Department of Statistics 2008

EU ETS Pricing Model

Tutor: Dr. Thorsten Rheinlander
Candidate Number: 51334

Submitted for the Master of Risk and Stochastics, London School of Economics

Abstract

This paper serves as a research proposal for deriving a pricing model and an optimal hedging strategy for the spot prices of carbon emissions under the EU Emission Trading Scheme (EU ETS for short). The model proposed in this here is an extension of the Çetin/Verschuere model, in their April 2008 paper Pricing and Hedging in Carbon Emissions. The proposal begins by explaining, in detail, the background of the carbon market under the EU ETS, its pricing system, its different components and its performance until this date, August 2008. The proposal then proceeds to a mathematical part summarising the main works of the Çetin/Verschuere paper, explaining the underlying assumptions of their model, identifying the model's weaknesses based on its assumptions, and giving the researcher guidelines and mathematical suggestions to improving the model. The research proposal's final part explains the methodology in simulating the variables of the original model, in order to give the researcher the key elements for simulating these variables, and new ones for the extended model.

Contents

Figures

Part I: Background of the EU ETS Carbon Market

Carbon finance is a system in which CO_2 (carbon dioxide gas) emissions are monetised into quantifiable units, known as carbon credits or carbon allowances, and traded amongst different entities, establishing a carbon market (as Wikipedia states, a market can be defined as 'any structure that allows buyers and sellers to exchange any goods, services and information'). The main goal of this system is to combat global warming in the most efficient and economical way possible. This is done by giving for each year pre-specified high CO_2 emitting entities emission level ceilings, which is represented by the number of carbon allowances given to them. This gives them the economic choice between spending the necessary expenses on switching to cleaner technologies –and hence not exceeding the emission cap – or buying carbon allowances from the market to offset the excess emissions (or a combination of the two).

Importantly, this system should be designed in such a way that these two choices are the most feasible options for the company to do so. For instance, this can be done by enforcing a penalty fee on entities exceeding its emission caps - which would usually be a more expensive option than buying extra carbon credits or investing in efforts to produce CO_2 below the cap level. It is also possible that policies could be placed to dissuade companies from not compensating for their excess emissions or taking the necessary measures to reducing them. Hence, on a large scale, the aim of the carbon market is to hold high carbon emitting companies responsible and to encourage them to switch to cleaner technologies (such as natural gas or investing in carbon sequestration technologies).

As the aim of this paper is to propose mathematical improvements on the Çetin/Verschuere model on the pricing of carbon allowances traded in the EU (European Union), known as EUAs, the main focus of this section is to explain the EU Emission Trading Scheme, known as EU ETS or ETS for short, especially with regards to the EUAs. I begin by giving a brief history of the establishment of the EU ETS, followed by explaining the trading mechanism and the structure of the scheme in each of its two trading periods until this date: August 2008. I then state the main lessons learnt in the first trading period of the ETS, future recommendations for subsequent ones and then I conclude.

A. History's Irony

In May 1992, the first international measure to address the problem of global warming took place at the Earth Summit in Rio de Janiero establishing a framework known as the UNFCCC (United Nations Convention on Climate Change). The idea of trading CO_2 emissions was first mentioned in this summit (Labatt and White 138) and one of the main goals of this convention was to 'oblige all its signatories to establish national programmes for reducing greenhouse gas emissions'

("MEMO/03/154"). This convention entered into force in 1994, and even though there were 189 countries voluntarily supporting this convention (Labatt and White 9), it was recognised that the UNFCCC would be insufficient to 'halt the increase in greenhouse gas emissions', especially on a global scale ("MEMO/03/154"). And so, in December 1997 'governments took a further step and adopted a protocol to the UNFCCC in the Japanese town of Kyoto'. This was known as the Kyoto Protocol, and it 'sets legally binding limits on greenhouse gas emissions in industrialised countries and envisages innovative market-based implementation mechanisms aimed at keeping the cost of curbing emissions low' ("MEMO/03/154").

The main drive behind the setting such a protocol is that it became more apparent (especially during the early nineties) that global warming is an existing phenomenon - with the potential to disrupt economies (including an estimated global cost of $7 trillion according to Sir Nicholas Stern mentioned in Chapter 1 of Labatt and White (6)) and human wellbeing (for the consequences and global impacts of climate change, I refer the reader to Chapter 8 of Labatt and White). More importantly, it has been increasingly shown that man is the cause of this event, and that if no action is taken immediately, the results will be majorly consequential and irreversible. Thus, the Kyoto Protocol was an attempt to make leader nations responsible for this by committing their governments to the reduction of anthropogenic (man-made) GHGs (greenhouse gases), which have been considered the most significant contributors to global warming.

Currently, the EU ETS is the EU's main market-based mechanism for the Kyoto Protocol, and hence the 'cornerstone of the EU's strategy for fighting climate change' making it 'the largest company-level scheme for trading in emissions of carbon dioxide [...and] the world leader in this emerging market' (Labatt and White 137). According to Directive 2003/87/EC of the EP (European Parliament), the Sixth Community Environment Action Programme of the EP is 'committed to achieving an 8% reductions in emissions of greenhouse gases by 2008 to 2012 compared to 1990 levels' which is an ambitious target as the Kyoto protocol had already provided a lower greenhouse gas ceiling (5.2%) for industrial countries at that time ("MEMO/03/154"). In addition to that, the Community, in the longer-term, is committed to reduce GHG emissions by 'approximately 70% compared to 1990 levels' ("Directive 2003/87/EC"). Until the present date, the carbon market under the EU ETS has been flourishing and has the potential to meet these environmental targets.

The irony behind this is that the 'innovative market-based implementation mechanisms' used for implementing the Kyoto Protocol – and through which the EU ETS is functioning - were American based and were reluctantly accepted by the EU, during the initial stages of the protocol. This was mainly because the EU had been more prone to using a Command and Control approach (Ellerman and Buchner; Labatt and White 141). Furthermore, the US withdrew in 2001 and on 31 May 2002, the EU and all its Member States ratified the Protocol, which came into effect 1 January

2005 (according to EU website sources), using the flexible trading mechanisms originally proposed by the US.

Next to the success of this flexible system and fact that the EU ETS is currently the dominant system in reducing GHGs in the EU, it is important to note that there are (and will be) also several other mechanisms and policies in the EU used for fighting global warming. For instance, there are taxations on goods and services with high CO_2 emissions (Labatt and White V), investments in renewable energies and use of fiscal policies (for more examples, see Directive 2003/87/EC paragraph 23). In fact, as stated in Labatt and White, the Protocol itself 'does not prescribe how emission reductions should be met. It does, however, propose three flexible mechanisms that are designed to help industrialised Kyoto signatory nations, known as Annex 1 countries, meet their emission reduction obligations: namely emissions trading schemes (ETS), Joint Implementation (JI), and the Clean Development Mechanism (CDM)' (10). Thus, one should keep in mind that the EU ETS is part of a bigger picture of creating a carbon constrained economy, and, as time goes by, it is expected that new policies and external forces will have an influence on its current components (including the prices of EUA contracts).

After having mentioned the main history and the ethos of the EU ETS, The next section explains the structure of the EU ETS scheme, with an emphasis on the trading of EUAs, and its different trading phases or periods: Phase 1 (2005-2007), Phase 2 (2008-20012), Phase 3 (2013-?).

B. The EU ETS Structure

The EU ETS is a cap-and-trade system in which participants are allowed to buy and sell carbon allowances electronically whose prices are based on the market forces of supply and demand. The reason why there are different prices for EUAs, and not one price, is because of the availability of different financial contracts (such as futures) for the buying and selling of EUAs in the market. As mentioned earlier, the timeline of this scheme is divided into consecutively exclusive phases. We are currently at the beginning of the second phase, which started in January of this year, 2008. For the first and second phase, each Member State is responsible for developing an NAP (national allocation plan) 'stating the total quantity of allowances that it intends to allocate for that period and how it proposes to allocate them,' as written in Article 9 of Directive 2003/87/EC. Thus, for each phase to present (Phase 1 and Phase 2), EU governments are responsible for distributing EUAs amongst targeted installations. Installations are high CO_2 emitting industries - such as mineral ore refiners and certain industries exceeding 20MW (for more examples, check Annex I of Directive 2003/87/EC).

By the end of each April, installations are required to surrender allowances equal to its CO_2 emissions of the previous year. The government is then responsible for cancelling these allowances and issuing new ones; for this phase and the previous one, generally, an equal number of allowances are and were issued annually (“MEMO/08/35”; Kruger, Slide 8). If the installation succeeds in emitting below its CO_2 cap, it has the ability to sell its excess EUAs at the market price. If, however, the allowances surrendered cannot cover its emissions, the installation will be -amongst other possible dissuasive measures taken by the government -fined a penalty fee (€40 and €100 per excess CO_2 tonne during the first phase and second phase respectively). Companies have the choice to paying this fee either every year or altogether at the end of the phase for all their excess emissions within that period (Seifert, Uhrig-Homburg, and Wagner 7). Paying the penalty fee, however, does not relieve the operator from its obligation to surrender its excess allowances; and so, it will be obliged to compensate for the missing allowances the subsequent year (“Directive 2003/87/EC”). Hence, installations are obliged to take the necessary measures not to exceed the CO_2 cap annually using the most cost-effective means (for a hypothetical example, see question 15 of MEMO/05/84). For this to work effectively, it is essential that NAPs have to ensure that the EUAs are scarce enough for the installations, in order to stimulate EUA trading and create an incentive to invest in cleaner technologies.

Next to EUAs, it is also possible, under the EU ETS, to partially offset excess carbon emission levels by investing in cleaner technology projects abroad. There are two flexible project-based mechanisms – which, were specifically proposed by the Kyoto Protocol (as mentioned earlier) and highlighted in Directive 2003/87/EC: the CDM (Clean Development Mechanism), which involves implementing projects in third countries, and the JI (Joint Implementation), which involves implementing projects in fellow industrialised Kyoto signatories (Annex I countries). These projects are advantageous as they encourage efficient allocation high level of expertise and environmentally friendly technologies to less advanced areas. Also, by performing these projects, operators (owners of installations) receive reduction units that can be converted into EUA units under the ‘Linking Directive’ (“MEMO/08/35”).

However, it is important to note these projects serve mainly as ‘supplemental to domestic action’ (“Directive 2003/87/EC”). Next to this, governments are instructed to hold reserve EUAs for new entrants as to encourage the addition of members to the CO_2 market. Therefore, the EU ETS is primarily focused on achieving its reduction levels domestically and incorporating as many major CO_2 emitters as possible.

After having mentioned the framework and the main mechanisms of the EU ETS, the next sections highlight the main differences between the scheme’s phases to date and the main experiences gained from them.

C. Phase 1 (2005-2007)

C1) Mechanism and Performance

This was the 'trial phase', where experience of optimising market efficiency was to be learnt by carrying it out. There were approximately 11,500 installations within 25 States of the EU (as Bulgaria and Romania were excluded in this phase) representing close to half of EU CO_2 emissions and 40% of total GHG emission and NAPs were submitted early 2004 ("MEMO/05/84"). JI ERUs (emission reduction units) were not to be included in this phase, while the CDM's CERs (certified emission reductions) were. About 2.2 billion allowances were issued annually (Kruger, 8) and with accordance to Directive 2003/87/EC, 95% of the allowances in this phase were to be issued free of charge. By the end of the period, excess allowances of installations would be deemed worthless – could not be 'banked'- unless the government had agreed to compensate them with an equivalent number of allowances for the new period.

Regarding the price performance of the EUAs during this period, the price 'increased more or less steadily' ("European Union") peaking in April, to about €30 according to ECX carbon prices. By near mid-May of the same year, prices plummeted to less than €10. According to sources, this was mainly due to the combined effect of the early releasing of national results and the spread of rumours that the EU carbon market was long. This means that the caps were generously set for the installations making the prices of EUAs significantly low. And so, as this became more apparent towards the end of the phase that EU caps, 2007-EUA-contracts gradually plummeted, reaching a near zero by December 2007.

C2) Lessons Learned from Phase 1

According to several recent sources, the main purpose the 'trial phase' served was the establishment of necessary infrastructure - and allowances units for the carbon market to function - rather than achieving environmental goals; there was a 1.9% rise of GHG emission during this phase according to Wikipedia. Also, by having some historical data and information, the EU and its Member States have attained a 'better feel' on the system and are now more able to improve the effectiveness and efficiency of the carbon market. The main lessons aimed to be incorporated in the Phase 2 are:

C2i) Proper emission level cap settings (not as generous as phase 1)

C2ii) Greater transparency (Labatt and White 231) and harmonisation between the different entities, as there were frequent delays in validating the NAPs and issuing EUAs ("MEMO/06/02"). The importance of having harmony and good intercommunication in such a system is reflected by the following quote: 'climate change must be addressed in a coordinated way if we are to have any chance of succeeding in reducing the risk in time' (Labatt and White 162).

D. Phase 2 (2008-2012)

D1) Mechanism and Performance

This is the first commitment period of the Kyoto Protocol, where the EU is obliged to reduce GHG emissions by 8%. This phase includes all 27 EU members plus 3 non-EU members: Norway, Iceland and Liechtenstein. There are currently more than 10,000 installations 'in the energy and industrial sectors' ("MEMO/08/35") (at the present time, there is a lack of sources stating the total number of installations). In this phase, JI ERUs are included and the NAP has 'to specify the maximum amount of JI and CDM credits that may be used for compliance purposes by installations under the ETS' ("MEMO/06/02"). Furthermore, aviation emissions are expected to be included by 2011, according to EU sources.

Also, as a precaution for having generous cap settings, allowances are on average are '6.5% below 2005 levels' ("MEMO/08/35") and, with accordance to Directive 2003/87/EC, 90% of the allowances in this phase were freely issued at the beginning of the period. Unlike the first phase, carbon allowances in this phase can be 'banked' to the subsequent trading periods. This means that excess carbon allowances of installations this phase will not be deemed worthless during the next one. Technically, the government does so by replacing the old EUAs with and an equal number of new EUAs.

D2) Lessons Learned from Phase 2

Until now, August 2008, there have not been any significant EUA price drops similar to the April 2006 plummet. The next section turns to the lessons learnt until now, followed by possible improvements for future trading periods.

E. Recommendations for the Next Phase

Until present date, the structuring of the transactions and trading mechanisms of the subsequent trading period's post 2012 are quite vague, especially due to the fact that carbon finance is such a new field. However, based on the experiences gained from the 'trial phase', marketing experts and EU economists/politicians have given recommendations of how the following phases should proceed.

For instance, Labatt and White recommend that the EU ETS should freely provide for the public important market information - for example on a quarterly basis as opposed to currently annual basis for the EU ETS issued (239). Furthermore, the public should be provided with the relevant information to buy wisely, as this would help optimise the transparency of the scheme and hence increase its effectiveness in reducing GHGs. Next to this, Labatt and White also emphasise on the importance of ensuring competition within the system, which would under capitalist theory optimise market pricing, effectiveness and efficiency.

Besides these recommendations and others written by scholars and marketing experts, an official proposal regarding the following phases has been written with a final decision on its approval/disapproval expected to be announced by 2009 (“MEMO/08/35”). These are the main recommendations for the future phases (Main Source: “MEMO/08/35”):

i) The trading periods would be eight years, instead of five, as stated by Directive 2003/87/EC. This would be useful as companies often require time ‘to make significant investment decisions in the line with their carbon abatement costs’ (Labatt and White 150).

ii) One system would issue carbon allowances to all EU ETS states instead of having NAPs for each country, increasing harmony and promoting fairness in trading among the different players in the system.

iii) Unlike the previous two phases, annual caps will be linearly decreasing every year, increasing the scarcity of carbon allowances, further motivating companies to emit less GHGs into the atmosphere.

iv) 40% of the allowances in phase 3 will be issued for free - as opposed to 95% in Phase 1 and 90% in Phase 2 – while the rest will be auctioned, as an aim to increase market efficiency and transparency by creating the ‘greatest incentive for investments in a low-carbon economy’(“MEMO/08/35”).

v) Including other GHGs (notwithstanding the fact that CO_2 is currently the main priority, as it constitutes about 77% of total GHG (Labatt and White 241) of which 80% of it is caused by industrial processes (30)), new industries (e.g. aluminium and ammonia producers) and opting out small installations – as to optimise market efficiency by focusing on the significant GHG producers.

F. Conclusion

Even though we have not yet reaped the environment benefits of the EU ETS, its very establishment and large-scale incorporation will eventually lead to a world of carbon constrained nations, given that it remains successful. Currently there is no other alternative way in minimising GHG emissions on a global scale. And so, by creating a market of ‘environmental virtue’, the EU ETS may be considered the most innovative system created until this date. By involving everyone in the system and giving them the freedom to pursue their interests under market system constraints, a global effort can be achieved.   As the father of capitalism once stated, regarding the establishment of competitive markets: ‘By directing that industry in such a manner as its produce may be of greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention' ("Biography"). Thus, it is to everyone's benefit to ensure the long-term success and prosperity of such a system.

In order to achieve this, the system has to be made as stable and fluid as possible (i.e. without significant disruptions). EUA prices play a significant role in this. This is because price glitches and shocks can significantly threaten several market entities, which, in turn, could threaten the market as a whole if not resolved. One can go as far as using the following analogy: 'blood is to body as EUA prices is to the EU ETS'. Therefore, it is crucial that companies are well equipped in implementing effective hedging strategies to counter significant EUA price movements. This is done through stochastic modelling. By explaining in detail the carbon market and its different mechanisms in this part, the next part turns to deriving a mathematical model in capturing the EUA prices. Part II begins by explaining the Çetin/Verschuere model, proceeds to identify its main weaknesses and gives key points in tackling these weaknesses and, hence, deriving an enhanced model.

Part II: Extending the Çetin/Verschuere Model

'A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time' ("Stochastic Modelling").

The model is based on a standard filtered probability space $(-, F, (F_t)_{t,0}, P)$, on which all subsequent stochastic processes are defined. The model consists of the carbon market variables:

1) The spot price of one allowance unit, EUA0, maturing at the end of the current year, denoted as P_t

2) The price of a forward contract of one allowance unit for the following year, EUA1, denoted as S_t

3) A stochastic parameter indicating the position of the market, long or short, denoted as θ_t

The objective of this model is to find a mathematical expression for pricing EUA0 contracts and finding its optimal hedging strategy. This is done by expressing P_t in terms of EUA1 contracts and with respect to the position of the market θ_t. Thus, it is assumed that S_t and θ_t are the only exogenous variables with respect to P_t. The following section highlights the main derivations of the model:

A. Mathematics of the Çetin/Verschuere Model

A1) EUA0 Prices at Maturity

As we are dealing with stochastic differential equations, it is imperative to derive expressions for initial and/or terminal conditions. In this case, we have the terminal condition given. P, at maturity time T, has the following expression:

$$P_T = \begin{cases} S_T + K, \text{if market is short} \\ 0 \qquad\quad , \text{if market is long} \end{cases}$$

Where K is the penalty fee per unit of non-surrendered carbon credit.

These values are based on the following assumptions:

(A1i) P is completely based on market forces with companies trying to maximise their profits and minimise their costs. And so, when the market is short and hence there is a scarcity of EUA0 credits, companies with excess EUA0 can sell these contracts to companies falling short of current year carbon allowances. Due to the fact that these companies will be required to surrender an equivalent amount of non-surrendered EUAs the following year, selling companies should not exceed the price ST+K. If this is the case, companies falling short would find it more economical to pay the penalty fee K and purchase EUA1 contracts instead. Thus, the market-equilibrium price is S_T+K, when the market is short.

(A1ii) Banking is not allowed, neither within nor between phases. And so, also with assumption (A1), if the market ends up long, EUA0 contracts will be deemed worthless at their maturity. This is because companies, in this case, will have met or emitted below their required targets and would find it pointless to buy carbon units with no future value for the following surrendering time period.

A2) The Mechanics of S and θ

The general expression of the EUA1 price process is:

$dS_t = S_t \mu(t, S_t, \theta_t)dt + S_t \sigma(t, S_t, \theta_t)dW_t$ ……………………………………***Equation 1***

with S_0=s, a constant; and W a standard Brownian motion which is independent of θ for all t P-almost surely.

Çetin and Verschuere make the following simplifications:

A2i) $\mu(t, S_t, \theta_t) = \mu + \alpha\theta_t$ where μ and α are constants

A2ii) $\sigma(t, S_t, \theta_t) = \sigma$ is a constant

A2ii) θ_t is a time-homogenous Markov chain such that:

1) $\theta_t \in \{-1,1\}$: -1 if the market is short, 1 if the market is long

2) R_t, the transition probability matrix, with $R_t(i,j)= P(\theta_t=j|\theta_0=i)$; Q, the generator matrix; and λ(i), the intensity of remaining in state $i \in \{-1,1\}$, are:

$\frac{dR_t}{dt} = R_t Q$, $Q = \begin{bmatrix} -\lambda(1) & \lambda(1) \\ \lambda(-1) & -\lambda(-1) \end{bmatrix}$ and λ(i) = λ

In general, if we assume that we have a generator matrix Q such that $\frac{dR_t}{dt} = R_t Q$, then a Markov process can be written as:

$\Theta_t = \Theta_0 + \int_0^t Q\Theta_s ds + N_t$ ……………………………………***Equation 2***

Where $\Theta = (\theta^1, \theta^2, ..., \theta^m)'$ and N is a martingale (Al Hussaini and Elliot 4).

Specifically in this model, we have $\theta_t = \theta_0 - 2\lambda\int_0^t \theta_s ds + N_t$

3) θ_t is observed twice: once before the maturity date of the EUA0 contracts and at the maturity date.

A3) Finding the Fair Price of EUA0 Contracts

As S has two sources of uncertainty, W and θ, we are in an incomplete market set up, implying that there can be several hedging strategies for the same derivative (Çetin and Verschuere, 6). This also means that there can be several equivalent martingale measures for the underlying asset corresponding to each hedging strategy. Hence, one seeks to find a method in choosing the martingale measure that gives the optimal hedging strategy for the asset. In this case, Çetin and Verschuere use the local-risk minimisation approach for obtaining the fair price of P_t and deriving its optimal trading strategy.

Let G denote the 'filtration modelling the information structure of the market' (9) such that

$$G_t = \begin{cases} F_t^S & for\ t < T \\ F_T^S \vee \sigma(\theta_T) & for\ t = T \end{cases}$$

As we do not see θ, except once, before maturity, we are interested in using a projection of it on the information we have: the data regarding the EUA1 prices. For this we create projection variable of θ, denoted by $\overline{\theta}$, such that:

$$\overline{\theta_t} := \begin{cases} E[\theta_t | F_t^S] \text{ for } t \le t_0 \\ E[\theta_t | F_t^S, \theta_{t_0} = j] \text{ for } t > t_0 \end{cases}$$ where t_0 is the time the true value of θ is announced.

Since $P_T = \begin{cases} S_T + K, \text{for } \theta_t = -1 \\ 0 \quad , \text{for } \theta_t = 1 \end{cases} = (S_T + K)1_{[\theta_T = -1]} = (S_T + K)(1 - \theta_T)/2$, then, under the local-risk minimisation approach - and assuming no arbitrage in the market - the fair EUA0 price is $\overline{P_t} := E^*[P_t | G_t] = E^*[(S_T + K)(1 - \theta_T)/2 | G_t]$ (under the minimal martingale measure P^*).

Through filtering theory, probability theory and the use of Itô's formula, Çetin and Verschuere derive the EUA0 fair price as:

$$\overline{P_t} = \begin{cases} Z_t^h + h(t, S_t, \overline{\theta_t}), & \text{for } t < t_0 \\ h(t, S_t, \overline{\theta_t}) - Z_t(S_t + K)/2, & \text{for } t \ge t_0 \end{cases}$$ ……………………………………***Equation 3***

where:

A3i) $Z_t = 1_{[t=T]}(\theta_T - \overline{\theta_T})$

A3ii) h(t, x, y) is a function mapping its parameters on to the real line solving the following PDE boundary problem:

$$h_t + \frac{1}{2}(\sigma x)^2 h_{xx} + \frac{1}{2}\left(\frac{\alpha(1-y^2)}{\sigma}\right)^2 hyy + \alpha x(1-y^2)h_{xy} - \left(2\lambda y + \frac{\alpha(1-y^2)}{\sigma}(\mu + \alpha y)\right)h_y = 0$$
$$h(T,x,y) = (x+K)(1-y)/2$$

where $h_t = \frac{\partial h(t,x,y)}{\partial t}$, $h_{xx} = \frac{\partial^2 h(t,x,y)}{\partial x \partial x}$, $h_{xy} = \frac{\partial^2 h(t,x,y)}{\partial x \partial y}$, $h_y = \frac{\partial h(t,x,y)}{\partial y}$

A3iii) $Z_t^h = E[h(t_0, S_{t_0}, \theta_{t_0}) - h(t_0, S_{t_0}, \overline{\theta_{t_0}})]\ \forall \mathrm{t} \le \mathrm{t}_0$

As for the optimal hedging strategy for EUA0 contracts, denoted as $\overline{\xi_t}$, Çetin and Verschuere derive its expression, under the specific case of no intermediate announcements (t_0=T):

$$\overline{\xi_t} = \frac{\partial h(t, S_t, \overline{\theta_t})}{\partial S_t} + \frac{\alpha\left(1-\overline{\theta_t}^2\right)}{\sigma^2 S_t}\frac{\partial h(t, S_t, \overline{\theta_t})}{\partial \overline{\theta_t}} \quad \forall t \in [0,T]$$

These are the main derivations of the model. The next section both identifies the main constraints of the model and gives the mathematical recommendations for improving it. One may consider this section as the heart of the research proposal within which lie the main technical guidelines for improving the model.

B. Proposed Improvements on the Çetin/Verschuere Model

'So we don't have to arrive at the correct asset allocation scheme, just a better one than most people currently have, and one based on plausible judgements. We have the tools already available for this. […] Successful models allow us to extend our knowledge and understanding into new areas, until eventually the weight of the initiating assumptions acts as a greater and greater drag on expansion, and a new model takes over, better able to deal with the problems on which the old model foundered' Barry du Toit, Principal Quantitative Analyst (3,5-6)

B1) The Advantages of the Current Model

More importantly than the model's ability to simulate prices close to real ones during the first phase is its very formulation. Because the carbon market is significantly young, there is scarce stochastic literature in pricing its assets and so, by providing eligible models for the market will positively contribute to producing improved ones in the future. Specifically, Çetin and Verschuere derive their model through a synthesis of several different disciplines, such as filtering theory, Markov processes and risk minimisation approaches. This allows specialists from different fields to contribute specific improvements upon the model. Thus, the main importance of this young model is to act as a general framework of study upon which other more sophisticated and powerful models can be derived.

Furthermore, by already having a successful model to work with, one is not confined to the task of creating superior models from scratch. Rather, by having a successful mathematical expression for the market's prices, it is far simpler to improve upon its mathematics (by adding more variable terms and/or relaxing the model's assumptions) and derive a better pricing model. Hence, the derivations of Çetin and Verschuere could be used as a good starting point for establishing a more effective model for the EU ETS.

B2) The Constraints of the Current Model

One can consider it an axiom that there is trade-off between having a simple model as opposed to having a realistic one. Most of the constraints in the following list have been mentioned by Çetin and Verschuere:

B2i) *Model does not incorporate banking*: This may have been suitable only for 2007 EUA0 contracts, however, as banking within and in-between phases is allowed, it is necessary to include that feature in the model.

B2ii) *Model has only one intermediate announcement at a predetermined date*: As explained in the first part, hearing rumours and receiving information about the market position can significantly alter the EUA prices overnight. However, even though it is recommended to have more than one announcement throughout the year, the number and dates of intermediate announcements are unknown.

B2iii) *There are only two EUA contracts traded in the market:* In reality there are several different EUA contracts issued with different maturity dates and from different indices. Thus, the model assuming simply one EUA0 contract for this year and only one specific type of future contract, EUA1, is highly simplistic. In reality, there are several –but not numerous- other types of contracts in the market, of which some mature at the same date (more than one EU0 contract). It is also expected that CER and ERU contracts will have an influence in reducing the EUA prices, since they provide the system with an alternative to EUAs. Thus, it is not realistic to assume that only one type of EUA price process influences the EUA0 price process.

B2iv) *Similar volatility across time*: The EUA1 Brownian motion component times a constant gives more or less homogeneous fluctuations until maturity. Assuming that carbon prices behave/will behave similarly to other financial assets and based on the 'stylized facts' of asset returns, these near consistent fluctuations of the EUA prices are unrealistic as there usually tends to be clustering of calm and violent movements in a financial market (Rheinlander, Asset Price 1).

B2v) *The transition probabilities of the market position parameter θ_t are symmetrical*: This implies an equal treatment in upper and lower movements of the EUA1 prices, S_t, where in reality, financial instruments tend to have a stronger lower pull. This is mainly because of people's psychology of assuming the worst case scenario. Hence, it is necessary to create an asymmetry in the drift term

$\mu(t, S_t, \theta_t)$ of the EUA1 model with a skew towards downward movements, in order to reflect people's risk adverse behaviour.

B3) Tackling the Weaknesses

The following list is an attempt to tackle the above mentioned constraints, and thereby improve the model. It is recommended to incorporate each modification at a time and then attempt to combine all these improvements in one final model.

B3i) <u>*Incorporating the concept of banking*</u>: No longer the value of the EUA0 price at maturity will be zero when the market is long. A good economic value of this price would be the expected price of the EUA1 during at its maturity when the market is short at that time. This is known as the indifference price, and the philosophy behind this is that a an entity would be interested in buying the EUA0 contracts, when the market is currently long, if they believe it will cover their requirement of surrendering carbon allowances the following year, if the market is short. Mathematically, Çetin and Verschuere propose: $P_T = \begin{cases} S_T + K, & \text{if market is short} \\ E[1_{[\theta_{2T}=-1]} \mathrm{S}_{2\mathrm{T}} \mid \mathrm{S}_\mathrm{T}, \theta_T = 1], & \text{if market is long} \end{cases}$

Clearly, this is under the assumption that there are only two types of EUA contracts with trading periods being of the same size (240 trading days in this model). One can relax this assumption and have two different trading period lengths resulting in the EUA0 price at maturity being:

$$P_{T1} = \begin{cases} S_{T1} + K, & \text{if market is short} \\ E[1_{[\theta_{T2}=-1]} \mathrm{S}_{\mathrm{T2}} \mid \mathrm{S}_{\mathrm{T1}}, \theta_{T1} = 1], & \text{if market is long} \end{cases}$$

where the first trading period is [0, T1] and the second being [T1, T2].

B3ii) <u>*Replacing one deterministic jump at a pre-specified date with random jumps at random times*</u>: It is possible that at some future time the market will be become absolutely transparent that one would not need a projected estimate of the market position. In this case, one could implement an improved version of the results obtained in Section 3 in the Çetin/Verschuere paper. However, at the present time we assume that the market position is officially announced once at maturity time, and hence, we have to model for that.

With exception of the intermediate announcement date, the model is continuous. One can include jumps by modifying the EUA1 process. This will not only tackle the issue of having random intermediate announcements, but it will also take into account random factors in the market which could have a role in significantly altering the price overnight (e.g. natural catastrophes, riots, etc.). For instance, the EUA1 price process in ***Equation 1*** can be written as:

$$dS_t = S_t \mu(t, S_t, \theta_t)dt + S_t \sigma(t, S_t, \theta_t)dW_t + df(X_t) \text{ ……………………………………} \textbf{\textit{Equation 4}}$$

where X_t is a Lévy process and the function f ensures that S_t is non-negative.

A possible candidate for X can be a compound Poisson process with $X_t = \sum_{i=1}^{N_t} Y_i$, where the Y_i's are iid (independently and identically distributed) and N is a Poisson process with a constant positive intensity. Since we are assuming that the carbon market abides with the 'stylized facts' of equity markets, it is suggestible to have the Y's skewed downwards, as to satisfy the gain/loss asymmetry property (Rheinlander, Asset Price 1). Other possible candidates can be Gamma and Inverse Gaussian processes. The advantage of these two types of processes over the compound Poisson process is that they are of a Type B Lévy process, which is more general, but also more challenging to implement.

Next to the process, finding a suitable function f could be challenging, as the researcher(s) would have to find such a function that would include negative jumps and avoid S_t hitting a negative value for all values of X. A possible candidate for f, based on speculation, could be:

$$f(X_t) = \begin{cases} \max(S_t + X_t, 0) & \text{if } \mathrm{X_t} \leq 0 \\ \gamma_t X_t & \text{if } \mathrm{X_t} > 0 \end{cases}$$ ……………………………………***Equation 5***

where γ_t is used to control the size of positive jumps at different time dates (based on the market's behavioural system at different time periods).

Also, as it is possible that there will be fixed time dates where announcements of the market position will take place – as suggested by scholars - one can easily incorporate this in f, by assigning the function with specific values when the market position is announced, at the pre-specified dates.

As a result of adding a function of a Levy process on to the EUA1 model, one would no longer be able to rely on the simple stochastic theory. Rather, one would resort to using more complex mathematical techniques in obtaining the fair price and an optimal hedging strategy for the EUA0 contract. For instance, Çetin and Verschuere's usage of the local-risk minimisation approach can lead to a problem known as the financial paradox, where it is possible to have a negative martingale measure (Wagih). Furthermore, if one attempts to tackle this issue, one would have to impose severe restrictions on the nature of the jumps (for instance $\Delta X_t > -1$), which would limit the realistic potential of the model.

A possible solution to this is to use the entropy approach instead, where the minimal entropy martingale density is expressed in the form of an exponential function (which is always positive). One then can proceed with the Föllmer-Sonderman decomposition of the claim - under the minimal entropy martingale measure -and proceed with calculations of its components. Thus, the entropy approach allows for jumps in the model and, at the same time, without strong restrictions on the jump sizes.

Moreover, the whole philosophy behind this approach is 'adding the least possible amount of information to the prior in order to reproduce today's observed option prices' (Rheinlander, Entropy Approach 6). This implies that one does not require a large amount of data in order to obtain the fair price and optimal hedging strategy under a new entropy martingale measure. Therefore, this approach aims for maximising efficiency while obtaining effective results.

Furthermore, unlike the local-risk minimisation approach, which focuses on short term optimisation, the entropy approach takes a global view on the pricing of financial assets, focusing on the entire time horizon. Because of this, and under certain assumptions, the entropy approach yields a better hedging strategy over the local-risk minimisation approach (Rheinlander, Office Meeting). Hence, usage of the entropy approach would be a more general and powerful method for the pricing of EUA0 contracts and obtaining its optimal hedging strategy.

B3iii) _Adding more EUA contracts_: As it is difficult in any model to calculate and simulate many interdependent variables, it is suggestible to 'start small' and then expand upon it. Naturally, these assumptions can be relaxed eventually and one can have a highly general model.

For instance, in this case, the researcher as a starting point can assume that all EUAi future prices are exogenous and follow the Çetin and Verschuere model with parameters μ^i, α^i and σ^i and that their Brownian motion components are independent as to reduce calculations. The researcher can also assume that there is only one EUA contract trading in each period including the current one (i.e. only one EUA0, EUA1, EUA2, etc.), and truncate long-term contracts from the equation (e.g. beyond 5). And so, ***Equation 4*** can be used to represent each EUAi contract in the form:

$$dS_t^i = S_t^i \mu^i (t, S_t^1, S_t^2, .., S_t^n, \theta_t) dt + S_t^i \sigma^i (t, S_t^1, S_t^2, .., S_t^n, \theta_t) dW_t^i + f^i(X_t^i), \forall i = 1,2,..., n$$

……………………………………***Equation 6***

and the EUA0 price could be derived as:

$$P_{T1} = \begin{cases} S_{T1} + K, & \text{if market is short} \\ E[\sum_{i=1}^{n} w_i 1_{[\theta_{Ti}=-1]} \mathrm{S}_{\mathrm{Ti}}^{\mathrm{i}} \big| \mathrm{S}_{\mathrm{T1}}^1, S_{T2}^2, ..., S_{Tn}^n, \theta_{T1} = 1], & \text{if market is long} \end{cases}$$

and hence making the EUA0 fair price:

$$\overline{P_t} = E^{Q^*}[S_{T1} 1_{[\theta_T \in \{-1,-2\}]} + E[\sum_{i=1}^{n} w_i 1_{[\theta_{Ti}=-1]} \mathrm{S}_{\mathrm{Ti}}^{\mathrm{i}} \big| \mathrm{S}_{\mathrm{T1}}^1, S_{T2}^2, ..., S_{Tn}^n, \theta_{T1} = 1] 1_{[\theta_T = 1]} \big| G_t]$$

……………………………………***Equation 7***

where: Q^* is the minimal martingale measure under the new model and w_i is the corresponding weight, summing up to one, indicating the strength of influence of the EUAith contract on the spot price. It is realistic to assume that EUA contracts maturing sooner would have a strong influence over ones maturing later due to having greater uncertainty the more forward we look. Hence, a possible candidate expression of w_i could be a linearly decreasing function, tending to zero the more forward we go.

One can then proceed by adding CER and ERU as exogenous variables to the model. However, as these two carbon reduction credits are still new, it would be suggestible to include these variables in the next phase, as to get a feel on their performances in the current one. One can also consider other non-asset exogenous variables such as percentage of auctioning, cap ceiling settings and other NAP factors. However, it is not possible to determine and include all the variables which explain the prices (it would not be stochastic then), and hence it is important to know when to draw the line. For example, one can use inferential statistics, multivariate analysis and/or Monte Carlo

B3iv) *Incorporating volatility clustering (proposed by Jensen)*: This can be done in several ways. For instance, one can have σ as a stochastic process in which one could include exogenous variables which are known to have a strong influence on the volatility of the carbon prices - such as hot summers and cold winters (Table 9.1 in Labatt and White show the main influencers of the carbon price (208)), which influence power consumption and thus the EUA prices, as proposed by fellow colleague Jensen. Another possibility is subordinating the Brownian motion with a positive-valued Lévy process as its arrival time. Possible candidates can be Gamma and Inverse Gaussian processes with parameters such that the model has the desired clustering behaviour.

B3v) *Making θ asymmetric*: Next to making the transition intensities λ of the Markov chain different, one can create asymmetry in market position transitions by having θ take several states (preferably an odd number as to automatically have more states in favour of one market condition over the other). For instance, one can have $\theta_t \in \{-2,-1,1\}$ where the negative values correspond to the market being short and the positive value corresponds to the market being long, as one would prefer assign a greater tendency for having the market short (worst case scenario) than having it long (best case scenario).

Certainly, one can further relax the assumptions of θ with the aim to obtain a more realistic picture. For example, θ can be a stochastic process (not necessarily Markov) taking discrete values within a set. Practically, however, one could find that many could probably be mathematically infeasible because of the extreme difficulty and tenaciousness in dealing with non-Markov processes.

Another example is having many θ's corresponding to many carbon markets. Presently, this is not an issue as there is only one dominant carbon market, but in some future time, it is quite possible to have different carbon assets inter-traded among different markets. In this case, one can make θ^i correspond to the market position for asset i^{th} asset and S in this case an n by m matrix: one dimension signifying the number of assets in the market and the other signifying the number of markets. Notwithstanding the fact that this example does not currently reflect the carbon market, it is often useful to take such ideas under consideration in an unexplored territory.

C. Conclusion

This part has given the researcher with the mathematical suggestions for extending the Çetin/Verschuere model. It is essential to take into account that the carbon market is quite new and that the recommendations mentioned here are based on the current status quo of the EU ETS. This could significantly change at some future date. For instance, it is possible that in the system would significantly change to the extent that EUA pricing models would have to be completely modified (or possibly rebuilt). It is also possible that governmental institutions impose certain restrictions, which have not been imposed before, on the trading of allowances. Another possibility is that financial assets in the carbon market do not follow the 'stylized facts' of equity market, and have their own usual trend that we are not currently aware of (due to a small amount of historical data and few financial instruments traded in the carbon market). Therefore, one must keep an open mind that creating models is an eternal process, and the objective is to continually improve upon the current ones.

And so, the researcher implementing this proposal should ensure that the modified model is open to further and improvement. It is imperative that the researcher be highly knowledgeable and up-to-date with the status quo of the market. As du Toit states: 'Part of what is required is simply described as applying one's mind, or developing a more reflective approach to risk analysis' (19). Thus, knowing how the market works and finding the mathematical means to explain and possibly predict its behaviour will help preserve the fluidity and performance of the system and its entities, prolonging the market's wellbeing.

In order to ensure that the model is doing so, it is necessary to be able to use the model to obtain numerical and graphical results, in order to compare with reality. Thus, it is imperative that the pricing model should include numerical simulations in order to demonstrate its relevance. The final part gives the main key points on how the model's variables were simulated and the main issues regarding the simulation variables of the new model.

Part III: Simulation of the Çetin/Verschuere Stochastic Variables

'[A] historical simulation provides an important reality check' (du Toit 18)

The aim of improving this model is to be able to use it to better represent the carbon market under the EU ETS. Not being able to do so, the model's practicality could be in question and making it simply existent within theoretical realm. Hence, next to formulating the mathematical expression of the optimal hedging strategy for the market prices, the researcher(s) is/are also expected to produce numerical and graphical results mimicking the behaviour of the EUA prices and calculating the fair spot price, EUA0. The following sections give an idea on how this is done in the Çetin/Verschuere model and how one could extend it further. It is important to note that the graphs shown in this part are for demonstrative purposes only as to illustrate key concepts and differences between alternate approaches. They do not represent the best fit for the market and it is the researcher's task to be able to do so under the improved model.

A. Simulating S

Çetin and Verschuere simulated S based on the following equation:

$dS_t = (\mu + \alpha\theta_t)S_t dt + \sigma S_t dW_t$ …………………………………….***Equation 8***

In order to do so, they had to simulate θ first. As θ is a two-state Markov chain with an exponential waiting time with parameter λ, one can make the variable toggle between the two states through generating exponentially random waiting times (to see how this can be done under MATLAB, see Annex). However, when θ is modified into multiple states and possibly a non-Markovian state process, the researcher will have to use more rigorous and technical state generation algorithms.

Next, one turns to numerical schemes, such as the Euler and Milstein scheme, as approximations, in order to simulate pathways of S. This is done by iterating the derivative expression of the stochastic variable a pre-fixed number of times, known as a time-step, across its time path. As for estimating the parameters of the equation, one can use Monte Carlo simulation to find the constant parameters that, on average, best fit the real path S. For example, this can be done by calculating for instance the minimal sum of squared errors.

This, however, can be extremely tenacious as one would need to simulate billions of different values of the constant parameters and simulate an average of several paths of S with each of these values. Nevertheless, as historical data are becoming more abundant in time, statistical estimation and time series theory will increasingly help relieve the need of permutation force. Thus, a large amount of effort lies in estimating the parameters alone.

The graphs shown here use the parameters given in the Numerical Study section of the Çetin/Verschuere paper. **Figure 1** shows two simulated paths of S_t, the first one is derived from ***Equation 8*** and the second one derived from the following equation:

$$dS_t = (\mu + \alpha\theta_t)S_t dt + \sigma S_t dW_t + df(X_t) \text{ ....} \textbf{\textit{Equation 9}}$$

where X_t is a compound Poisson process and f is as in ***Equation 5***. See Annex for the computer code used to simulate the results.

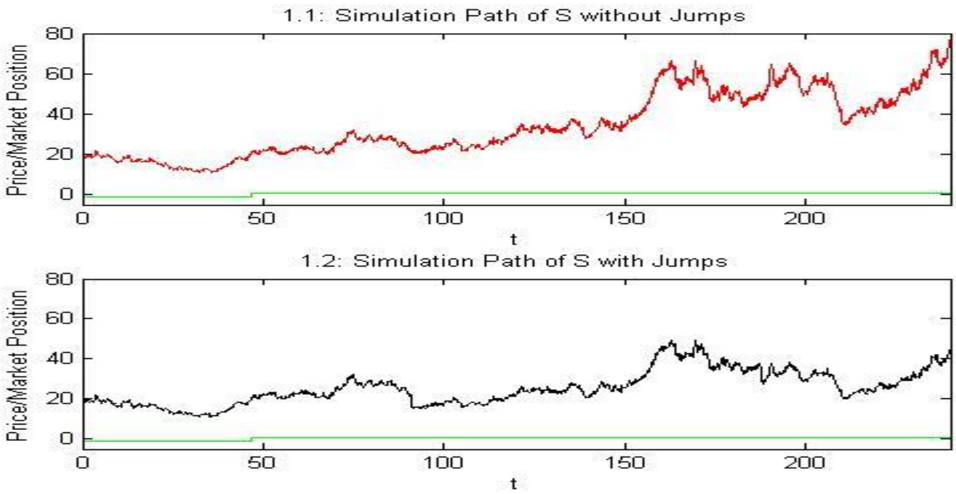

Figure 1: Simulated path of S, with and without jumps, relative to the market position (green)

As one can see that the graphs are quite similar, with the main differences triggered by two jump events in this path (with approximate jump times of 91 and 188, and jump sizes of -5.4 and -7.9, respectively). As financial markets display sudden negative jumps in stock prices, it is realistic to have the same phenomenon in carbon markets –due to unexpected announcements or events – as mentioned in Part II. As mentioned earlier, this is simply an illustration as it is not expected that the common parameters between the new and the original model will be equal. It will be researcher's task modify the EUA future price equation(s) accordingly and estimate its/their parameters under the enhanced model. The next step is to simulate the EUA fair spot price.

B. Simulating the Fair Price of P

As the EUA0 price is the central variable of the model, its estimation and simulation will be the most complex. There are two main methods one can calculate and simulate the fair EUA0 price in the Çetin/Verschuere model:

B1) Calculating the Variables of Equation 3:

As a reminder this is based on the mathematical derivations of the Cetin/Verschuere model and it is expected that the researcher will be required to calculate different – and more complex - mathematical variables of the enhanced model. The purpose of the following list is to involve the researcher with the current methodologies used in the EUA0 price process in ***Equation 3***. Hopefully, there would be some commonalities between calculating the variables of this model and of the new one.

B1i) _Simulating $\overline{\theta}$ and Calculating Z_: $\overline{\theta}$ can be simulated using numerical schemes on its differential form decomposition (under the minimal martingale measure P^*).

$d\overline{\theta_t} = -(2\lambda\overline{\theta_t} + \frac{\alpha}{\sigma^2}(1-\overline{\theta_t}^2)(\mu + \alpha\overline{\theta_t}))dt + \frac{\alpha}{\sigma}(1-\overline{\theta_t}^2)dW_t^*$, with $\overline{\theta_0} = 2p-1$ (extracted from Theorem 4.2 of the Cetin/Verschuere paper). **Figure 2** displays this, using the parameters given in the Numerical Study section of Çetin and Verschuere's paper.

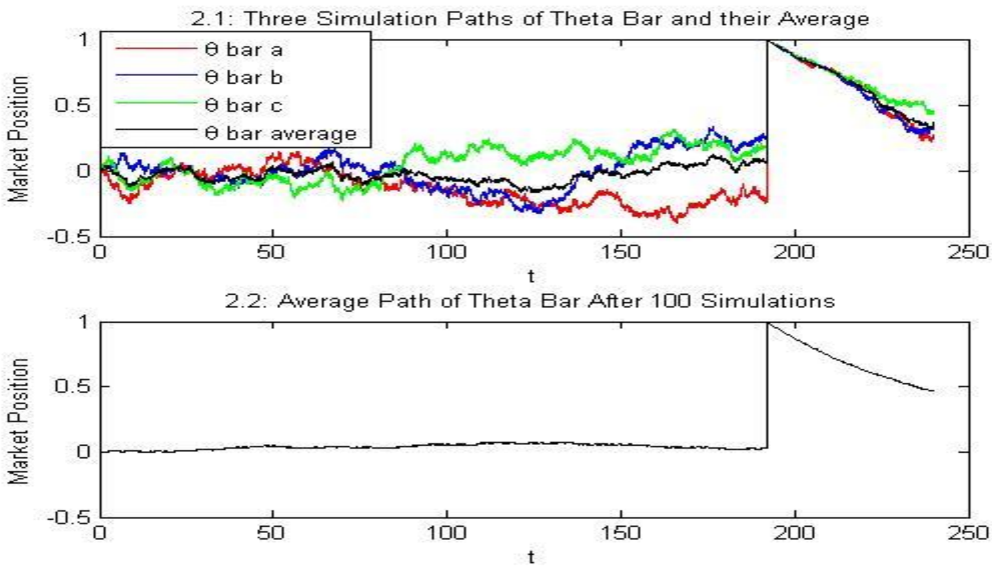

Figure 2: Simulated paths of $\overline{\theta}$

The 'high jump' in the graphs reflects the intermediate announcement at t_0 =192 of a long market position (by setting $\overline{\theta_{t_0}} = 1$ in this case), which was incorporated within the numerical scheme used for deriving $\overline{\theta}$. However, in the new model, one will have to treat both the market position and frequency of intermediate announcements as random.

B1ii) _Finding h and Z^h_: As there is no analytical solution for its PDE, one will have to turn to numerical methods and grid solutions in order to find h, as indicated by Çetin and Verschuere (14). Once obtained, one can then plug in the values of S_t and $\overline{\theta}$, calculated from the previous step, and find the values of h.

One can then proceed to calculate Z^h using Monte Carlo simulation, by obtaining an average from multiple simulated paths of the h's under for each time t. This however can have high computational complexity (i.e. it takes a lot of time for the algorithm to compute, which is mainly due to a high number of iterations) that would require a fast computer, or even a supercomputer, in order to obtain reliable results of Z^h.

Therefore, this method is implemented by solving every variable obtained from the final expression of $\overline{P_t}$, for all t, within the trading period [0,T]. Also, as the calculation of h can be significantly complex, one can turn to more practical methods such as Monte Carlo simulation, which is the second method.

B2) Monte Carlo Simulation of $E^*[(S_T + K)(1-\theta_T)/2|G_t]$***:***

The task is simple: First simulate many θ's and their corresponding S's, take their average and calculate P_0. Reset the initial values of θ and S for the next time-step unit, simulate many θ's and their corresponding S's again, take their average and calculate P_1, and so on until maturity time T. Even though this method is straight forward and, by far, simpler than the first one the main constraint lies in its significantly high computational complexity, especially if one aims to obtain effective results. However, optimistically, in a vast world of technological advancements and superfast computation devices, one expects this issue to become less an issue in the future.

A great advantage of this is step, especially in an IT oriented age, one can apply the Monte Carlo simulation of more complex conditional expectation expressions with only computational complexity as the significant issue. For instance, with supercomputer, one could calculate the stochastic variables of the process and then use Monte Carlo to simulate the EUA0 price process in ***Equation 7***. Hence, Monte Carlo offers a highly convenient alternative to simulating highly mathematically complex variables. This notion is reaffirmed as technology increases computational speed.

C. Conclusion

This part has presented the main methodologies of simulating stochastic variables under the Çetin/Verschuere framework. However, since the current model is based on simplistic assumptions, it is obvious that the modified version will be highly challenging, both practically and theoretically. However, as the literature and market grows, one can expect to acquire better tools and methodologies to developing models that help promote environmental virtue.

Annex

```
%Using MATLAB
%Simulating one path of EUA1 price S with jumps and without jumps, and its corresponding theta
>> T=240; h=10;                                                     %Trading period and number of increments within one time unit
>> theta=-ones(1,h*T+1); Swithjump=20*ones(1,h*T+1);                %Setting the stochastic variables
>> Swithoutjump=Swithjump;
>> X=zeros(1,h*T+1);                                                %Setting compound Poisson process
>> theta0=-1; S0=20;
>> mu=0.4; alpha=-0.5;sigma=1;lambda=2;                             %Setting the parameters of S
>> lambdapoiss=1/150; mujumpsize=-7; sigmajumpsize=2; gamma=1/2;    %Setting the parameters  for compound Poisson function
>> poissjumpnum=floor(poissrnd(lambdapoiss*T));
>> poisssize=mujumpsize+sigmajumpsize*randn(1,poissjumpnum);
>> jumptime=rand(1,poissjumpnum)*T;
>> changedate=h*ceil(exprnd(T/lambda));                             %Point where the market position switches
>> dt=1/(T*h); timestep=1/dt;
>> for j=1:timestep
t=j/h;                                                              %time
theta(1)=theta0; Swithoutjump(1)=S0;Swithjump(1)=S0;                %Setting the initial values
if j+1==changedate
theta(j+1)=-theta(j);
changedate=changedate+h*ceil(exprnd((T-changedate)/lambda));        %Setting a new date where market position switches
else
theta(j+1)=theta(j);
end
for n=1:poissjumpnum
if jumptime(n)<t
X(j+1)=X(j+1)+poisssize(n);
end
end
dw=sqrt(dt)*randn; %Setting Brownian motion increment
Swithoutjump(j+1)=Swithoutjump(j)+(mu+alpha*theta(j))*Swithoutjump(j)*dt+sigma*Swithoutjump(j)*dw;     %Euler method for Cetin/Verschuere model
nonjumppart=Swithjump(j)+(mu+alpha*theta(j))*Swithjump(j)*dt+sigma*Swithjump(j)*dw;                    %Euler method for Poisson incorporated model
jumppart=X(j+1)-X(j);
if jumppart<0
Swithjump(j+1)=max(nonjumppart+jumppart,0); %Avoid S being negative
else
Swithjump(j+1)=nonjumppart+gamma*jumppart;%Control non-negative jumps
end
end
```

Works Cited

Part I: Background of the EU ETS Carbon Market

"Biography of Adam Smith". The Library of Economics and Liberty: The Concise Encyclopedia of Economics. David R. Henderson. 2002. Liberty Fund. 20 July 2008 <http://www.econlib.org/library/Enc/bios/Smith.html>

"Directive 2003/87/EC of the European Parliament and of the Council of 18 October 2003: Establishing a Scheme for Greenhouse Gas Emission Allowance Trading within the Community and Amending Council Directive 96/61/EC". Europa. Official Journal of the European Union. 24 Aug. 2008 < http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:32003L0087:EN:NOT >.

Ellerman, A. Denny and Barbara K. Buchner. "The European Union Emissions and Trading Scheme: Origin, Allocation, and Early Results". Oxford Journals. Association of Environmental and Resource Economists. 2007. 26 July 2008 <http://reep.oxfordjournals.org/cgi/content/full/1/1/66>.

"European Union Emission Trading Scheme". Wikipedia. 2008. 29 July 2008 < http://en.wikipedia.org/wiki/European_Union_Emission_Trading_Scheme#cite_note-12>.

"Futures: Historical Data – ECX EUA Futures Contracts". European Climate Exchange. 6 Aug. 2008 <http://www.europeanclimateexchange.com/default_flash.asp>.

Kruger, Dina. " Lessons Learned from EU Emissions Trading Scheme (ETS)". PowerPoint. Naruc Winter Meeting. 19 Feb. 2008.

"MEMO/03/154". Europa. Kyoto Protocol. 29 July 2008 < http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/03/154&format=HTML&aged=0&language=EN&guiLanguage=en >.

MEMO/05/84. Europa. Questions and Answers on Emissions Trading and National Allocation Plans. 8 March 2005. 23 Aug. 2008 <http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/05/84&format=HTML&aged=1&language=EN&guiLanguage=en>.

"MEMO/06/02". Questions and Answers on National Allocation Plans for 2008-2012. Europa. 9 Jan 2006. 23 Aug. 2008 <http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/06/2&format=HTML&aged=0&language=EN&guiLanguage=en>.

"MEMO/08/35" Europa. Questions and Answers on the Commission's Proposal to Revise the EU Emissions Trading System. 23 Jan. 2008. 26 July 2008 .<http://europa.eu/rapid/pressReleasesAction.do?reference=MEMO/08/35&format=HTML&aged=0&language=EN&guiLanguage=en>.

Labatt, Sonia and Rodney R. White. Carbon Finance: The Financial Implications of Climate Change. New Jersey: John Wiley & Sons, Inc., 2007.

"Market". Wikipedia. 27 July 2008 <http://en.wikipedia.org/wiki/Market>.

Seifert, Jan et.al. Dynamic Behavior of CO_2 Spot Prices - A Stochastic Equilibrium Model.Ed. Chair of Financial Engineering and Derivatives. Universität Karlsruhe (TH). 18 May 2006. 24 Aug. 2008 < http://derivate.fbv.uni-karlsruhe.de/downloads/Paper/Article%20Dynamic%20CO2%20Spot%20Prices%20-%201%5B1%5D.09.pdf >.

Part II: Extending the Çetin/Verschuere Model

Al Hussaini, Ata N. and Robert J. Elliott. Markov Bridges and Enlarged Filtrations. Sep. 1989. Statistical Society of Canada. 19 Aug. 2008 < http://www.jstor.org/sici?sici=0319-5724(198909)17%3A3%3C329%3AMBAEF%3E2.0.CO%3B2-E >

Çetin, Umut and Michel Verschuere. Pricing and Hedging in Carbon Emissions Markets. 20 April 2008. 23 Aug. 2008 < http://stats.lse.ac.uk/cetin/files/carbon_filter9.pdf >.

du Toit, Barry. Risk, Theory, Reflection: Limitations of Stochastic Model of Uncertainty in Financial Risk Analysis. Ed. Risk Worx. June 2004. 18 Aug 2008 < http://www.riskworx.com/insights/theory/theory.pdf>.

Jensen, Henrik Nygaard. Personal Meeting. Risk and Stochastics Student. 16 Aug. 2008.

Rheinlander, Thorston. Asset Price Modelling by Exponential Lévy Processes. ST 432.2 Lecture Notes. Lent Term 2007. The London School of Economics and Political Science.

Rheinlander, Thorston. Entropy Approach to Option Pricing and Calibration. ST 432.2 Lecture Notes. Lent Term 2007 The London School of Economics and Political Science.

Rheinlander, Thorston. Office Meeting. 6 Aug. 2008. The London School of Economics and Political Science. 2007.

"Stochastic Modelling". Wikipedia. 18 Aug. 2008 <http://en.wikipedia.org/wiki/Stochastic_modeling>.

Wagih, Basil Karim. ST433: Project. Class Notes. Thorston Rheinlander. The London School of Economics and Political Science. Lent Term 2008.

Part III: Simulation of the Çetin/Verschuere Stochastic Variables

Çetin, Umut and Michel Verschuere. Pricing and Hedging in Carbon Emissions Markets. 20 April 2008. 23 Aug. 2008 < http://stats.lse.ac.uk/cetin/files/carbon_filter9.pdf >.

du Toit, Barry. Risk, Theory, Reflection: Limitations of Stochastic Model of Uncertainty in Financial Risk Analysis. Ed. Risk Worx. June 2004. 18 Aug 2008 < http://www.riskworx.com/insights/theory/theory.pdf>.