Fall 2020 Computer Science 220
Program assignment 3

Learning objectives:

1. Develop a simple Python program that does arithmetic
2. Practice accumulating sequences

Part 1: Programming Problems (60pts)

"Pi (π) is one of the most important and fascinating numbers in mathematics. Roughly 3.14, it is a constant that is used to calculate the circumference of a circle from that circle's radius or diameter. It is also an irrational number, which means that it can be calculated to an infinite number of decimal places without ever slipping into a repeating pattern. This makes it difficult, but not impossible, to calculate precisely" [1].

One way to determine the value of pi is using an infinite series. Similar to Newton's method for approximating square roots, the series approaches the real value of pi as more terms are included.

In this assignment, you are going to calculate pi using 3 different series, given the number of terms provided by the user. In addition to the result, your program should display the approximation error—how close your approximation is to the actual value of pi. Have your functions subtract the result from the value of `math.pi` to see how close it is.

1. Wallis product.

Write a function `pi_wallis(n)` that approximates the value of pi using the Wallis product:

$$\frac{\pi}{2} = \frac{2}{1} * \frac{2}{3} * \frac{4}{3} * \frac{4}{5} * \frac{6}{5} * \frac{6}{7} * \frac{8}{7} * \dots$$

Note that the above formula yields π / 2.

2. Gregory-Leibniz series.

Write a function, `pi_gregory(n)`, to approximate the value of pi by adding the terms of the Gregory-Leibniz series, given below.

$$\frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \cdots$$

Think carefully about how to switch between + and – for each term. There are multiple ways to accomplish this.

3. Nilakantha Series.

Write a function `pi_nilakantha(n)` that approximates the value of pi using the Nilakantha series given below:

$$3 + \frac{4}{2*3*4} - \frac{4}{4*5*6} + \frac{4}{6*7*8} - \frac{4}{8*9*10} + \frac{4}{10*11*12} - \frac{4}{12*13*14} + \cdots$$

Note denominators that are the product of three consecutive integers which increase with every new iteration.

4. Euler's method.

Write a function `pi_euler(n)` that approximate the value of pi using Euler's method when Leonhard Euler solved the Basel problem:

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots$$

Note that the above formula yields $\pi^2/6$.

File to be submitted:
`pi.py`

Part 2: Number Conversion (40pts)

1. Answer the following questions in this document. (16pts)
   a. How many possible values can a signed 16 bit number represent? What are the minimum and maximum values? (6pts)
   b. Assuming a signed integer is stored in *n* bytes, what are the minimum and maximum values that can be represented? Write your answer as a power of 2. (5pts)
   c. Assuming a signed integer is stored in 2 bytes, what are the minimum and maximum values that can be represented? Write your answer as a power of 2 *and* in binary representation. (5pts)

2. For the following unsigned binary/decimal conversions, please show the conversion steps. (12pts)
   For example: 0b1101 = 8 + 4 + 1 = 13
   34 = 32 + 2 = 0b100010

   Convert the following binary integers to decimal. (2pts each)
   a. `0b100110`
   b. `0b10100110`
   c. `0b1001010`

Convert the following decimal integers to binary. (2pts each)

d. `117`
e. `52`
f. `20`

3. Convert the following signed decimal integers to 8-bit binary (parts a-c), and binary integers to decimal (parts d-f). Please show yours steps. (12pts)
   a. `-41`
   b. `84`
   c. `-69`
   d. `0b00111101`
   e. `0b11111101`
   f. `0b10110010`

File to be submitted:

`HW3.docx`

Policies:

Please see the homework policy to review all policies for all homework submissions in CSCI220.