

ACTSC 831 Project, Fall 2008

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1 Introduction

1.1 Scenario

An insurer and a reinsurer are both covering claim amounts of a certain portfolio. Let N denote the number of claims occurred and Y_k denote the size of the k th claim and S denote the total amount of claims. Thus, S can be expressed as $\sum_{k=1}^N Y_k$. The insurer is required to cover all claims that are less than or equal to a specified deductible d . Hence, the total amount required to be covered by the insurer, known as the insurer's retained loss, can be expressed as:

$$S_I(d) = \sum_{k=1}^N \min(Y_k, d)$$

And so, the reinsurer's loss, known as ceded loss (being the difference between the total amount of claims and the insurer's retained loss) can be expressed as:

$$S_R(d) = \sum_{k=1}^N (Y_k - d)_+$$

1.2 Assumptions

The individual claim amounts, Y_k , are positive, independent of the number of claims, N , and i.i.d., with a distribution $F(y)$ and a positive mean μ . We also assume that the deductible d is positive. We also assume that the reinsurer and insurer, respectively, charge premiums of the following form:

$$c_R(d) = (1 + \theta_R)E[S_R(d)]$$

$$c_I(d) = (1 + \theta_I)E[S_I(d)] - c_R(d)$$

where θ_I and θ_R are constants such that $0 < \theta_I < \theta_R \in \mathbb{R}$

1.3 Preliminary Derivations

Let $f(y)$ denote the pf of the Y_k 's, $f_{I(d)}(y)$ denote the pf of $I(d) := \min(Y_1, d)$ and $f_{R(d)}(y)$ denote the pf of $R(d) := (Y_1 - d)_+$. Based on the given assumptions above, one obtains the following results:

$$\begin{aligned} 1. \quad & E[S_I(d)] = E[N]E[I(d)] \text{ and} \\ & E[S_R(d)] = E[N]E[R(d)] = E[N]\{E[S] - E[I(d)]\} \end{aligned}$$

$$\begin{aligned} 2. \quad f_{I(d)}(y) &= \begin{cases} f(y) & \text{if } y < d \\ 1 - F(d) & \text{if } y = d \end{cases} \\ f_{R(d)}(y) &= \begin{cases} F(d) & \text{if } y = 0 \\ f(y + d) & \text{if } y > 0 \end{cases} \end{aligned}$$

$$\begin{aligned}
3. \quad E[I(d)] &= \begin{cases} 1_{\{d \neq 1\}} \sum_{k=1}^{\lfloor d \rfloor - 1} kP[Y_1 = k] + \lfloor d \rfloor (1 - P[Y \leq \lfloor d \rfloor]) & \text{if the Y's are discrete} \\ \int_0^d yf(y) dy + d(1 - F(d)) & \text{if the Y's are continuous} \end{cases} \\
E[R(d)] &= \begin{cases} \sum_{k=\lfloor d \rfloor}^{\infty} kP[Y_1 = k] - \lfloor d \rfloor (1 - P[Y \leq \lfloor d \rfloor]) & \text{if the Y's are discrete} \\ \int_d^{\infty} yf(y) dy - d(1 - F(d)) & \text{if the Y's are continuous} \end{cases}
\end{aligned}$$

1.4 Goals

1. Find the optimal d 's which maximise the insurer's and reinsurer's survival probabilities for specific distributions.
2. Find the optimal d which minimises the reinsurer's VaR and CTE for specific distributions.

2 Goal One: Maximising Survival Probabilities for the Insurer and the Reinsurer

2.1 Objective

With the aid of a software package, one aims to calculate:

$$\max_{d>0} Pr\{S_I(d) \leq u_I + c_I(d)\} \text{ and } \max_{d>0} Pr\{S_R(d) \leq u_R + c_R(d)\} \quad (1)$$

where u_I and u_R are the initial reserves of the insurer and the reinsurer respectively.

Assumptions:

1. d is discrete, taking values from 1 to 199.
2. $u_I = 1.5E[S_I(\infty)]$ and $u_R = 1.5E[S_R(0)]$
3. The Y_k 's are discrete with $pfPr\{Y_1 = k\} = 1/200$ for $k = 1, 2, \dots, 200$
4. $\theta_I = 0.3$ and $\theta_R = 0.35$
5. N is a member of the $(a, b, 0)$ class

We will examine three different distributions of N lying in that class, with specified values for a and b , but first, one can obtain the following expressions, based on the above mentioned assumptions:

1. $S_I(\infty) = \sum_{k=1}^N \min(Y_k, \infty) = \sum_{k=1}^N Y_k = S$, and

$$S_R(0) = \sum_{k=1}^N (Y_k - 0)_+ = \sum_{k=1}^N Y_k = S$$

2. $E[S] = E[S_I(\infty)] = E[S_R(0)] = E[Y_1]E[N] = 100.5E[N]$

$$3. f_{I(d)}(y) = \begin{cases} 1/200 & \text{if } y < d \\ \frac{201-d}{200} & \text{if } y = d \\ 0 & \text{otherwise} \end{cases}$$

$$f_{R(d)}(y) = \begin{cases} d/200 & \text{if } y = 0 \\ 1/200 & \text{if } 0 < y \leq 200 - d \\ 0 & \text{otherwise} \end{cases}$$

4. From the previous derivations and assumptions, we get:

a) $u_I = u_R = 150.75E[N]$

b)

i. $C_I(d) = 1.3E[N]E[I(d)] - 1.35E[N]\{E[Y_1] - E[I(d)]\}$

$$= E[N]\{2.65(E[I(d)]) - 135.675\}$$

$$= E[N]\{2.65(\sum_{k=1}^{d-1} k/200 + d \sum_{k=d}^{200} 1/200) - 135.675\}$$

$$= E[N]\{2.65(\frac{d(d-1)}{400} + \frac{d(201-d)}{200}) - 135.675\}$$

$$= E[N]\{\frac{53}{8000}(d(401-d)) - 135.675\}$$

$$\text{ii. } C_R(d) = 1.35E[N]E[R(d)] = 1.35E[N]\{\sum_{k=d+1}^{200}(k-d)/200\}$$

$$= E[N]\{\frac{27}{4000}[\sum_{k=1}^{200-d} k]\}$$

$$= E[N]\{\frac{27}{8000}(201-d)(200-d)\}$$

c) From **a)** and **b)**, **(1)** becomes:

$$\max_{d>0} Pr\{S_I(d) \leq E[N]\{\frac{53}{8000}(d(401-d)) - 15.075\}\} \text{ and}$$

$$\max_{d>0} Pr\{S_R(d) \leq E[N]\{\frac{27}{8000}(201-d)(200-d) + 150.75\}\} \quad \textbf{(2)}$$

5. And so, the main task at hand is to calculate $Pr\{S_I(d) = k\}$ and $Pr\{S_R(d) = k\}$ for k 's up to and including the floor value of the right-hand-side (as the sums only take discrete values) of the expressions in **(2)**, for each possible value of d . Thus, the next task is to find a mathematical expression for $Pr\{S_I(d) = k\}$ and $Pr\{S_R(d) = k\}$. Luckily, since N is assumed to be a member of the $(a, b, 0)$ class, one can turn to Panjer's formula:

a) Let $g_k := Pr\{S_I(d) = k\}$, we have:

$$g_0 = Pr\{\sum_{k=1}^N \min(Y_k, d) = 0\} = E[f_{I(d)}(0)^N] = Pr\{N = 0\}, \text{ and}$$

$$\begin{aligned} g_k &= \frac{1}{1-af_{I(d)}(0)} \sum_{j=1}^k (a + \frac{bj}{k}) f_{I(d)}(j) g_{k-j} \\ &= \sum_{j=1}^{\min(k,d)} (a + \frac{bj}{k}) f_{I(d)}(j) g_{k-j} \end{aligned}$$

b) Let $h_k := Pr\{S_R(d) = k\}$, we have:

$$h_0 = Pr\{\sum_{k=1}^N (Y_k - d)_+ = 0\} = E[f_{R(d)}(0)^N] = E[(\frac{d}{200})^N], \text{ and}$$

$$\begin{aligned} h_k &= \frac{1}{1-af_{R(d)}(0)} \sum_{j=1}^k (a + \frac{bj}{k}) f_{R(d)}(j) h_{k-j} \\ &= \frac{200}{200-ad} \sum_{j=1}^{\min(k, 200-d)} (a + \frac{bj}{k}) f_{R(d)}(j) h_{k-j} \\ &= \frac{1}{200-ad} \sum_{j=1}^{\min(k, 200-d)} (a + \frac{bj}{k}) h_{k-j} \end{aligned}$$

Hence, the objective becomes finding:

$$\begin{aligned} \max_{d>0} \sum_{k=0}^{\lfloor A(d) \rfloor} g_k, \text{ with } A(d) &= E[N]\{\frac{53}{8000}(d(401-d)) - 15.075\} \\ \max_{d>0} \sum_{k=0}^{\lfloor B(d) \rfloor} h_k, \text{ with } B(d) &= E[N]\{\frac{27}{8000}(201-d)(200-d) + 150.75\} \quad \textbf{(3)} \end{aligned}$$

The following three subsections examine specific distributions of N and calculate **(3)** for each case.

2.2 Case I: $N \sim b(15, 0.6)$

In this case, we have $E[N] = 9$ and, for a binomial distribution, a and b in Panjer's formula are $\frac{-p}{1-p}$ and $(n+1)\frac{p}{1-p}$ respectively. Therefore, we have $a = -1.5$ and $b = 24$. Because we are dealing with a binomial case with a maximum number of 15, a maximum claim amount of 200, and hence a maximum total amount of 3000, one can begin by simply examining the cases for which $S_I(d)$ and $S_R(d)$ can never exceed the floor values $\lfloor A(d) \rfloor$ and $\lfloor B(d) \rfloor$ respectively. If this is true, then one can find deductible values, d , such that:

$\sum_{k=0}^{\lfloor A(d) \rfloor} g_k = 1$ and $\sum_{k=0}^{\lfloor B(d) \rfloor} h_k = 1$. This is indeed the case here. The following are the optimal deductible values for each party:

a) Insurer $d \in \{1, 2, \dots, 163\}$: One finds that for all these cases $\max S_I(d) \leq \lfloor A(d) \rfloor$, implying that the insurer has an exact 100 percent survival probability.

b) Reinsurer $d \in \{81, 82, \dots, 199\}$: Similar to the insurer's case, one finds that $\max S_R(d) \leq \lfloor B(d) \rfloor$, which implies that the reinsurer has an exact survival probability of one using these values of d .

The following cases require more computation, as the number of claims are not restricted there.

2.3 Case II: $N \sim NB(3, 3)$

We have $E[N] = 9$ (same value as in binomial), $a = 0.75$ and $b = 1.5$. The following results are obtained with the aid of the MATLAB (see Appendix for the source code):

a) Insurer: $g_0 = 1/64$ and $\max_{d>0} \sum_{k=0}^{\lfloor A(d) \rfloor} g_k \approx 1$ when $d = 1$.

b) Reinsurer: $h_0 = (4 - 3d/200)^{-3}$ and $\max_{d>0} \sum_{k=0}^{\lfloor B(d) \rfloor} h_k \approx 1$ when $d = 198$.

2.4 Case III: $N \sim Pois(9)$

Same as the previous two cases, $E[N] = 9$, $a = 0$ and $b = 1.5$. With the aid of MATLAB (see Appendix), the following results were obtained:

a) **Insurer:** $g_0 = e^{-9}$ and $\max_{d>0} \sum_{k=0}^{\lfloor A(d) \rfloor} g_k \approx 1$ when $d = 2$.

b) **Reinsurer:** $h_0 = e^{-9 \frac{(200-d)}{200}}$ and $\max_{d>0} \sum_{k=0}^{\lfloor B(d) \rfloor} h_k \approx 1$ when $d = 172$.

This section has shown that, using the above mentioned cases, the insurer and reinsurer have contrasting preferences for choosing optimal values of d : For the insurer, it is optimal to have d low; for the reinsurer, it is optimal to have d high. The following section tackles the second goal of this project.

3 Goal Two: Minimising the VaR and CTE for the Reinsurer

3.1 Objective

One aims to find an expression for: $\min_{d>0} VaR_{S_R(d)}(\alpha)$ and $\min_{d>0} CTE_{S_R(d)}(\alpha)$, with $0 < \alpha < 1$ (4)

In order to do so, one must derive the probability distribution of $S_R(d)$. Before doing so, and proceeding to deriving (4), we make the following assumptions

Assumptions:

1. $N \sim NB(1, \beta)$ and $Y_k \sim exp(1/\mu)$
2. $E[P_R(d)] \geq \delta$, where $0 < \delta < \theta_R \mu \beta$. This implies that:

$$E[P_R(d)] = E[c_R(d) - S_R(d)] = \theta_R E[S_R(d)] \geq \delta$$

Now we turn to deriving the probability distribution of the reinsurer's survival probability.

3.2 Finding the CDF of $S_R(d)$

This can be done using moment and probability generating functions. Let $Z := (Y_1 - d)_+$, $M_X(t) := E[e^{tX}]$ and $P_X(t) := E[t^X]$. Now, based on the independence assumption mentioned at the beginning of the project, we have $M_{S_R(d)}(t) = P_N(M_Z(t))$. Since N is negative binomial, we have $P_N(\zeta) = \frac{1}{1-\beta(\zeta-1)}$. Based on Result 2 in 1.3:

$$M_Z(t) = e^0[1 - e^{-d/\mu}] + \frac{1}{\mu} \int_0^\infty e^{zt} e^{-\frac{z+d}{\mu}} = 1 - e^{-d/\mu} + \frac{e^{-d/\mu}}{\mu(1/\mu - t)} = \frac{1 - \mu t(1 - e^{-d/\mu})}{1 - \mu t}$$

Thus, using algebraic manipulation, one finds $M_{S_R(d)}(t) = \frac{1 - \mu t}{1 - (1 + \beta e^{-d/\mu}) \mu t}$.

This can be broken down into $\frac{1}{1 + \beta e^{-d/\mu}} + \frac{\beta e^{-d/\mu}}{1 + \beta e^{-d/\mu}} (1 - (1 + \beta e^{-d/\mu}) \mu t)^{-1}$.

Finally, one can simply see this MGF as a weighted sum of two distributions. Thus, one finds:

$$\begin{aligned} F_{S_R(d)}(x) &= \frac{1}{1 + \beta e^{-d/\mu}} F_1(x) + \frac{\beta e^{-d/\mu}}{1 + \beta e^{-d/\mu}} F_2(x) \\ &= \frac{1}{1 + \beta e^{-d/\mu}} + \frac{\beta e^{-d/\mu}}{1 + \beta e^{-d/\mu}} \left(1 - \exp\left(-\frac{x}{\mu(1 + \beta e^{-d/\mu})}\right)\right) \end{aligned}$$

$$= 1 - \frac{\beta e^{-d/\mu}}{1 + \beta e^{-d/\mu}} \exp\left(-\frac{x}{\mu(1 + \beta e^{-d/\mu})}\right), \forall x \geq 0$$

with a singleton at zero: $Pr\{S_R(d) = 0\} = P_N(Pr\{Z = 0\}) = \frac{1}{1 + \beta e^{-d/\mu}}$

Thus, $E[P_R(d)] = \theta_R E[S_R(d)] = \theta_R \mu \beta e^{-d/\mu} \geq \delta$. Combining this with the fact that:

$0 < \delta < \theta_R \mu \beta$, we deduce that $0 < E[S_R(d)] < \theta_R \mu \beta \forall d$, and thus, there are no restrictions for values of d in calculating the reinsurer's VaR and CTE . Next, we turn to derive the expressions in (4).

3.3 Finding the Minimum VaR and Corresponding d

First we examine $VaR_{S_R(d)}(\alpha)$:

$$VaR_{S_R(d)}(\alpha) = \inf\{x : Pr\{S_R(d) \leq x\} \leq 1 - \alpha\} \text{ (} x \text{ is non-negative)}$$

$$= \inf\{x : \frac{\beta e^{-d/\mu}}{1 + \beta e^{-d/\mu}} \exp\left(-\frac{x}{\mu(1 + \beta e^{-d/\mu})}\right) \leq \alpha\}$$

$$= \inf\{x : -\frac{x}{\mu(1 + \beta e^{-d/\mu})} \geq \ln\left(\frac{\alpha[1 + \beta e^{-d/\mu}]}{\beta e^{-d/\mu}}\right)\}$$

(for x to exist: $\frac{\alpha[1 + \beta e^{-d/\mu}]}{\beta e^{-d/\mu}} \leq 1$ must be true for values of d)

$$= \inf\{x : x \leq -\mu(1 + \beta e^{-d/\mu}) \ln\left(\frac{\alpha[1 + \beta e^{-d/\mu}]}{\beta e^{-d/\mu}}\right)\}$$

Hence, by the definition of infimum, $VaR_{S_R(d)}(\alpha) = -\mu(1 + \beta e^{-d/\mu}) \ln\left(\frac{\alpha[1 + \beta e^{-d/\mu}]}{\beta e^{-d/\mu}}\right)$,

where d must satisfy $\frac{\alpha[1 + \beta e^{-d/\mu}]}{\beta e^{-d/\mu}} \leq 1$ (5)

By examining the influence of d on $VaR_{S_R(d)}(\alpha)$, one may find the optimal retention value d^* that minimises (5). Indeed, by increasing d from zero, until the expression in the log-function tends to 1 (and hence (5) tends zero), one finds that:

$$\frac{\alpha[1 + \beta e^{-d^*/\mu}]}{\beta e^{-d^*/\mu}} = 1 \Leftrightarrow d^* = -\mu \ln\left[\frac{\alpha}{\beta(1 - \alpha)}\right]$$

Finally, we now can derive the optimal d to minimise the insurer's CTE .

3.4 Finding the Minimum CTE and Corresponding d

We begin by deriving $CTE_{S_R(d)}(\alpha)$:

For reducing notation, let $X := S_R(d)$ and $A := VaR_X(\alpha)$.

$$\begin{aligned}
CTE_X(\alpha) &= E[X|X > A] = \int_A^\infty \frac{xf(x)}{Pr\{X>A\}} dx = \frac{1}{\alpha} \int_A^\infty xf(x) dx \\
&= \frac{\beta e^{-d/\mu}}{\alpha\mu(1+\beta e^{-d/\mu})^2} \int_A^\infty x \exp\left(-\frac{x}{\mu(1+\beta e^{-d/\mu})}\right) dx \\
&= \mu(1+\beta e^{-d/\mu}) \left\{1 - \ln\left[\frac{\alpha(1+\beta e^{-d/\mu})}{\beta e^{-d/\mu}}\right]\right\}
\end{aligned}$$

Similar to $VaR_X(\alpha)$, $CTE_X(\alpha)$ approaches zero as d approaches one from the left. Thus, the optimal retention level d^* in this case is also such that:

$$d^* = -\mu \ln\left[\frac{\alpha}{\beta(1-\alpha)}\right]$$

Hence, the optimal retention level d^* is the same for minimising both the VaR and the CTE for the reinsurer.

4 Conclusion

By examining claims following distributions, one notes significant differences between each model. It is essential to be aware of the assumptions at hand, and the environment in which this model is to serve, in order to formulate both realistic and effective probability distributions. This will protect the main parties involved and hence, optimise harmony in the system.

5 Appendix

Source code for calculating g_k . Warning: The following programs take several minutes to run.

```
%ACTSC 831 Project Code for Insurer
Exp_N=9;
Bin(1)=0.4^15; a(1)=-1.5; b(1)=24; %Binomial initial setup
NB(1)=(1/4)^3; a(2)=0.75; b(2)=1.5;%Negative Binomial initial setup
Pois(1)=exp(-9); a(3)=0; b(3)=9; %Poisson initial setup

MAX_Pr_SI_Bin_d=0; %Maximum Probability of SI set to zero for binomial
MAX_Pr_SI_NB_d=0; %Maximum Probability of SI set to zero for negative binomial
MAX_Pr_SI_Pois_d=0;%Maximum Probability of SI set to zero for Poisson
MAX_NB_d=0; %Initial setup for optimal d for negative binomial
MAX_Pois_d=0; %Initial setup for optimal d for Poisson
Limit=199;
for d=1:Limit
    A(d)=floor(Exp_N*(53/8000*(d*(401-d))+15.075)); %floor(UI + CI(d))
    Pr_SI_Bin(d)=Bin(1); %Initial setup for sum of insurer probability for a binomial sum
    Pr_SI_NB(d)=NB(1); %Initial setup for sum of insurer probability for a negative binomial sum
    Pr_SI_Pois(d)=Pois(1);%Initial setup for sum of insurer probability for a Poisson sum
    for k=1:A(d)
        Bin(k+1)=0;
        NB(k+1)=0;
        Pois(k+1)=0;
        for j=1: min(k,d)
            if j ~d f_j=1/200;
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else f_j=(201-d)/200;
end
Bin(k+1)=Bin(k+1)+(a(1)+b(1)*j/k)*f_j*Bin(k+1-j); %Panjer's formula for binomial
NB(k+1)=NB(k+1)+(a(2)+b(2)*j/k)*f_j*NB(k+1-j); %Panjer's formula for Negative binomial
Pois(k+1)=Pois(k+1)+(a(3)+b(3)*j/k)*f_j*Pois(k+1-j);%Panjer's formula for Poisson
end
Pr_SI_Bin(d)=Pr_SI_Bin(d)+ Bin(k+1); %Sum of g_k's for binomial
Pr_SI_NB(d)=Pr_SI_NB(d)+ NB(k+1); %Sum of g_k's for negative binomial
Pr_SI_Pois(d)=Pr_SI_Pois(d)+ Pois(k+1);%Sum of g_k's for Poisson
end
if MAX_Pr_SI_NB_d<Pr_SI_NB(d) MAX_NB_d=d; %Finding maximum d for negative binomial
end
if MAX_Pr_SI_Pois_d<Pr_SI_Pois(d) MAX_Pois_d=d;%Finding maximum d for Poisson
end
MAX_Pr_SI_Bin_d=max(MAX_Pr_SI_Bin_d,Pr_SI_Bin(d)); %Current maximum with Pr(SI<=A) for binomial
MAX_Pr_SI_NB_d=max(MAX_Pr_SI_NB_d,Pr_SI_NB(d)); %Current maximum with Pr(SI<=A) for negative binomial
MAX_Pr_SI_Pois_d=max(MAX_Pr_SI_Pois_d,Pr_SI_Pois(d));%Current maximum with Pr(SI<=A) for Poisson
end
%ACTSC 831 Project Code for Reinsurer
Exp_N=9;
a(1)=-1.5; b(1)=24; %Binomial initial setup
a(2)=0.75; b(2)=1.5;%Negative Binomial initial setup
a(3)=0; b(3)=9; %Poisson initial setup

MAX_Pr_SR_Bin_d=0; %Maximum Probability of SR set to zero for binomial
MAX_Pr_SR_NB_d=0; %Maximum Probability of SR set to zero for negative binomial
MAX_Pr_SR_Pois_d=0;%Maximum Probability of SR set to zero for Poisson

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MAX_NB_d=0; %Initial setup for optimal d for negative binomial
MAX_Pois_d=0;%Initial setup for optimal d for Poisson
Limit=199;
for d=1:Limit
    B(d)=floor(Exp_N*(27/8000*(d*(401-d))+150.75));%floor(UR + CR(d))
    Bin(1)=(0.4+3*d/1000)^15; %h_0 for binomial
    NB(1)=(4-3*d/200)^-3; %h_0 for negative binomial
    Pois(1)=exp(-9*(200-d)/200);%h_0 for Poisson
    Pr_SR_Bin(d)=Bin(1); %Initial setup for sum of reinsurer probability for a binomial sum
    Pr_SR_NB(d)=NB(1); %Initial setup for sum of reinsurer probability for a negative binomial sum
    Pr_SR_Pois(d)=Pois(1);%Initial setup for sum of reinsurer probability for a Poisson sum
    for k=1:B(d)
        Bin(k+1)=0;
        NB(k+1)=0;
        Pois(k+1)=0;
        for j=1: min(k,200-d)
            Bin(k+1)=Bin(k+1)+1/(200-a(1)*d)*(a(1)+b(1)*j/k)*Bin(k+1-j); %Panjer's formula for binomial sum
            NB(k+1)=NB(k+1)+1/(200-a(2)*d)*(a(2)+b(2)*j/k)*NB(k+1-j); %Panjer's formula for negative binomial sum
            Pois(k+1)=Pois(k+1)+1/(200-a(3)*d)*(a(3)+b(3)*j/k)*Pois(k+1-j);%Panjer's formula for Poisson sum
        end
        Pr_SR_Bin(d)=Pr_SR_Bin(d)+ Bin(k+1); %Sum of h_k's for binomial
        Pr_SR_NB(d)=Pr_SR_NB(d)+ NB(k+1); %Sum of h_k's for negative binomial
        Pr_SR_Pois(d)=Pr_SR_Pois(d)+ Pois(k+1);%Sum of h_k's for Poisson
    end
    if MAX_Pr_SR_NB_d<Pr_SR_NB(d) MAX_NB_d=d; %Finding maximum d for negative binomial
    end
    if MAX_Pr_SR_Pois_d<Pr_SR_Pois(d) MAX_Pois_d=d;%Finding maximum d for Poisson

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end
MAX_Pr_SR_Bin_d=max(MAX_Pr_SR_Bin_d,Pr_SR_Bin(d));    %Current maximum with Pr(SI<=B) for binomial
MAX_Pr_SR_NB_d=max(MAX_Pr_SR_NB_d,Pr_SR_NB(d));      %Current maximum with Pr(SI<=B) for negative binomial
MAX_Pr_SR_Pois_d=max(MAX_Pr_SR_Pois_d,Pr_SR_Pois(d)); %Current maximum with Pr(SI<=B) for Poisson

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