Lab3

April 28, 2022

1 Funkcje sklejane

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

1.1 Bartosz Kucharz

2 Funkcja zadana do analizy

$$k = 3m = 0.2f(x) = \sin\left(\frac{xk}{\pi}\right)e^{\frac{-mx}{\pi}}$$

[2]:
```
def f8(x):
    k=3
    m=0.2
    return np.sin(x*k/np.pi)*np.exp(-m*x/np.pi)
```

[3]:
```
def get_nodes(function, n, nodes_func):
    x = nodes_func(-np.pi*np.pi, 2*np.pi*np.pi, n)
    y = np.vectorize(f8)(x)
    return x, y

def  uniform_nodes(a, b, n):
    return np.linspace(a, b, n)
```

3 Sześcienna funkcja sklejana

[4]:
```
def spline3(x, y, n, boundary_condition='free boundary'):
    h = x[1:] - x[:-1]
    matrix = np.zeros((x.shape[0], x.shape[0]))
    for i in range(x.shape[0]-2):
        matrix[i+1][i:i+3] = np.array([h[i], 2*(h[i]+h[i+1]), h[i+1]])

    delta = (y[1:] - y[:-1])/h
    free = np.zeros((x.shape[0]))
    free[1:-1] = delta[1:] - delta[:-1]
    free = free.reshape((-1))
```

```python
if boundary_condition == 'free boundary':
    matrix[0][0] = 6
    matrix[-1][-1] = 6
elif boundary_condition== 'clamped boundary':
    d = 1e-6
    s_1 = (f8(x[0]+d) - y[0])/(d)
    matrix[0][:2] = np.array([-2*h[0], -h[0]])
    free[0] = s_1 - (y[1] - y[0])/h[0]

    s_1 = (y[-1] - f8(x[-1]-d))/(d)
    matrix[-1][-2:] = np.array([h[-2], 2*h[-2]])
    free[-1] = s_1 - (y[-1] - y[-2])/h[-2]
else:
    raise ValueError("Wrong boundary condition")
    return


result = np.linalg.solve(matrix, free)
s_2 = 6*result


C = y[1:]/h - s_2[1:]*h/6
D = y[:-1]/h - s_2[:-1]*h/6

def s(nx):
    s = s_2[i]/(6*h[i])*np.power(x[i+1] - nx, 3)
    s += s_2[i+1]/(6*h[i])*np.power(nx-x[i], 3)
    s += C[i]*(nx-x[i]) + D[i]*(x[i+1] - nx)
    return s

new_x = np.linspace(x[0], x[-1], n)
new_y = np.empty(new_x.shape)


for i in range(x.shape[0]-1):
    mask = ((new_x >= x[i]) & (new_x <= x[i+1]))
    new_y[mask] = np.vectorize(s)(new_x[mask])

return new_x, new_y
```

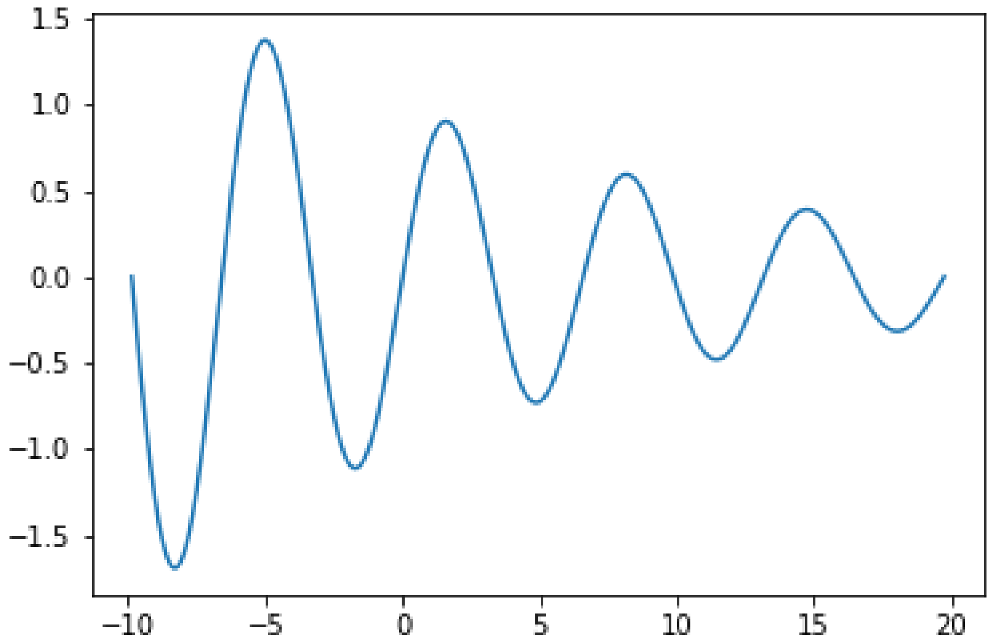
4 Kwadratowa funkcja sklejana

[20]:
```python
def spline2(x, y, n, boundary_condition):
    matrix = np.zeros((x.shape[0], x.shape[0]))
    free = np.empty((x.shape[0]))

    np.fill_diagonal(matrix, 1)
    np.fill_diagonal(matrix[:-1, 1:], 1)

    h = (x[1:] - x[:-1])

    free[:-1] = 2*(y[1:]-y[:-1])/h

    matrix[-1,:].fill(0)
    if boundary_condition == 'free boundary':
        matrix[-1][0] = 1
        free[-1] = (y[1]-y[0])/h[0]
    elif boundary_condition== 'clamped boundary':
        d = 1e-6
        s_1 = (f8(x[0]+d) - y[0])/(d)
        matrix[-1][0] = 1
        free[-1] = s_1
    else:
        raise ValueError("Wrong boundary condition")
        return

    m = np.linalg.solve(matrix, free)

    C = y[:-1] + m[:-1]*h/2

    def s(nx):
        s = -m[i]*np.power(x[i+1] - nx, 2)/(2*h[i]) + m[i+1]*np.power(nx -␣
↪x[i], 2)/(2*h[i]) + C[i]
        return s

    new_x = np.linspace(x[0], x[-1], n)
    new_y = np.empty(new_x.shape)


    for i in range(x.shape[0]-1):
        mask = ((new_x >= x[i]) & (new_x <= x[i+1]))
        new_y[mask] = np.vectorize(s)(new_x[mask])

    return new_x, new_y
```

5 Wykres funkcji do analizy

[22]:
```
X, Y = get_nodes(f8, 1000, uniform_nodes)
plt.plot(X, Y)
```

[22]: [<matplotlib.lines.Line2D at 0x7f33330ebdc0>]



6 Wyniki interpolacji funkcją sklejana 3-ego stopnia

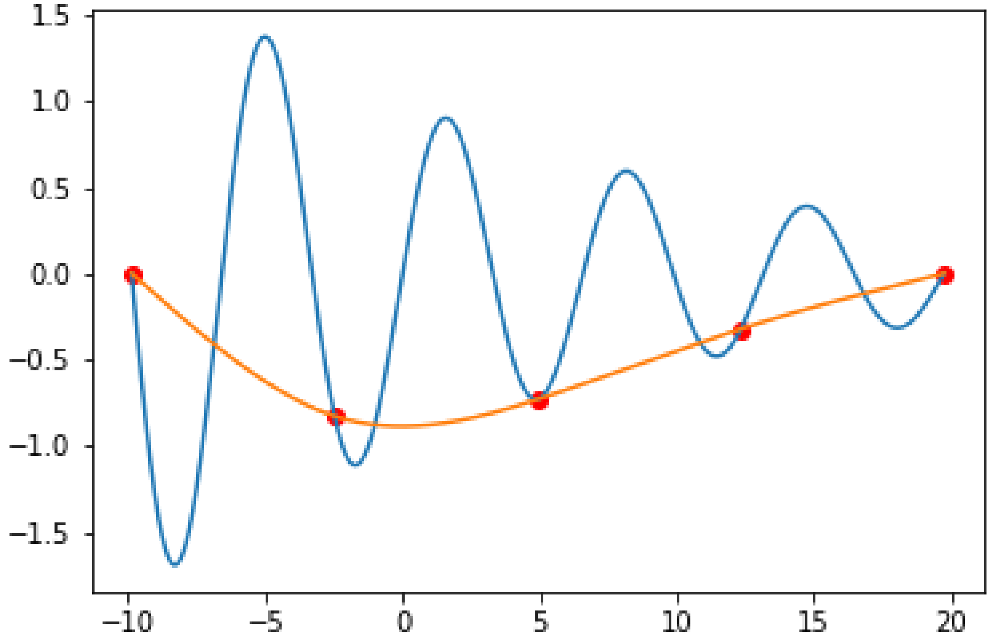
6.1 Warunek brzegowy: free boundary

$$s''(x_1) = s''(x_n) = 0$$

6.1.1 Liczba węzłów: 5

[27]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 5, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

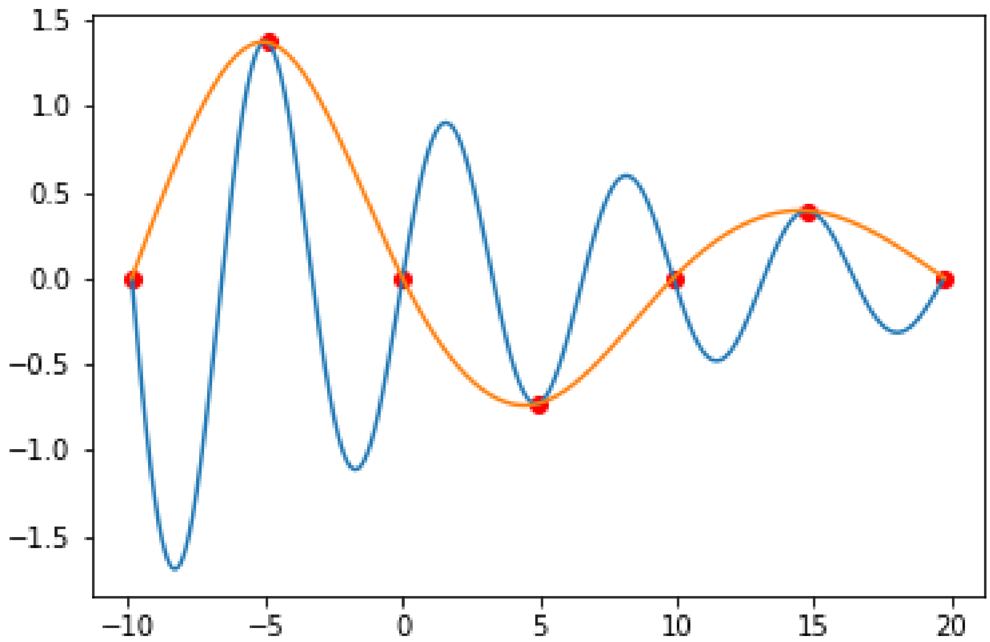
[27]: [<matplotlib.lines.Line2D at 0x7f3333046220>]



6.1.2 Liczba węzłów: 7

[30]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 7, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

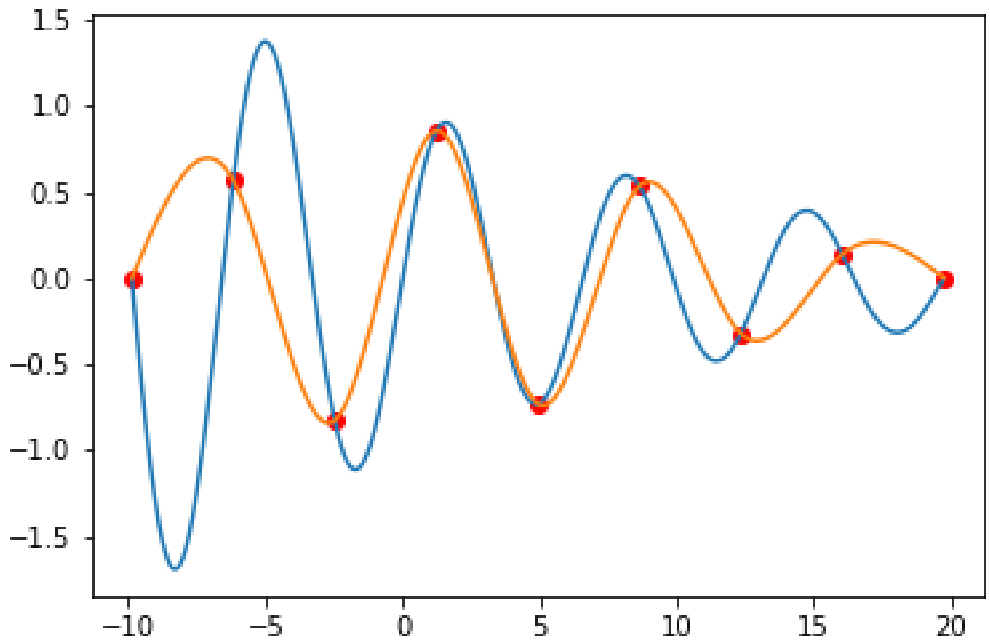
[30]: `[<matplotlib.lines.Line2D at 0x7f3332f23400>]`



6.1.3 Liczba węzłów: 9

[60]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 9, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

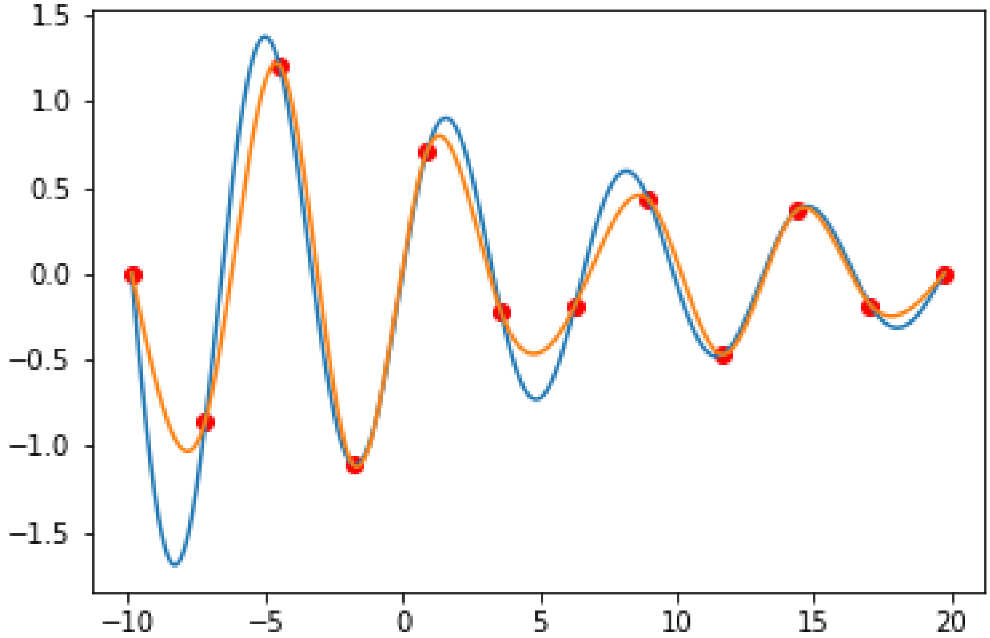
[60]: `[<matplotlib.lines.Line2D at 0x7f3332526cd0>]`



6.1.4 Liczba węzłów: 12

[38]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 12, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

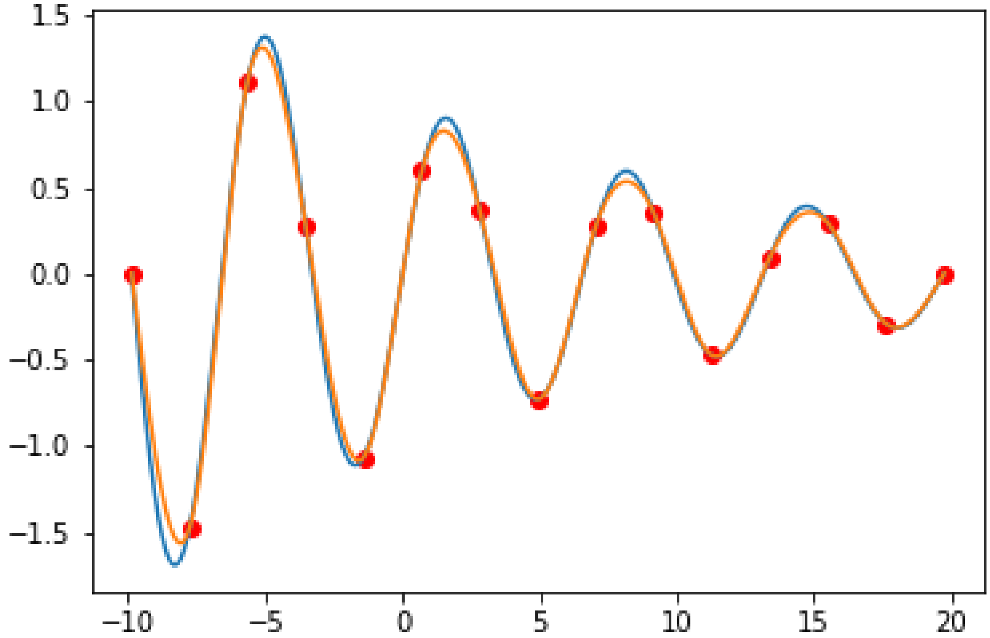
[38]:
```
[<matplotlib.lines.Line2D at 0x7f3332cc52b0>]
```



6.1.5 Liczba węzłów: 15

[40]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 15, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

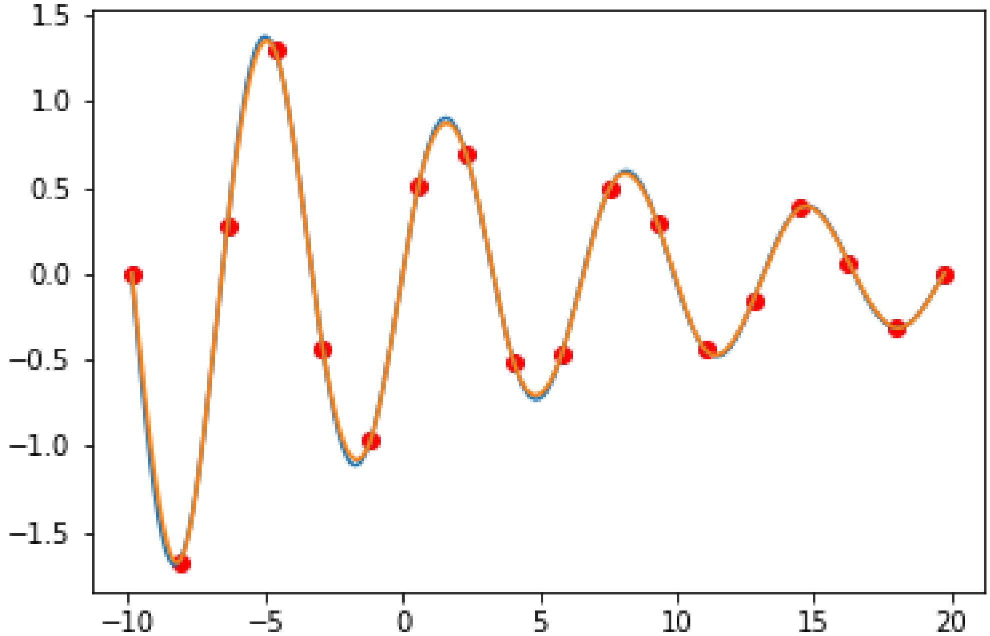
[40]:
```
[<matplotlib.lines.Line2D at 0x7f3332c339a0>]
```



6.1.6 Liczba węzłów: 18

[48]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 18, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

[48]: [<matplotlib.lines.Line2D at 0x7f33329c9850>]



6.2 Warunek brzegowy: clamped boundary

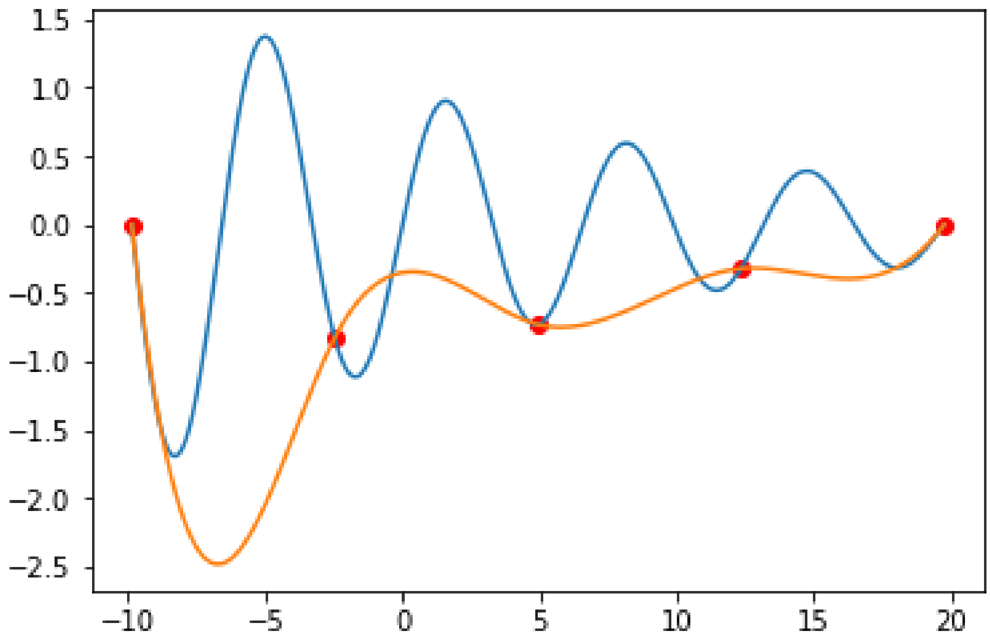
$$s'(x_1) = f_1' s'(x_n) = f_n'$$

W punktach brzegowych pochodne funckji sklejanej są równe pochodnym funkcji interpolowanej. Oznacza to że w tych punktach funkcja interpolująca i interpolowana są do siebie styczne.

6.2.1 Liczba węzłów: 5

[50]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 5, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

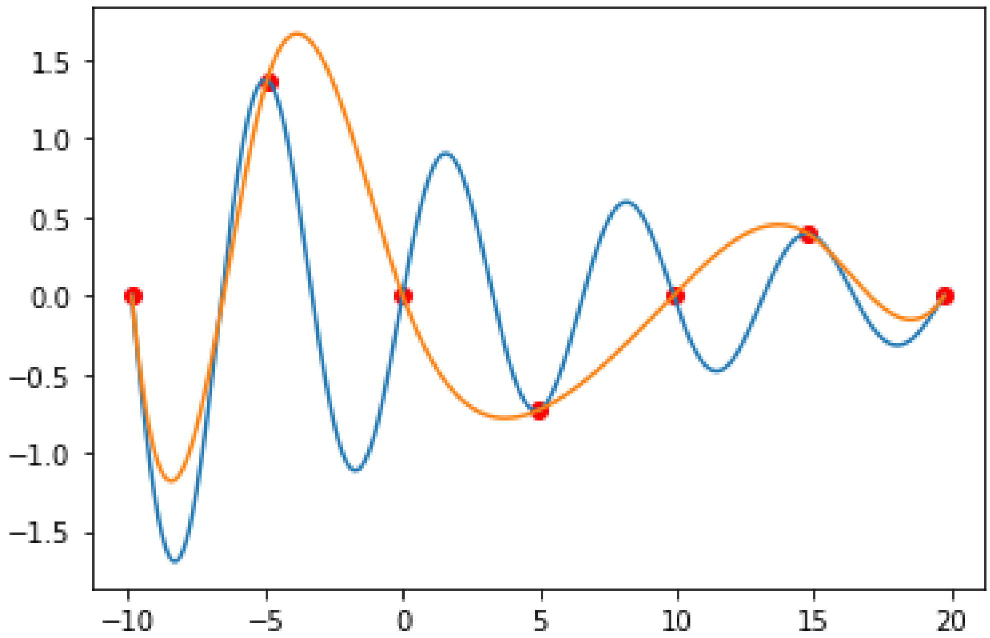
[50]: [<matplotlib.lines.Line2D at 0x7f3332931e80>]



6.2.2 Liczba węzłów: 7

[51]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 7, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

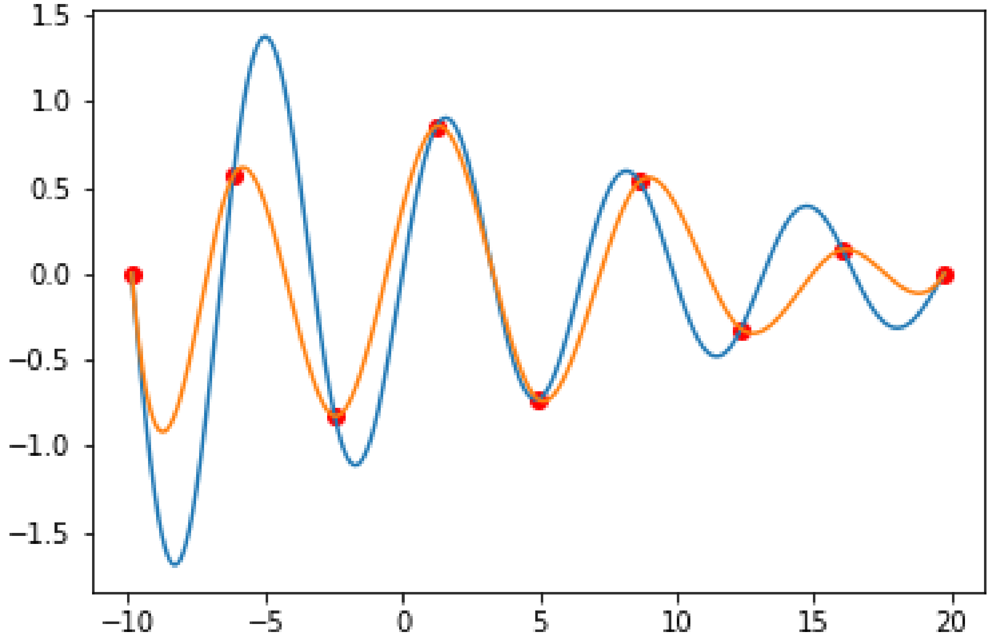
[51]: [<matplotlib.lines.Line2D at 0x7f33328b1310>]



6.2.3 Liczba węzłów: 9

[59]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 9, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

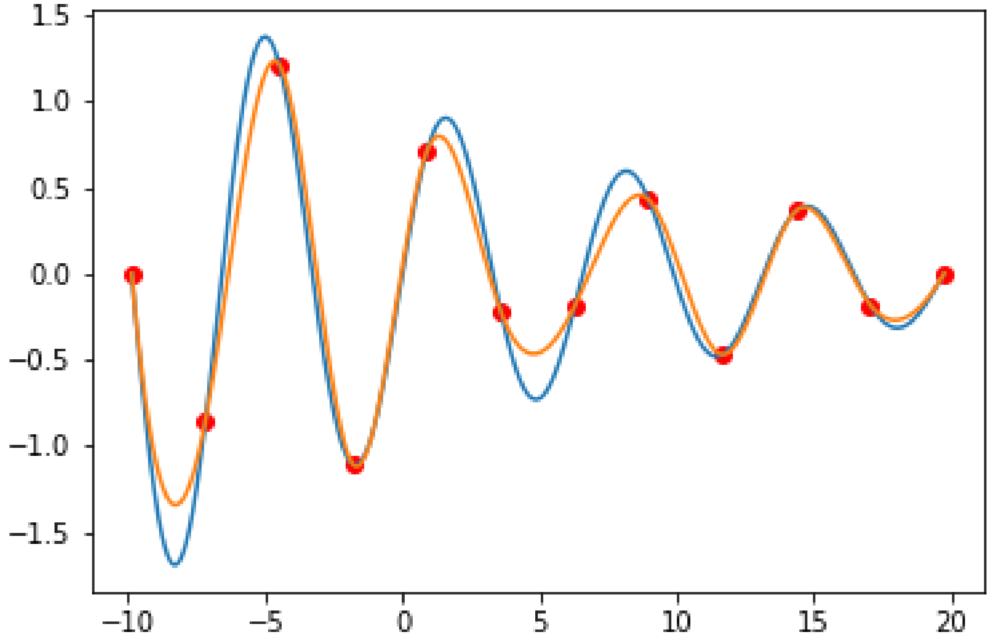
[59]: [<matplotlib.lines.Line2D at 0x7f33325c2850>]



6.2.4 Liczba węzłów: 12

[57]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 12, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

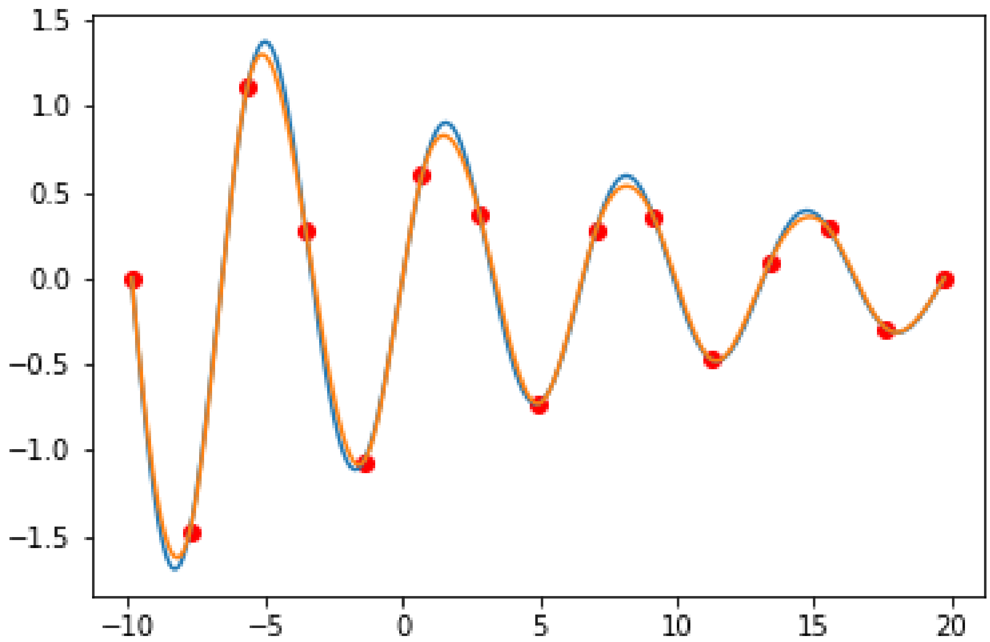
[57]: [<matplotlib.lines.Line2D at 0x7f3332660880>]



6.2.5 Liczba węzłów: 15

[53]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 15, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

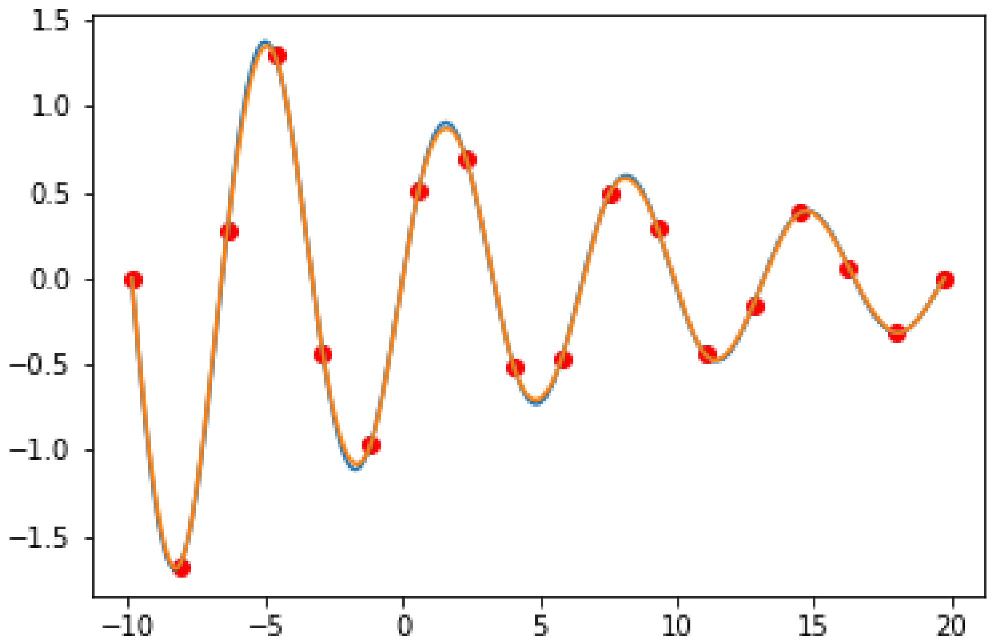
[53]:
```
[<matplotlib.lines.Line2D at 0x7f3332796280>]
```



6.2.6 Liczba węzłów: 18

[54]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 18, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline3(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

[54]: [<matplotlib.lines.Line2D at 0x7f33327809a0>]



7 Wyniki interpolacji funkcją sklejana 2-ego stopnia

7.1 Warunek brzegowy:

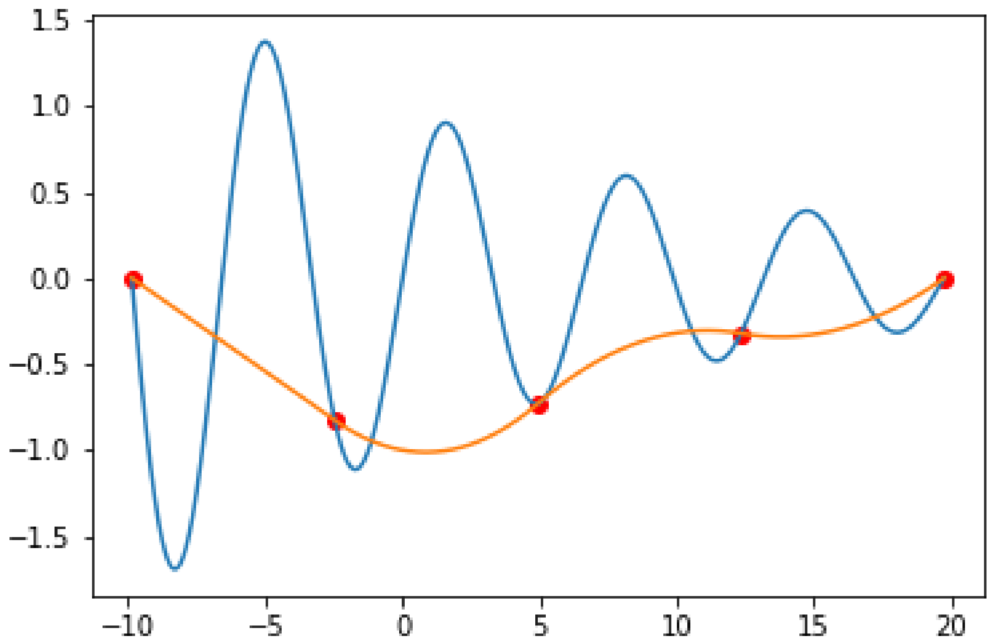
$$s'(x_1) = 0$$

Za pierszy spline przyjmujemy funkcje liniową.

7.1.1 Liczba węzłów: 5

[62]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 5, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

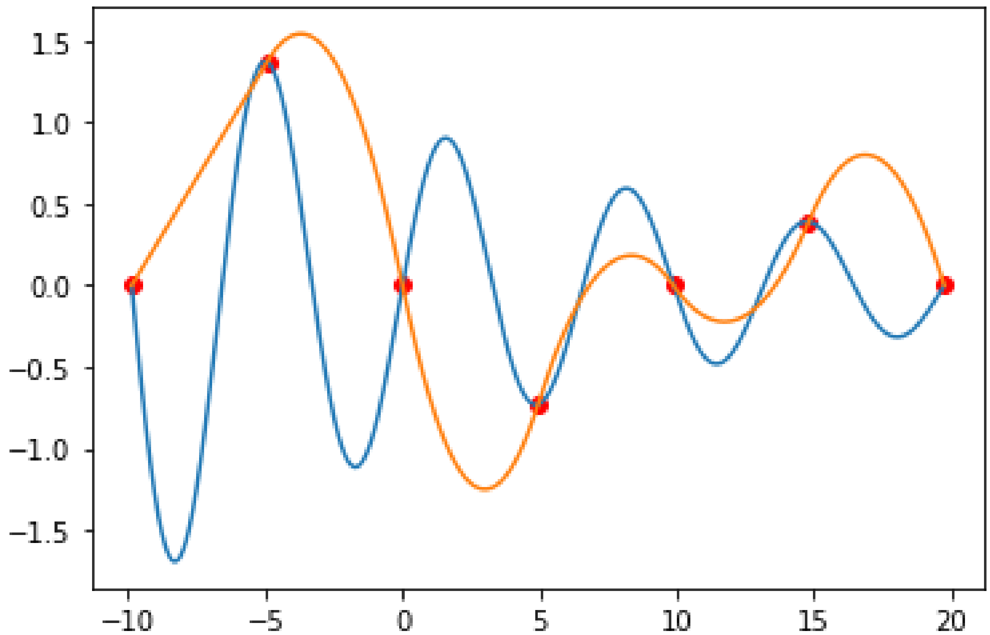
[62]:
```
[<matplotlib.lines.Line2D at 0x7f33324916a0>]
```



7.1.2 Liczba węzłów: 7

[64]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 7, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

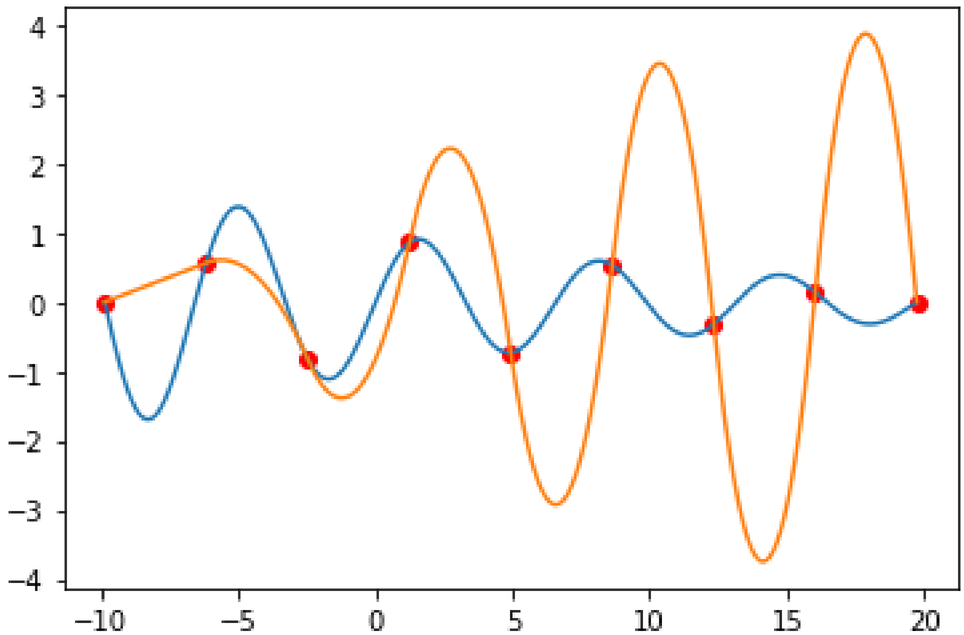
[64]: `[<matplotlib.lines.Line2D at 0x7f333247ccd0>]`



7.1.3 Liczba węzłów: 9

[65]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 9, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

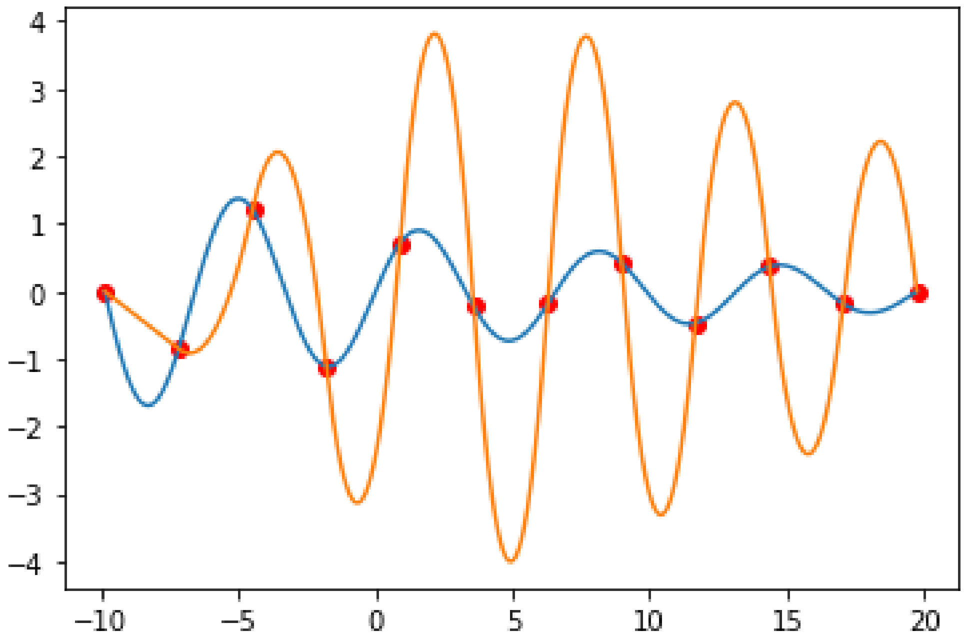
[65]: [<matplotlib.lines.Line2D at 0x7f33323f5550>]



7.1.4 Liczba węzłów: 12

[66]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 12, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

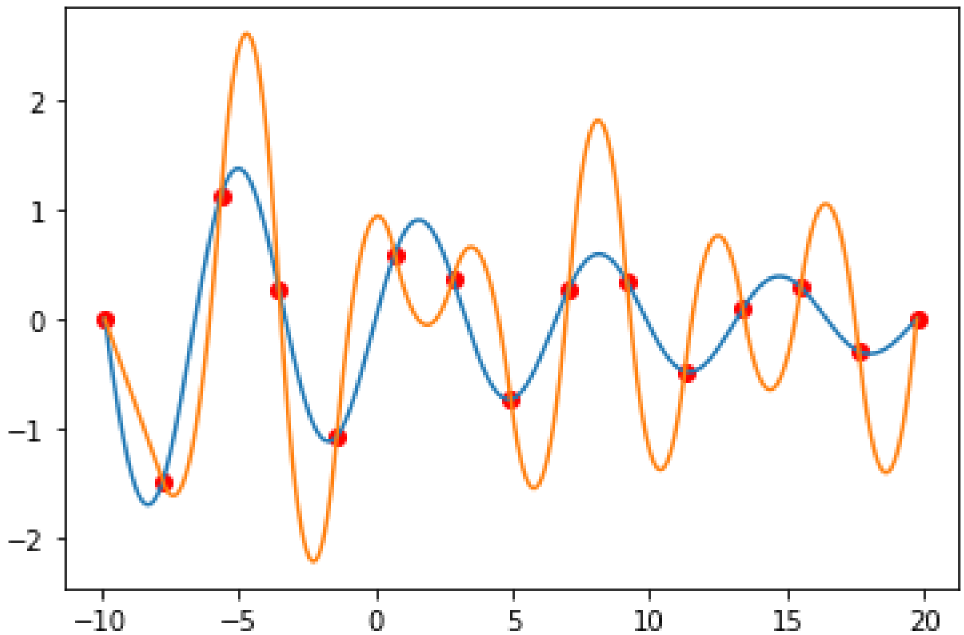
[66]: [<matplotlib.lines.Line2D at 0x7f33323679a0>]



7.1.5 Liczba węzłów: 15

[67]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 15, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

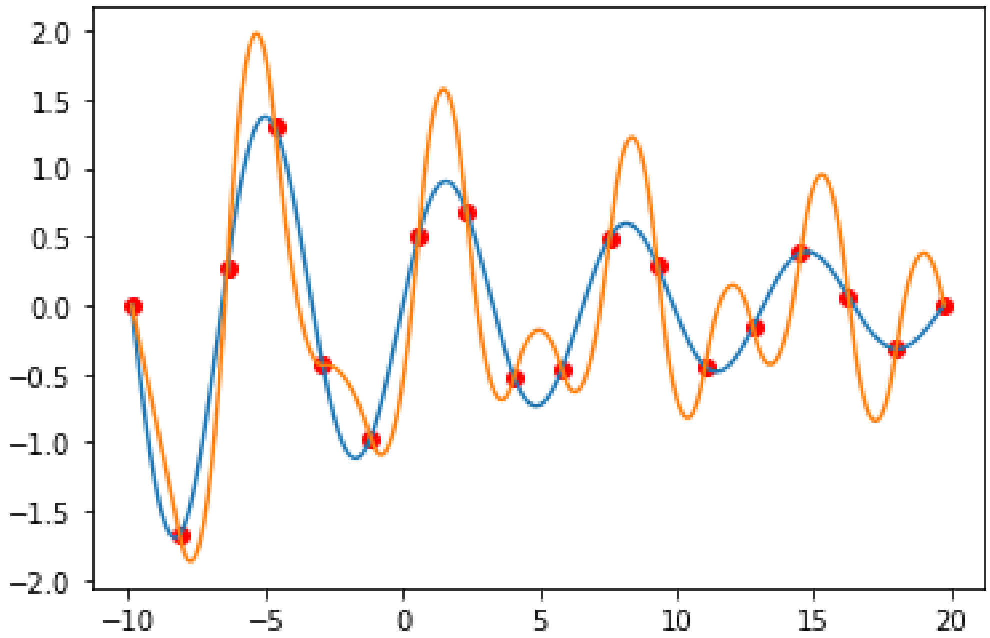
[67]: [<matplotlib.lines.Line2D at 0x7f33322d7be0>]



7.1.6 Liczba węzłów: 18

[88]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 18, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

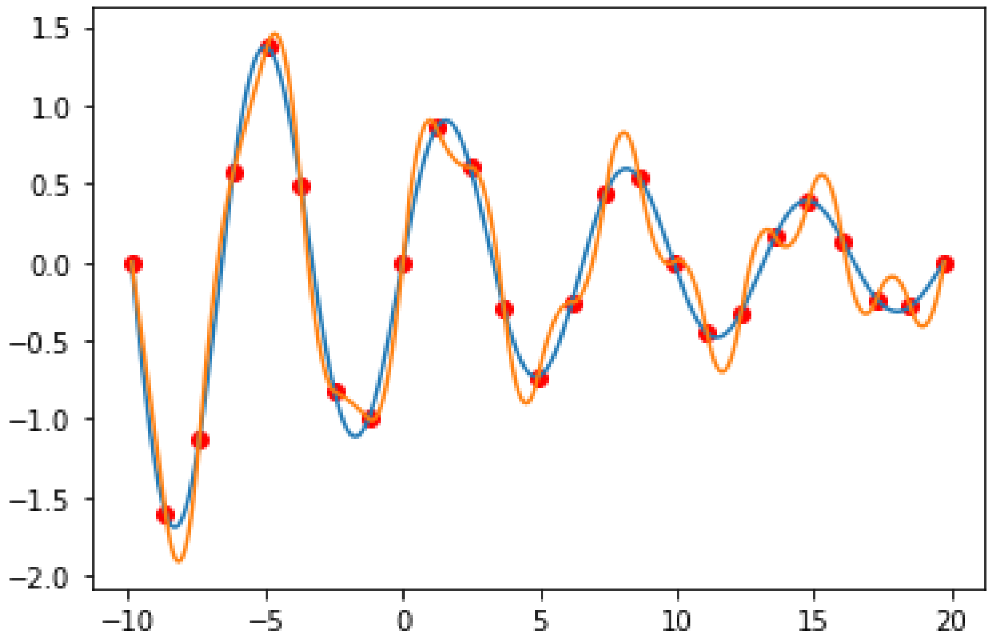
[88]:
```
[<matplotlib.lines.Line2D at 0x7f3331b34cd0>]
```



7.1.7 Liczba węzłów: 25

[91]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 25, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'free boundary')
plt.plot(nx, ny)
```

[91]: [<matplotlib.lines.Line2D at 0x7f3331993b50>]



7.2 Warunek brzegowy: clamped boundary

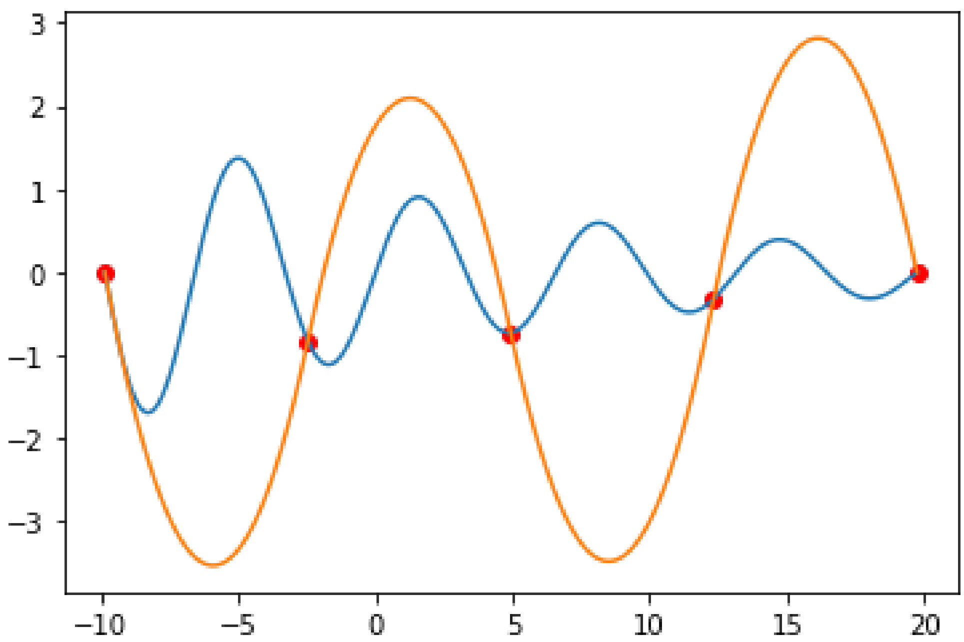
$$s'(x_1) = f'_1$$

W pierszym węźle funkcja interpolująca i interpolowana są do siebie styczne.

7.2.1 Liczba węzłów: 5

[72]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 5, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

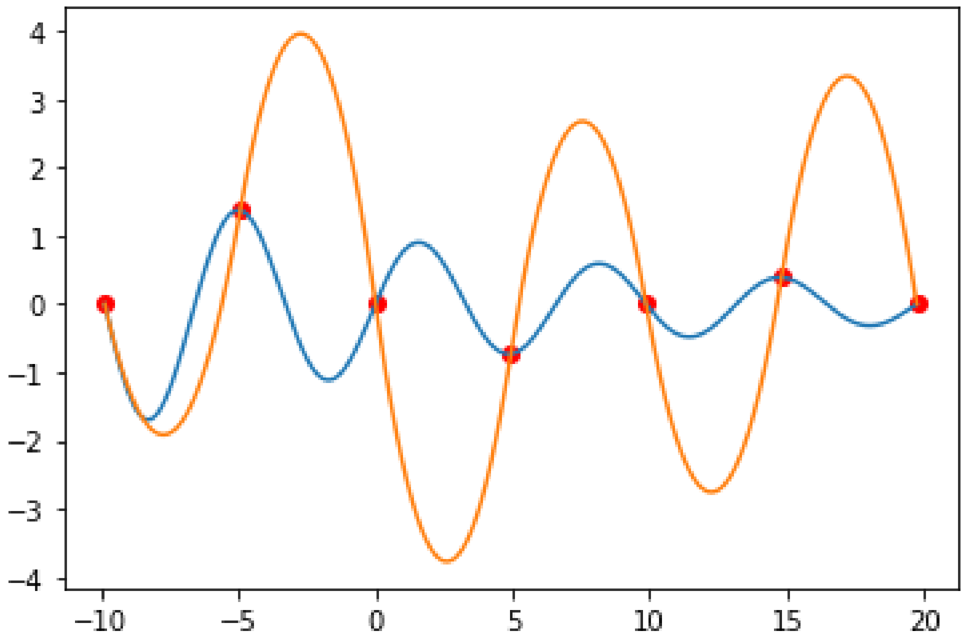
[72]: [<matplotlib.lines.Line2D at 0x7f33321b39a0>]



7.2.2 Liczba węzłów: 7

[73]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 7, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

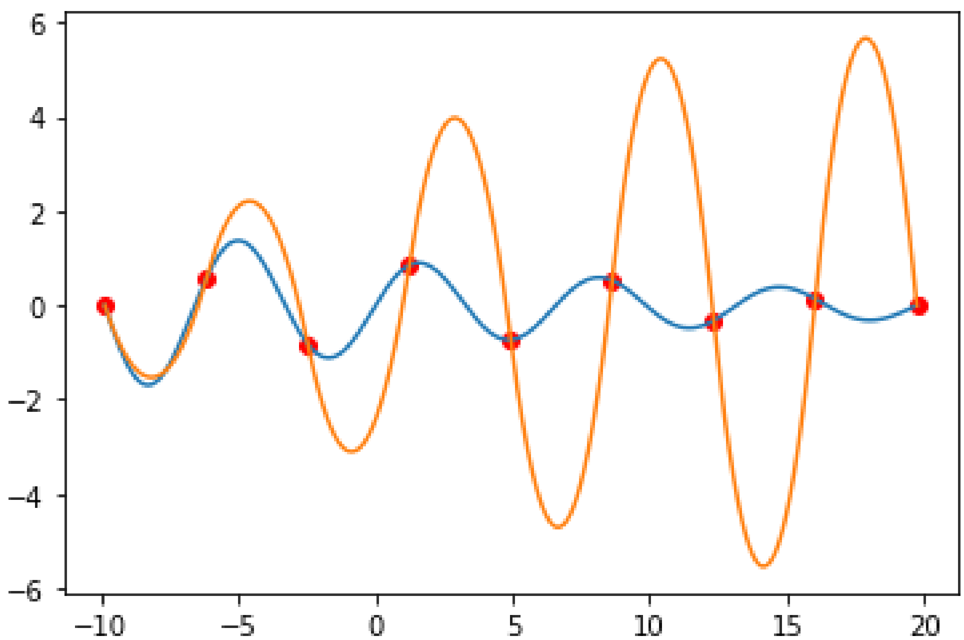
[73]:
```
[<matplotlib.lines.Line2D at 0x7f3332128250>]
```



7.2.3 Liczba węzłów: 9

[74]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 9, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

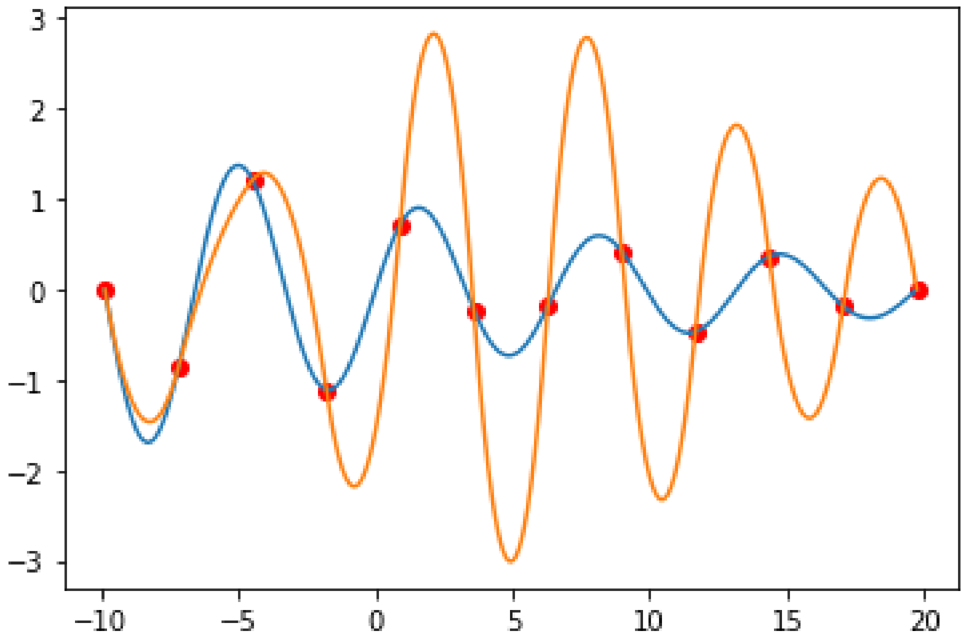
[74]: `[<matplotlib.lines.Line2D at 0x7f333209a5b0>]`



7.2.4 Liczba węzłów: 12

[75]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 12, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

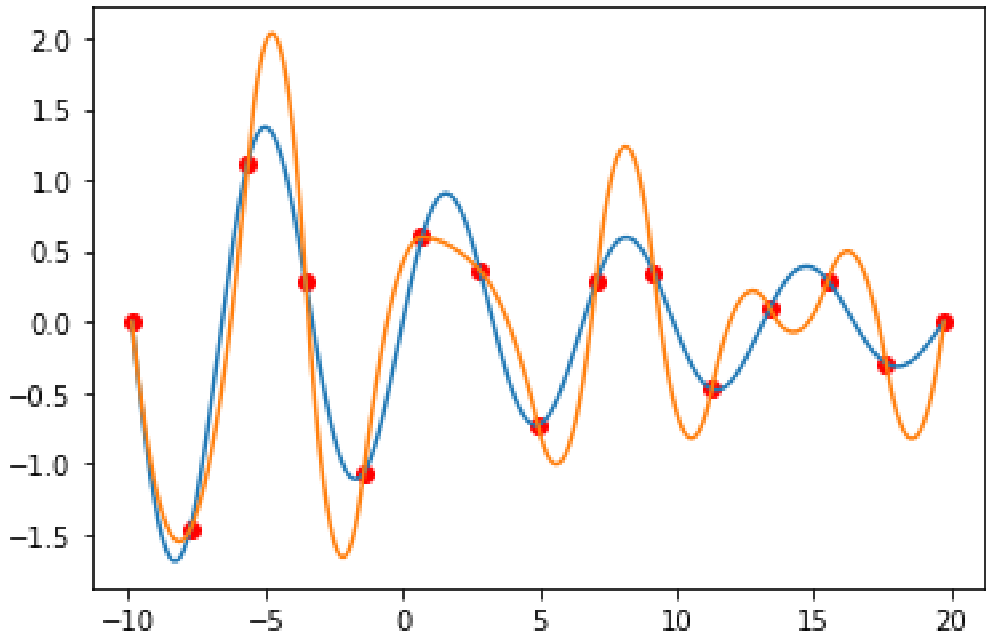
[75]: [<matplotlib.lines.Line2D at 0x7f3332086bb0>]



7.2.5 Liczba węzłów: 15

[76]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 15, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

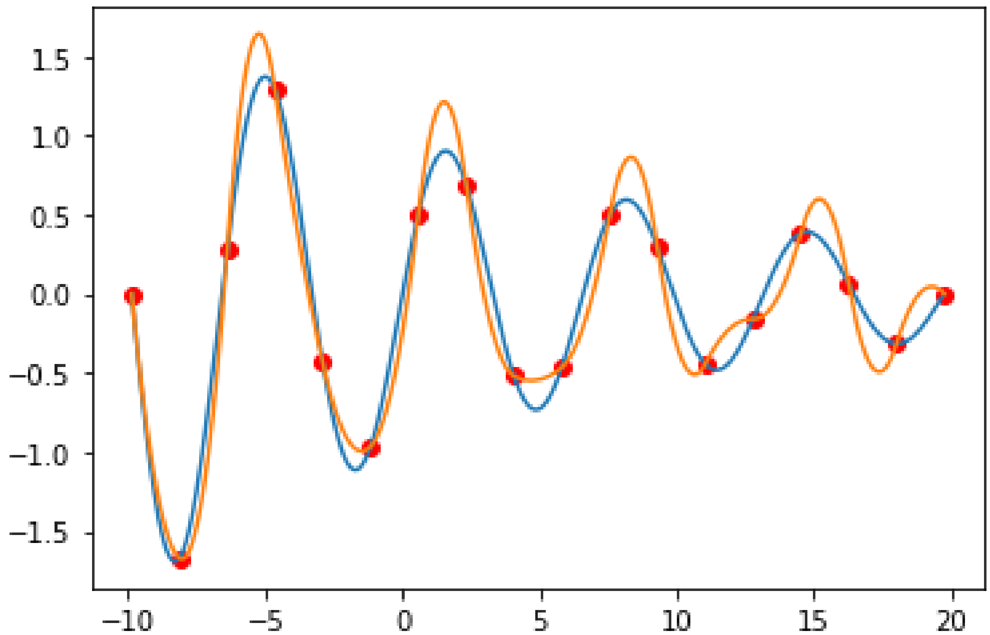
[76]:
```
[<matplotlib.lines.Line2D at 0x7f3331ffe430>]
```



7.2.6 Liczba węzłów: 18

[82]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 18, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

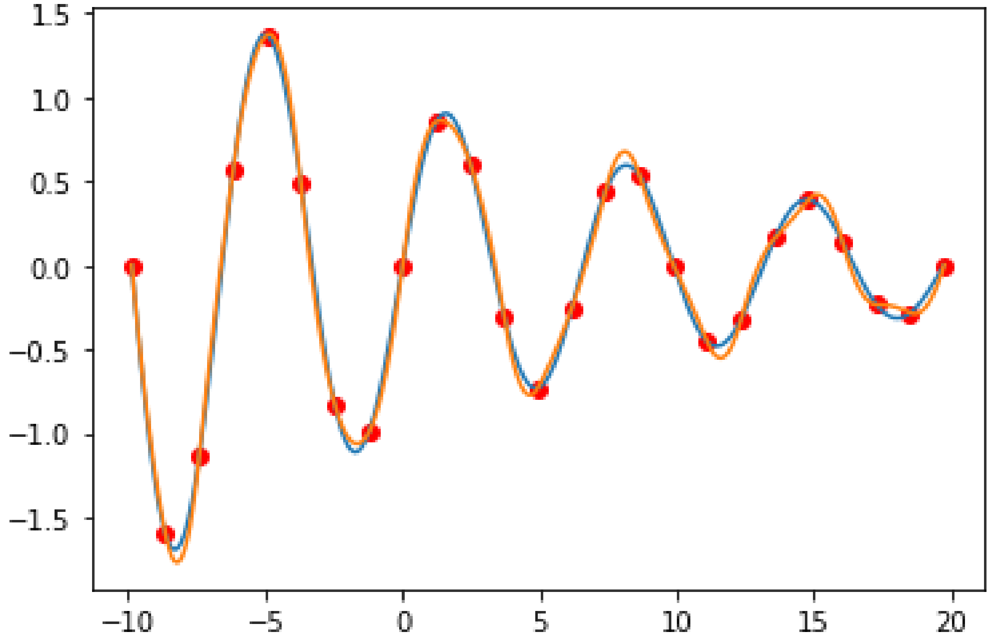
[82]:
```
[<matplotlib.lines.Line2D at 0x7f3331d89610>]
```



7.2.7 Liczba węzłów: 25

[87]:
```
plt.plot(X, Y)
x, y = get_nodes(f8, 25, uniform_nodes)
plt.scatter(x, y, c='r')

nx, ny = spline2(x, y, 1000, 'clamped boundary')
plt.plot(nx, ny)
```

[87]: [<matplotlib.lines.Line2D at 0x7f3331b4c460>]

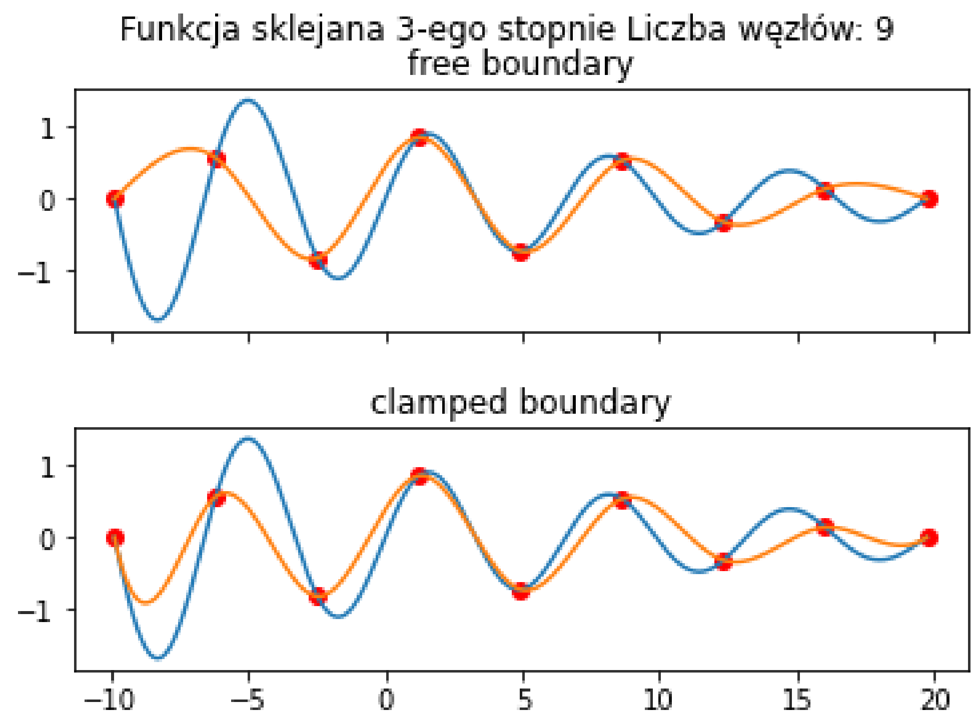


8 Różnice w interpolacjach

8.1 Ze względu na warunek brzegowy

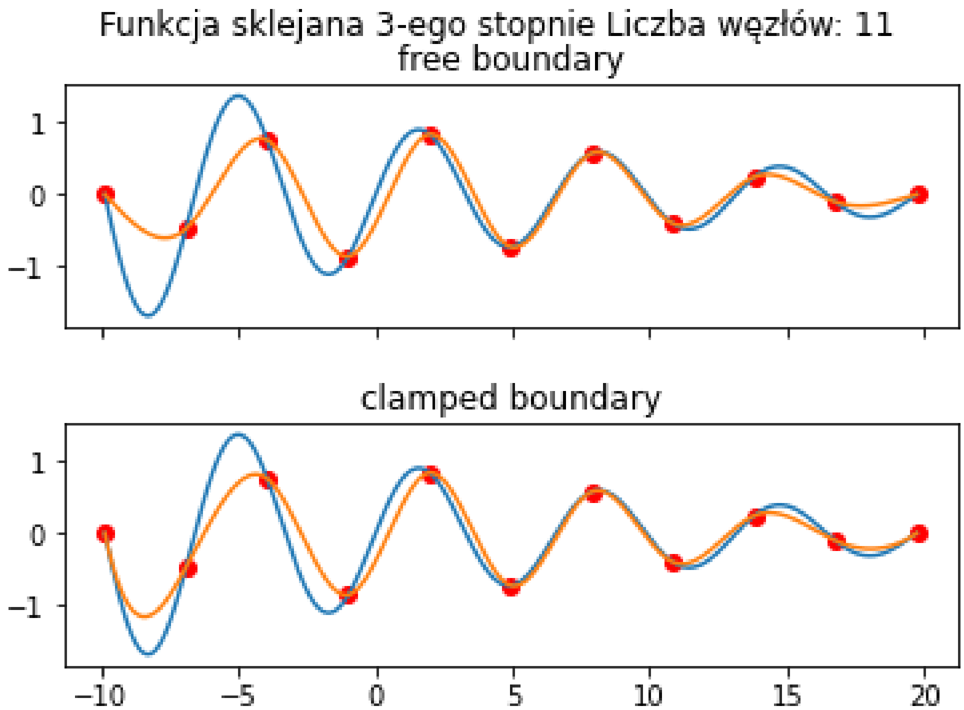
[119]:
```
fig = plt.figure()
gs = fig.add_gridspec(2, hspace=0.4, wspace=0)
(ax1, ax2) = gs.subplots(sharex='col', sharey='row')
ax1.plot(X, Y)
ax2.plot(X, Y)
x, y = get_nodes(f8, 9, uniform_nodes)
ax1.scatter(x, y, c='r')
ax2.scatter(x, y, c='r')
nx, ny = spline3(x, y, 1000, 'free boundary')
ax1.plot(nx, ny)
nx, ny = spline3(x, y, 1000, 'clamped boundary')
ax2.plot(nx, ny)
fig.suptitle("Funkcja sklejana 3-ego stopnie Liczba węzłów: 9")
ax1.set_title("free boundary")
ax2.set_title("clamped boundary")
```

[119]:
```
Text(0.5, 1.0, 'clamped boundary')
```



[122]:
```
fig = plt.figure()
gs = fig.add_gridspec(2, hspace=0.4, wspace=0)
(ax1, ax2) = gs.subplots(sharex='col', sharey='row')
ax1.plot(X, Y)
ax2.plot(X, Y)
x, y = get_nodes(f8, 11, uniform_nodes)
ax1.scatter(x, y, c='r')
ax2.scatter(x, y, c='r')
nx, ny = spline3(x, y, 1000, 'free boundary')
ax1.plot(nx, ny)
nx, ny = spline3(x, y, 1000, 'clamped boundary')
ax2.plot(nx, ny)
fig.suptitle("Funkcja sklejana 3-ego stopnie Liczba węzłów: 11")
ax1.set_title("free boundary")
ax2.set_title("clamped boundary")
```

[122]: Text(0.5, 1.0, 'clamped boundary')

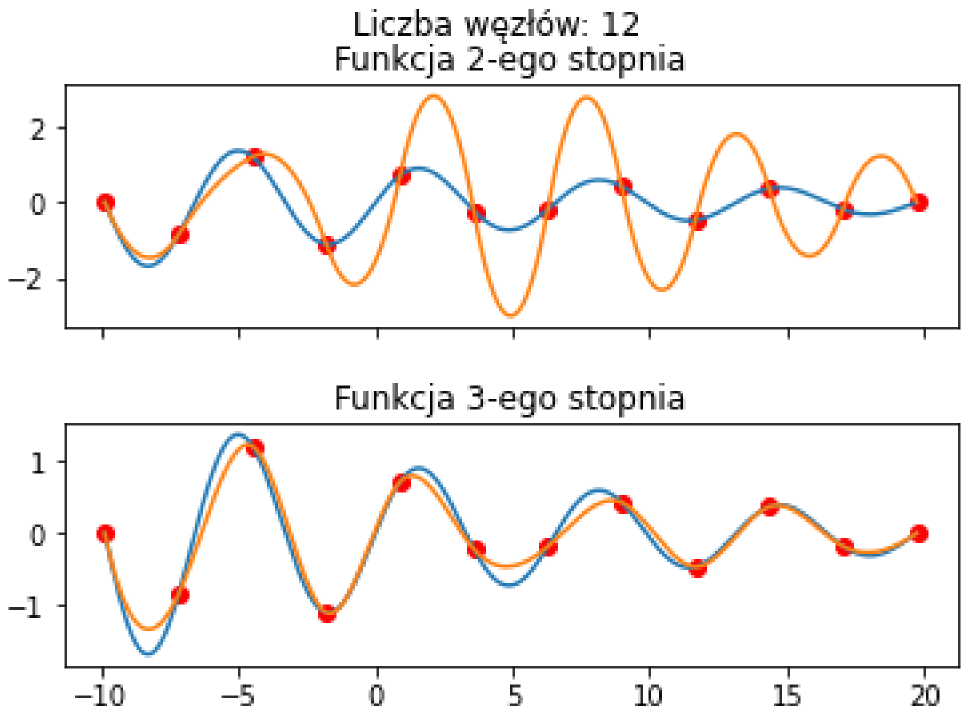


W powyższych przykładach można zauważyć, że wybór warunku brzegowego "clamped boundary" wpłynął pozytywnie na dokładność interpolacji na brzegach funkcji.

8.2 Ze względu na stopień funkcji sklejanej

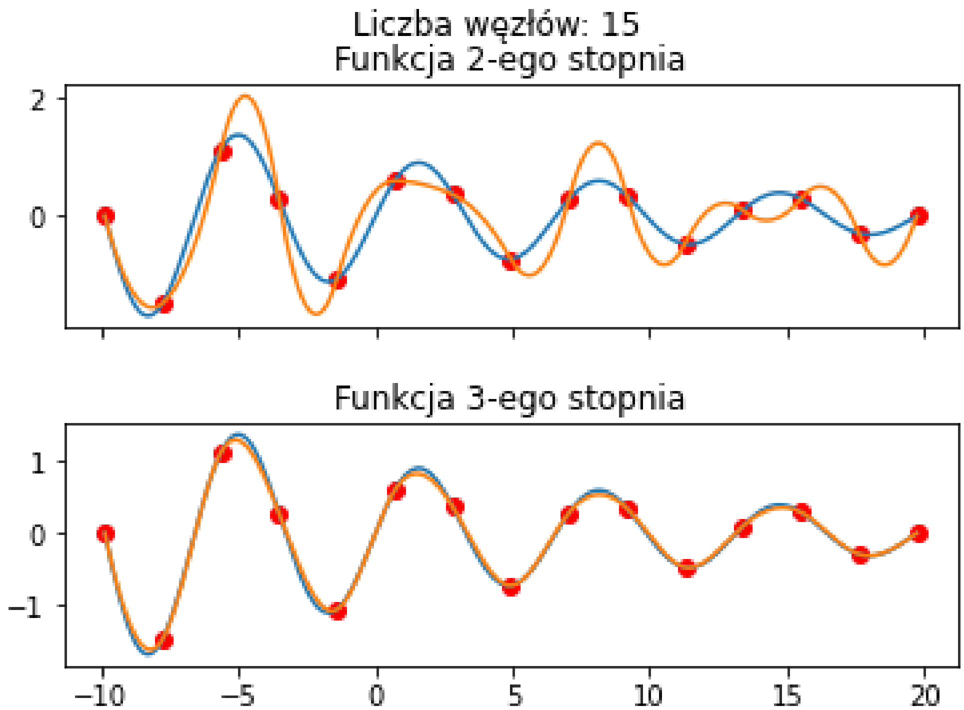
[146]:
```
fig = plt.figure()
gs = fig.add_gridspec(2, hspace=0.4, wspace=0)
(ax1, ax2) = gs.subplots(sharex='col', sharey='row')
ax1.plot(X, Y)
ax2.plot(X, Y)
x, y = get_nodes(f8, 12, uniform_nodes)
ax1.scatter(x, y, c='r')
ax2.scatter(x, y, c='r')
nx, ny = spline2(x, y, 1000, 'clamped boundary')
ax1.plot(nx, ny)
nx, ny = spline3(x, y, 1000, 'clamped boundary')
ax2.plot(nx, ny)
fig.suptitle("Liczba węzłów: 12")
ax1.set_title("Funkcja 2-ego stopnia")
ax2.set_title("Funkcja 3-ego stopnia")
```

[146]:
```
Text(0.5, 1.0, 'Funkcja 3-ego stopnia')
```



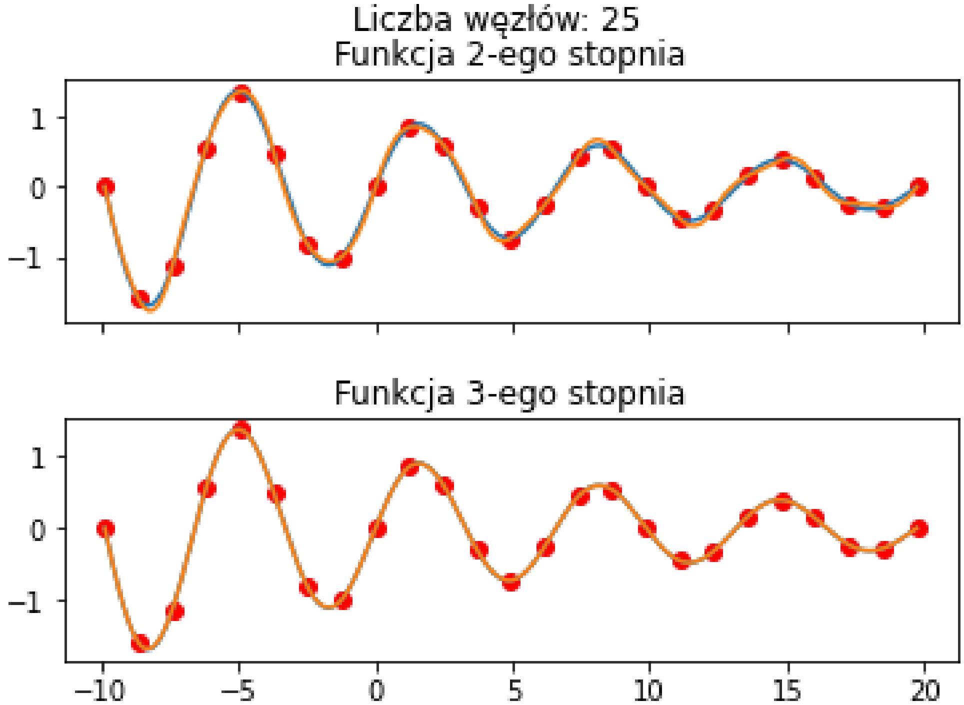
[147]:
```
fig = plt.figure()
gs = fig.add_gridspec(2, hspace=0.4, wspace=0)
(ax1, ax2) = gs.subplots(sharex='col', sharey='row')
ax1.plot(X, Y)
ax2.plot(X, Y)
x, y = get_nodes(f8, 15, uniform_nodes)
ax1.scatter(x, y, c='r')
ax2.scatter(x, y, c='r')
nx, ny = spline2(x, y, 1000, 'clamped boundary')
ax1.plot(nx, ny)
nx, ny = spline3(x, y, 1000, 'clamped boundary')
ax2.plot(nx, ny)
fig.suptitle("Liczba węzłów: 15")
ax1.set_title("Funkcja 2-ego stopnia")
ax2.set_title("Funkcja 3-ego stopnia")
```

[147]:
```
Text(0.5, 1.0, 'Funkcja 3-ego stopnia')
```



[162]:
```
fig = plt.figure()
gs = fig.add_gridspec(2, hspace=0.4, wspace=0)
(ax1, ax2) = gs.subplots(sharex='col', sharey='row')
ax1.plot(X, Y)
ax2.plot(X, Y)
x, y = get_nodes(f8, 25, uniform_nodes)
ax1.scatter(x, y, c='r')
ax2.scatter(x, y, c='r')
nx, ny = spline2(x, y, 1000, 'clamped boundary')
ax1.plot(nx, ny)
nx, ny = spline3(x, y, 1000, 'clamped boundary')
ax2.plot(nx, ny)
fig.suptitle("Liczba węzłów: 25")
ax1.set_title("Funkcja 2-ego stopnia")
ax2.set_title("Funkcja 3-ego stopnia")
```

[162]: `Text(0.5, 1.0, 'Funkcja 3-ego stopnia')`



W powyższych przykładach można zauważyć, że interpolacja funkcją sklejaną 3-ego stopnia jest dokładniejsza od funkcji 2-ego stopnia. Spline 3-ego stopnia całkiem dokładnie zinterpolował zadaną funkcję już dla 15 węzłów. Natomiast interpolując funkcją 2-ego stopnia podobną dokładność otrzymano dopiero dla 25 węzłów.