normal state. However, when we instead adopted a double SVI approach to fit one curve to the left wing and another to the right we found a significant improvement in the fit. The fit is shown below and although the shape of the volatility curve is captured, the fit is not differentiable at the point where the two parameterizations are joined. Therefore, the resulting probability density function is not continuous being that it is the second derivative of this combined fit.

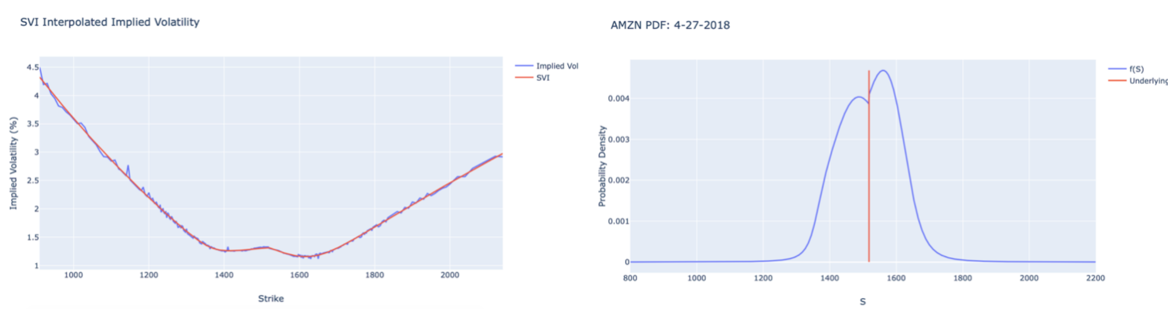

Figure 6: Double SVI Fit and PDF

Despite achieving an effective fit, we were unable to find a meaningful result from the risk neutral probability density achieved from the Breeden Litzenberger formula as this density shown above omits the point of discontinuity at which the two parameterizations meet. However, by omitting this point we achieve the bimodal distribution that we were looking for with the right peak of the distribution being approximately equal to the level that AMZN stock closed at following the earnings announcement. Furthermore, this was an encouraging result that leads us into the final iteration of this model where we smooth over the double SVI fit with a cubic spline fit to ensure the result is continuous and differentiable at every point. With the cubic spline method being composed of piecewise third order polynomials between each knot that is fed into the model, if we select a sufficient number of knots to fit the cubic spline we are guaranteed the continuous and differentiable fit to the implied volatility curve that we require. We were able to determine this optimal number of points to be fed into the model by manually altering the knots used until we achieved a sufficient result. The resulting fit for April 27 is shown below as well as the resulting continuous and bimodal probability density function.

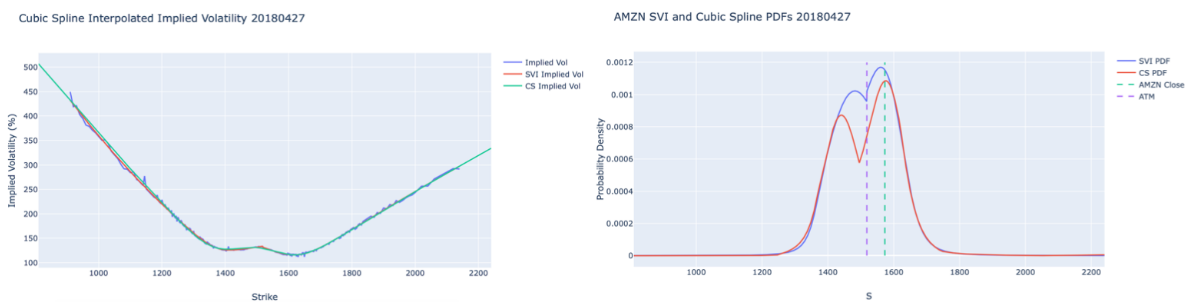

Figure 7: SVI vs. Cubic Spline PDF

The figure on the right compares the probability density functions of the cubic spline fit compared to that of the double SVI fit with the purple dashed line representing the at-the-money level at the time the data was taken and the green being the price AMZN closed at following their earnings announcement. There are several takeaways from this result, namely the superiority of the cubic spline fit as we can see the distribution from the SVI model is not continuous at the at-the-money point whereas the cubic spline result is continuous at every point. Furthermore, the two distributions are similar with the cubic spline displaying a more pronounced bimodal distribution with the higher of the peaks essentially exactly where AMZN closed after its earnings announcement. After completing this process for the first maturity we were able to develop a more general approach that provides a sufficient fit to any maturity when the data is fed into the program, and moreover, can be applied to option data for any equity or maturity to facilitate future research to validate our research on a wider variety of equities. Applying this program to the remaining maturities yields the following result:

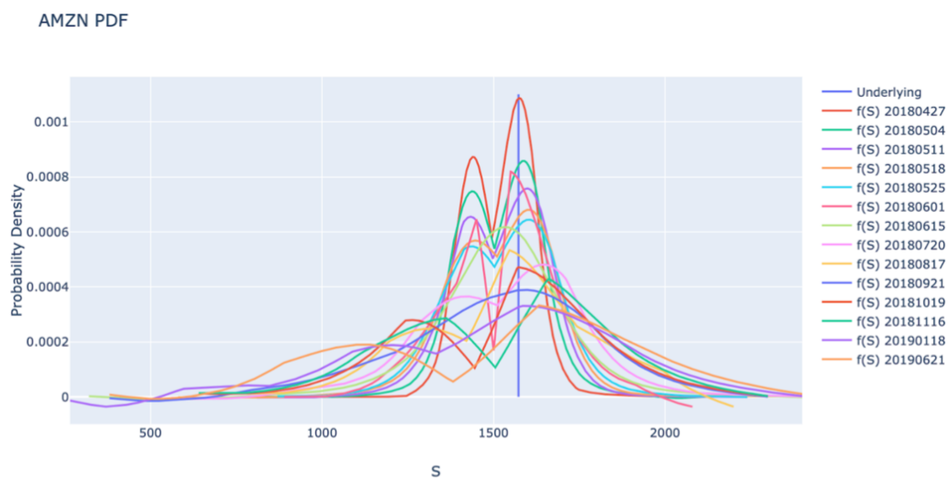

Figure 8: Double SVI Fit PDF for all maturities

As we would expect, the shorter maturities have the highest peaks being that with more time to maturity there is more uncertainty around the realization of the stock price. This uncertainty is reflected in the flatter distributions that exhibit greater excess kurtosis. We found these characteristics and results to be consistent for each model as we will go on to show in the following sections.

5.2 SVI Wing Cubic Spline Fit

This model goes beyond trying to fit a cubic spline over the entire implied volatility curve. This "raw" cubic spline fit was able to give a continuous and differentiable fit on the implied volatility curve, but problems arose when observing the risk-neutral density curve. The W-shape was captured effectively, but there tended to be fat tails in the distribution that should not have occured. A solution to this problem was to apply an SVI fit to the outer portions of the implied volatility curves, or "wings", and proceed with the cubic spline fit over the entire curve. This results in smoother wings for the cubic spline function to work with, thus giving a better fit.

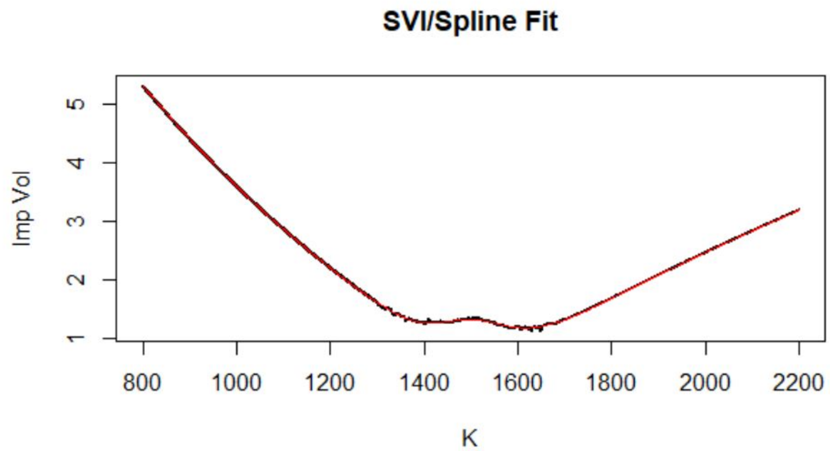

Figure 9: SVI Wing Cubic Spline Fit

In Figure 9, the black line represents the implied volatility curve after an SVI fit was applied to the wings. One can see that there is still some jaggedness that exists in the W-shape while the wings look fairly straight. The red line represents the cubic spline over this SVI wing implied volatility curve. It can be observed that the SVI wing cubic spline fit does a great job in closely tracking the values in the wings while smoothing out the W-shape curve in the model. This method also does a great job of preserving the W-shape, which is very important because a bad fit will cause a normal distribution as opposed to the bimodal distribution that we are looking for. By taking the second derivative of the option prices calculated using the SVI wing cubic spline fit, we can plot the risk-neutral PDF for the option maturity using the Breeden Litzenberger formula.

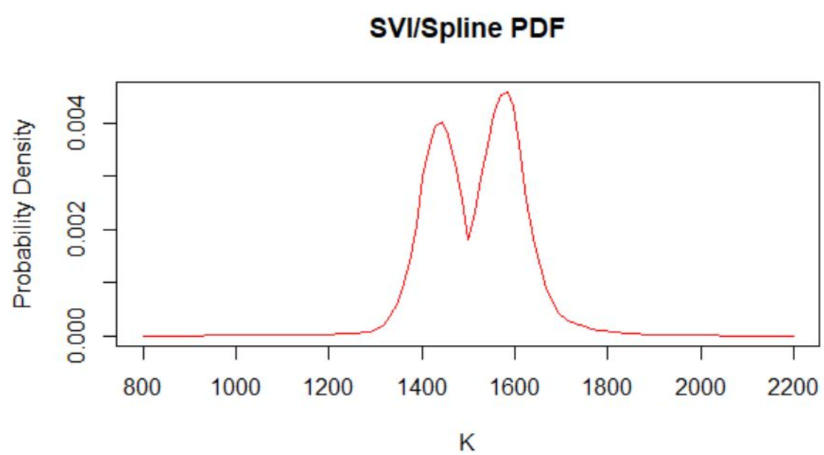

Figure 10: SVI Wing Cubic Spline PDF

Figure 10 shows the risk-neutral density of the set of options on the April 27, 2018 maturity. This plot perfectly exhibits the bimodal distribution that we are looking for.

The function is also continuous and differentiable, ensuring that there are no jumps in the data. The tails of this distribution are close to zero, ensuring that the probabilities are reasonable at each given possible strike.

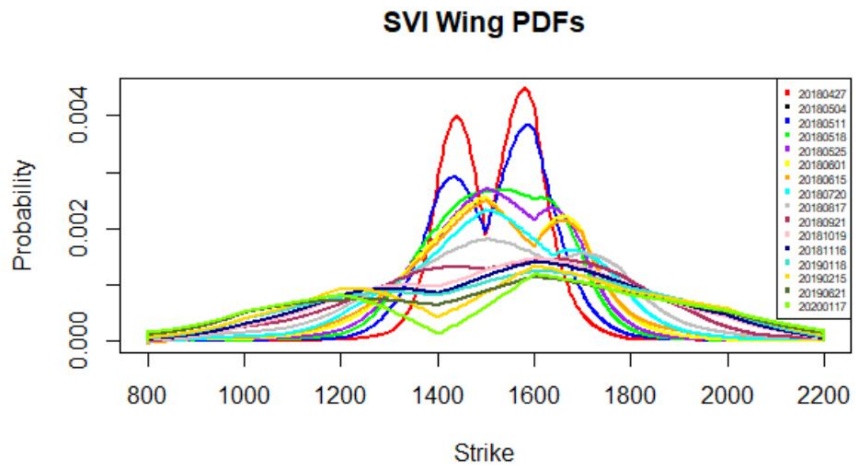

Figure 11: SVI Wing Cubic Spline PDF for all maturities

Figure 11 shows what the risk-neutral PDFs for all maturities look like overlaid with one another. As the time to maturity increases, the peaks of the probability distribution get smaller and smaller and the size of the tails get larger and larger. This makes sense because the stock price can take on many more values as time continues to progress, thus flattening out the distribution. However, the most important observation is that each probability distribution remains continuous and bimodal.

5.3 Single SSVI Fit

Our very first implementation of SSVI was a single SSVI fit to capture the entire W-shaped implied volatility curve using the SSVI equation:

$$\sigma^2(z) = \sigma_0^2[\frac{1}{2}(1 + s_2 z) + \sqrt{\frac{1}{4}(1 + s_2 z)^2 + \frac{1}{2}c_2 z^2}]$$

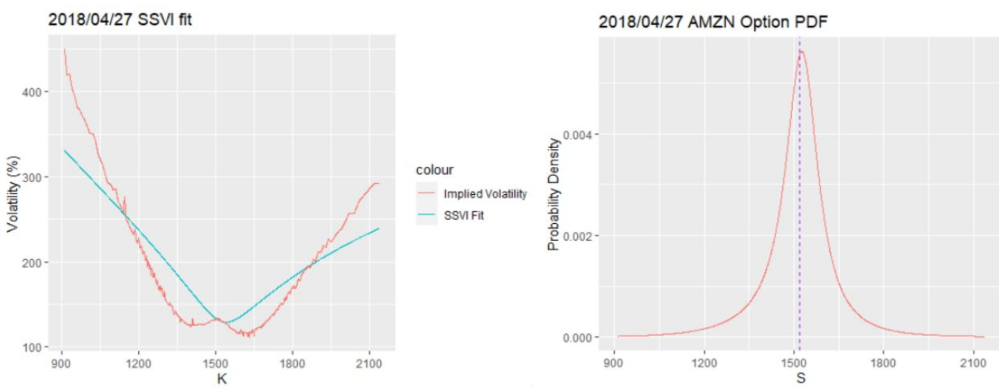

Figure 12: The Single SSVI Fit and PDF.

The single SSVI parameterization fit fails to capture the wings, the inflection points and changes in convexity as shown in the figure below and its accompanied risk neutral probability density function. The inflection point in the Implied volatility curve results in a higher ATM volatility parameter input in the SSVI mode. Thus, the SSVI is unable to capture the wings. Moreover, the property of SSVI is such that it allows for a singular curvature, thus it cannot capture the dips around the inflection point. As a result yields a probability density function that is more indicative of the normal distribution with a single peak.

5.4 Double SSVI Fit

To solve the issues faced with the single SVI fit, we implemented a double SSVI fit. The double SSVI fit will have one fit on each half of the SSVI curve. Thus, we can reproduce W-shape. We also had to account for the change in the normalized strike parameter (z) of the SSVI strike, so we implemented two minimums (m) for each side of the curve to shift the SSVI to the right and left.

$$\sigma^2(z) = \sigma_0^2[\frac{1}{2}(1 + s_2(z - m)) + \sqrt{\frac{1}{4}(1 + s_2(z - m))^2 + \frac{1}{2}c_2(z - m)^2}]$$

We also implemented an optimization technique to determine the best values for each of the SSVI parameters (0 , s2and c2) and the minimums m , which reduces minimize the mean square error between the actual implied volatility and the modeled implied volatility.

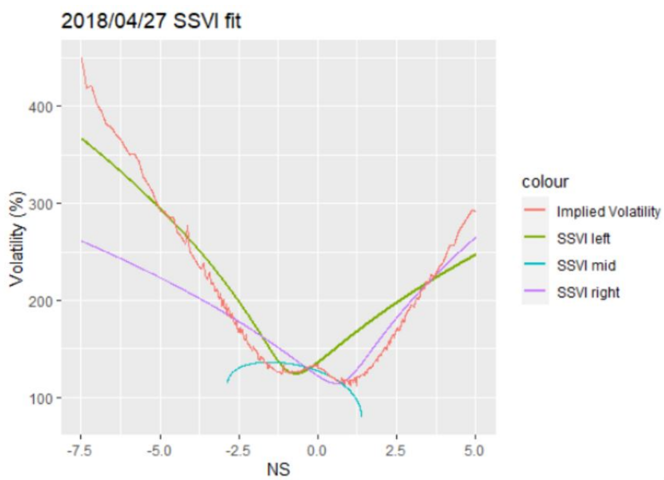

Figure 13: The double SSVI raw fits

Combining the SSVI right and SSVI left, we get the following fit, and resulting risk-neutral distribution:

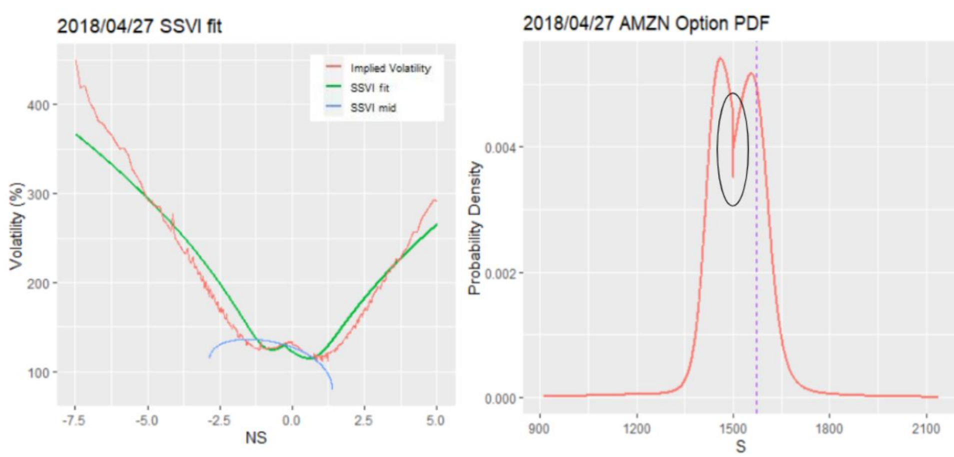

Figure 14: The Double SSVI Fit and PDF

The double SSVI approach does a much better job of capturing the W-shaped implied volatility curve. We are also able to observe the bimodal distribution of stock prices. However, there are a few issues we faced here. The first issue is that when the SSVI is optimizing for the curvature parameter, it will impact the slope. Thus, we see that the green fit is still not the most accurate in the wings. Moreover, the risk-neutral probability of the green line SSVI fit is not continuous because there is a jump where we combine the SSVI right and SSVI left fits. Hoping to solve this problem, we implemented the SSVI mid fit. We replaced the two outlier values (as can be seen in the "pdf" column in the snippet of data below), with the "pdf_mid" values. The combination and resulting data is in the "pdf_com" column below.

pdf	pdf_mid	pdf_com
0.004616494	0.003508535	0.004616494
0.004600053	0.003513117	0.004600053
-6.882961334	0.003517647	0.003517647
6.611789458	0.003522126	0.003522126
0.003966542	0.003526553	0.003966542
0.003982045	0.003530928	0.003982045
0.003997537	0.003535252	0.003997537

Figure 15: Risk-neutral density Data

However, even with these more reliable values, the discontinuity persisted.

5.5 SSVI Wing Cubic Spline Fit

To further improve this model, we looked into combining the cubic spline model with the SSVI model. Our most efficient and accurate results were produced from the SSVI wing Cubic Spline Model. The SSVI Wing Cubic Spline model addresses the issues faced when implementing SSVI to interpolate the entire W-shaped implied volatility curve. We describe the SSVI Wing Cubic Spline Fit in two steps.

Step 1 To ensure better fit in the wings of the W-shaped implied volatility curve, we focused on using SSVI to replicate the wings. We took two break points, namely K = \$1300 and \$1700. Below \$1300 and above \$1700 is where we observed the wings begin to take shape. The first SSVI is fit onto the left side of the curve, from K = \$1300 and below. The second SSVI is fit on the right side of the curve, from K = \$1700 and above. The original implied volatility data points for K in (\$1300,\$1700) are retained. To improve the fits in the wings we adjusted the parameters σ_0, s_2 and c_2 for each maturity by using an optimization function to minimize the mean square error between the modeled implied volatility and the raw combined implied volatility data. Moreover, we also adjusted the initial SSVI equation by considering the new minimum starting point. Thus, the new equation is as follows, where m is the normalized strike price at K=\$1300 and K=\$1700 for the left and right curves, respectively.

$$\sigma^2(z) = \sigma_0^2[\frac{1}{2}(1 + s_2(z - m)) + \sqrt{\frac{1}{4}(1 + s_2(z - m))^2 + \frac{1}{2}c_2(z - m)^2}]$$

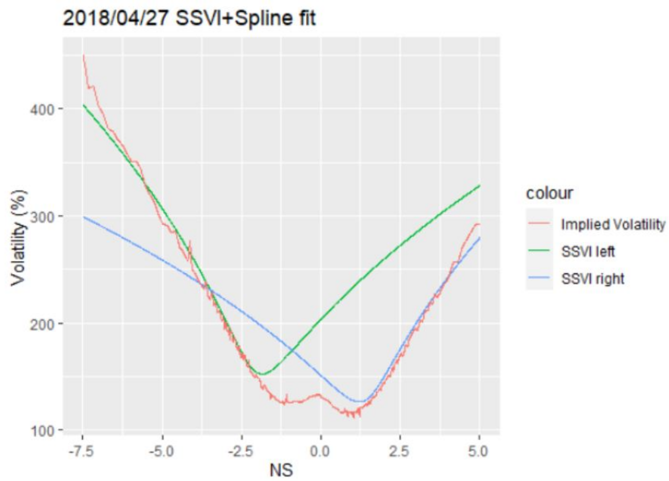

Figure 16: Double SSVI Wings

After fitting the wings, we combined the left side wing with the initial data for K in ($1300,$17000), and did the same with the right side wing. Thus, there is a point of discontinuity at K = $1300 and $1700. The combined graph is as follows:

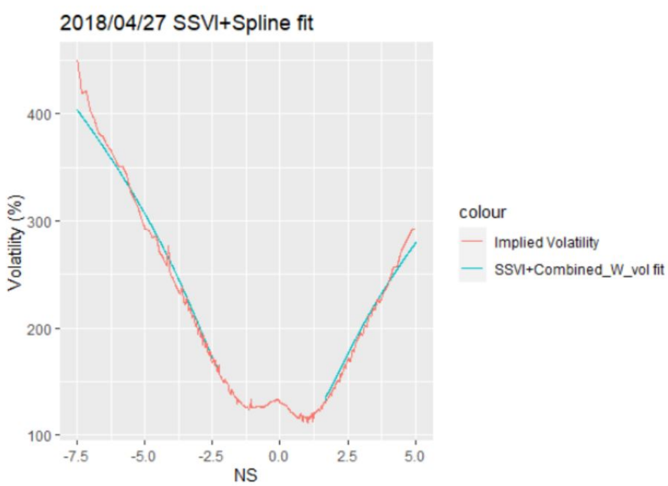

Figure 17: Double SSVI Wings & Original Data combined fit

Step 2 Next, we determined the easiest and most efficient way to ensure the points at $1300 and $1700 are differentiable is by fitting a cubic spline over the entire fit. Thus, we name this model SSVI Wing Cubic Spline Fit. We tried other methods of ensuring continuity at these boundary points, however we determined this was an equally accurate, but more efficient method than enforcing continuity at just two individual points.Thus, we implemented the cubic spline model to fit to the step 1 fit we obtained. While implementing cubic spline, we noticed setting the number of knots was a key factor in obtaining the most smooth risk-neutral density results. Thus, we manually adjusted the cubic spline knots for the SSVI wing Cubic Spline curve. Going forward, an optimization technique can be studied to make the selection process more efficient.

The following graphs represent the step 2 fits. The red lines are the SSVI Wing Cubic Spline model fits, where the dots represent the knots in each fit. The dark green line represents the raw call and put combined implied volatility data.

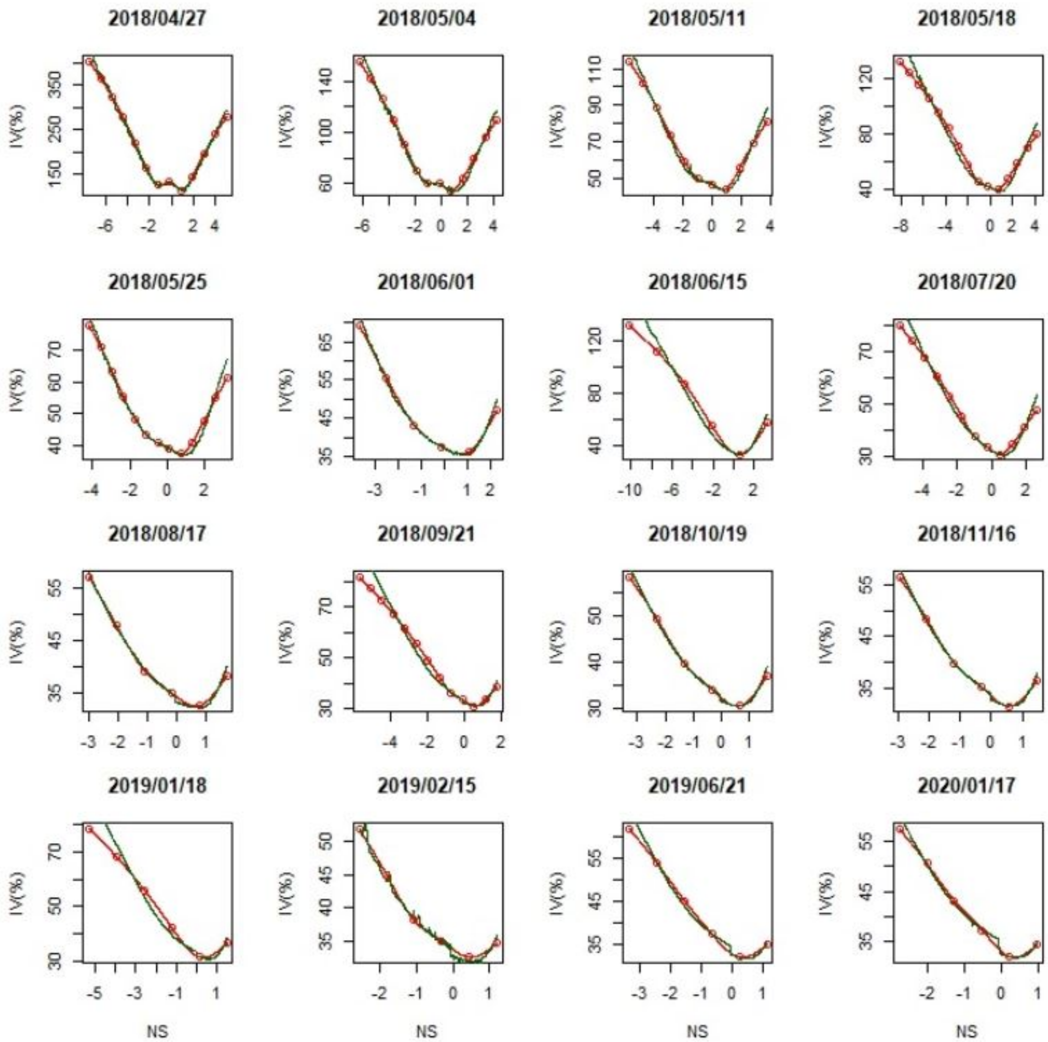

Figure 18: SSVI Wing Cubic Splie Fits

Analysis As seen in Figure 18, overall the SSVI Wing Cubic Spline model is much more improved than we had initially seen. We clearly see the W-shape curve, especially in the short dated options. The W-shape is more defined in the underlying data. The dots represent the number of knots used in the cubic spline interpolation. The number of knots were chosen manually by trial and error to ensure that the fits are not over or under fitted.

The dynamics of the implied volatility curve can also be observed using the modeled fits, and they do stay true to our initial observations of the raw data. As the maturity of the Amazon option data increases, the W-shaped curve becomes less pronounced. Rather, the W-shape becomes wider, thus looking increasingly like a smirk or smile for long data options.

Risk Neutral Density Using the Breeden Litzenberger formula, we obtained the distribution of stock prices. The red lines represent the risk-neutral density function for each stock price. The purple dashed line represents the stock price following earnings announcements at \$1572.62/share.

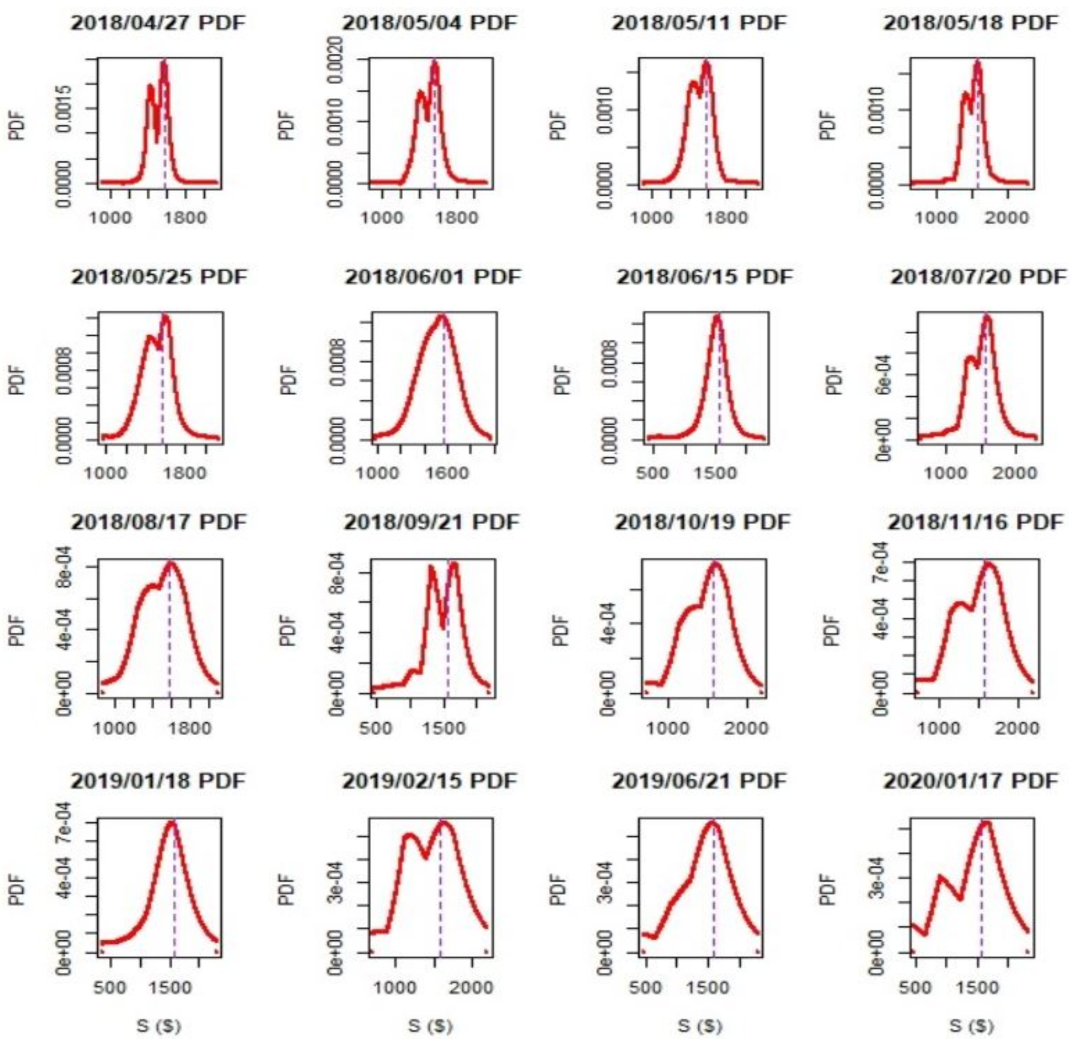

Figure 19: SSVI Wing Cubic Spline PDFs

Analysis From Figure 19, we see they the fits are able to capture the bimodal distribution fairly accurately. They are continuous and differentiable. Moreover, the short-dated maturities (around the first five maturities) display a very clear bimodal distribution as we hoped to achieve. The peaks of these maturities coincide with the closing price of Amazon on April 27, 2018 at \$1572.62. Thereafter, we start to see more jaggedness and some graphs do not show a bimodal curve. This is due to longer-dated options not depicting a W-shaped smile as strongly as the short-data options as we saw in the fits. Thus, the graphs portray a wider or less defined bimodals distribution, with a lower density peak at the closing price of \$1572.62.

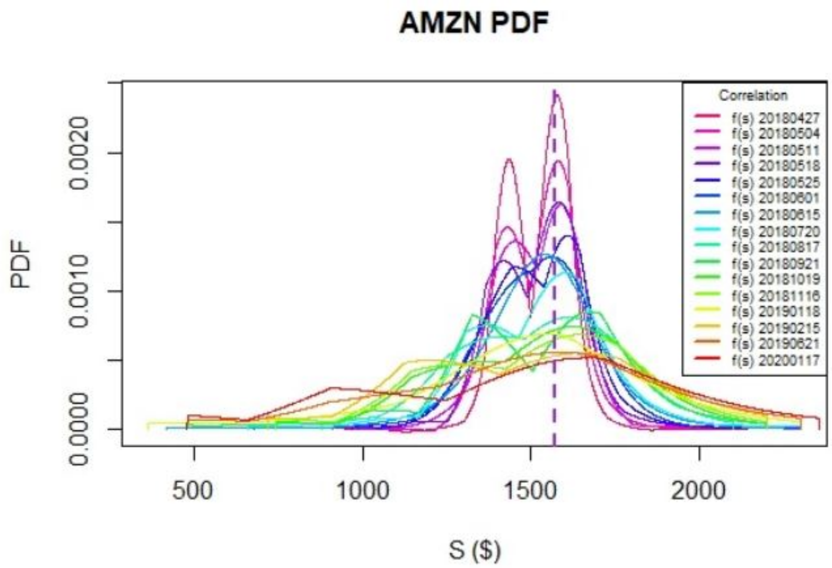

Figure 20: SSVI Wing Cubic Spline PDFs for all maturities

Figure 20 clearly portrays the relationship between the different maturities. The earliest maturities have the most clearly defined bimodal distribution of stock prices, and highest probability densities for the event of under-performing and over performing due to corporate earnings announcements. Moving forward, the densities at the S=$1572.62 decrease. The distributions also widen due to the W-shaped curve being less pronounced. The graph below can be used for an easy comparison of the various maturities risk-neutral probability graphs.

6 Trading Strategy and Improved Pricing Methods

The risk-neutral density distribution of stock prices along with the expected value of the stock price can provide investors with valuable information to facilitate decision making. The simplest trading strategy is to enter long positions on stocks whose expected share price is greater than the current market price and short those with lower expectations as a greater expectation reflects a higher probability of beating earnings while a lower expectation suggests the opposite. The expected Amazon stock prices calculated using the three methods above yielded the results.

Cubic Spline Double SVI Fit

	Maturity	E[S]	Maturity	E[S]
0	2018-04-27	1520.522454	2018-08-17	1528.012168
1	2018-05-04	1521.228683	2018-09-21	1532.040556
2	2018-05-11	1522.747693	2018-10-19	1530.365743
3	2018-05-18	1527.978573	2018-11-16	1529.376044
4	2018-05-25	1525.219392	2019-01-18	1542.005351
5	2018-06-01	1521.301758	2019-02-15	1513.123232
6	2018-06-15	1533.139335	2019-06-21	1528.309534
7	2018-07-20	1535.757050	2020-01-17	1527.983312

Figure 21

SVI Wing Cubic Spline Fit

	Maturity	Expected Price	Maturity	Expected Price
1	2018-04-27	1523.23	2018-08-17	1548.62
2	2018-05-04	1523.49	2018-09-21	1560.11
3	2018-05-11	1510.94	2018-10-19	1551.67
4	2018-05-18	1532.12	2018-11-16	1549.27
5	2018-05-25	1516.52	2019-01-18	1540.45
6	2018-06-01	1535.92	2019-02-15	1522.62
7	2018-06-15	1552.75	2019-06-21	1519.92
8	2018-07-20	1561.24	2020-01-17	1510.66

Figure 22

SSVI Wing Cubic Spline Fit

	Maturity	E[S]	Maturity	E[S]
0	2018-04-27	1518.359	2018-08-17	1520.826
1	2018-05-04	1518.384	2018-09-21	1510.865
2	2018-05-11	1518.776	2018-10-19	1521.076
3	2018-05-18	1518.077	2018-11-16	1516.902
4	2018-05-25	1519.604	2019-01-18	1495.683
5	2018-06-01	1519.505	2019-02-15	1492.744
6	2018-06-15	1520.365	2019-06-21	1481.133
7	2018-07-20	1519.239	2020-01-17	1480.165

Figure 23

The stock price of Amazon at the time the data was taken was \$1517.96. All of the three methods returned an expected stock price higher than this price. As a result, this would be a signal to take a long position in Amazon. As mentioned previously, the price of Amazon after the quarterly earnings announcement was \$1572.62. Therefore, the trading strategy yielded a profit of \$54.66 per share in this instance.

Furthermore, this discrepancy between the expectation and the market price suggests there is arbitrage in the market and creates potential for improved pricing methods. Earnings announcements essentially always result in significant jumps in the stock price either up or down depending whether or not the company beat earnings. Therefore, by incorporating this research we may be able to develop a more robust pricing method that can mitigate the jump in stock price following earnings announcements to make markets more efficient and less volatile. While our initial attempt at pricing AMZN with these distributions yielded a similar price to the market price at the time the data is taken, varying by less than 1%, it is possible that there exists a more robust technique that would yield a different result that would reflect a truer representation of the value of the underlying stock.

7 Future Steps

A future step that can be taken is to apply our research to other equities besides Amazon. It would be very interesting to observe the different sectors of the stock market to ensure that the W-shape implied volatility curve holds consistently prior to company earnings. Some automation will be required to the code to ensure that the cubic spline will contain the proper knots needed to apply a fit over the market data. This is a crucial step because if the option data of a different stock doesn't need as many knots as Amazon did, then this will create overfitting for the risk-neutral PDF of the different stock. Conversely, if the implied volatility curve needs more knots than the amount that Amazon had, then the cubic spline may completely miss the W-shape observed in the middle of the implied volatility curve of the different stock. Lastly, a more intensive trading strategy can be created using the risk-neutral PDFs. While the strategy of taking a long/short position in the stock when the expected stock price is greater/less than the current stock price works sufficiently, it is basic in nature and can be expanded upon in order to maximize returns. Our attempts at finding a more accurate price for the underlying can also be expanded on to potentially make markets more efficient around earnings announcements when they tend to see high levels of volatility.

8 Conclusion

The options market provides relevant information to predict the performance of US equities, particularly around earnings announcements. Our project demonstrates how one can quite accurately obtain the distribution of stock prices around earnings announcements by modeling the W-shape implied volatility curve. Through this project, we were able to complete the three main objectives, namely, explaining the dynamic of the implied volatility smile shape around earnings announcements, modeling the W-shape curve with absence of arbitrage by interpolating strike prices, and determining the stock distribution around earnings announcements. We outlined and implemented three implied volatility models to obtain accurate results. Through the use of stochastic volatility inspired (SVI) parametric models, simple SVI (SSVI) models, and cubic spline interpolation, we replicated the expected bimodal risk-neutral probability distribution of stock prices around earnings announcements.

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