

# Bholanath Kumbhakar

## Curriculum Vitae

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🌐 [bkumbhakar.github.io](https://bkumbhakar.github.io)

### Education

- December 2018- Current **Research Scholar**, Department of Mathematics, Indian Institute of Technology, Roorkee
- **Research Area:** Differential Inclusion
  - **Supervisor:** Prof. Dwijendra Narain Pandey
- July 2016- May 2018 **M. Sc. in Mathematics**, CGPA: 8.38 The University of Burdwan, Burdwan, West Bengal, India
- July 2013- May 2016 **B.Sc. in Mathematics**, Percentage: 79.875%, Jagannath Kishore College, Purulia, West Bengal, India
- 2013 **Higher Secondary in Science**, Percentage: 77.2%, Purulia Jilla School, Purulia, West Bengal, India
- 2011 **Class 10th**, Percentage: 81.875%, Ladhurka High School (H.S), Purulia, West Bengal, India

### List of Publications

- Kumbhakar, Bholanath, and Dwijendra Narain Pandey. "Approximate controllability of evolution hemivariational inequalities in Banach spaces." *Journal of Differential Equations* 410 (2024): 346-381.
- Bholanath Kumbhakar, Dwijendra Narain Pandey. Approximate controllability of nonconvex valued semilinear differential inclusion. *Evolution Equations and Control Theory*. doi: 10.3934/eect.2024047
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey. "Approximate controllability of semilinear differential inclusion with nonlocal conditions." 2023 Proceedings of the Conference on Control and its Applications (CT). Society for Industrial and Applied Mathematics, 2023.

### List of Preprints

- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, Exact Controllability of Evolution Hemivariational Inequalities in Banach spaces (submitted).
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey,  $L^p$ - null controllability of abstract differential inclusion with the nonlocal condition, Revision submitted in the Journal "Mathematical Control and Related Fields."
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey, Approximate Controllability of Nonconvex valued Semilinear Differential Inclusions with Nonlocal Conditions, (Submitted)
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey,  $L^p$ - Exact controllability of abstract differential inclusion with nonlocal condition, (Revision submitted in the Journal "Topological Methods in Nonlinear Analysis").
- Kumbhakar, Bholanath, and Dwijendra Narain Pandey. "Approximate Controllability of Fractional Hemivariational Inequalities in Banach Spaces." arXiv preprint arXiv:2408.05536 (2024).

- Kumbhakar, Bholanath, and Dwijendra Narain Pandey. "Optimal Control and Approximate controllability of fractional semilinear differential inclusion involving  $\psi$ -Hilfer fractional derivatives." arXiv preprint arXiv:2306.01352 (2023).

## Research Interest

### ○ Differential Inclusion

Differential inclusions constitute an active branch of the general theory of differential equations. Differential inclusions, as opposed to ordinary differential equations, encompass a broader range of possible trajectories, representing phenomena involving uncertain inputs, discontinuities, and various types of constraints. Because of this adaptability, differential inclusions are particularly effective at capturing situations in which the evolution of a system is subject to ambiguous or non-deterministic influences. As a result, they are used in various fields, including control theory, economics, ecology, and engineering, where uncertainty, variability, and discontinuities are common.

### ○ Control Theory

The controllability problem may be formulated roughly as follows. Consider an evolution system either described in terms of partial differential equations (PDEs) or ordinary differential equations (ODEs) on which we are allowed to act by means of a suitable choice of control (the right-hand side of the system, the boundary conditions, etc.). Given a time interval  $I := [0, a]$  and initial and final states, the goal is to determine whether there exists a control driving the given initial data to the given final ones in time  $a$ . If the system can be steered from any given initial state to any given final state in finite time using control input(s), then we say that the system is exactly controllable. If the system can be steered from any given initial state to the origin, then we say the system is null controllable. On the other hand, approximate controllability means we can steer the state of a system from any given initial state to arbitrarily close to a desired final state using control input(s) rather than reaching the exact final state.

### ○ Hemivariational Inequality

Hemivariational inequalities were first introduced by Panagiotopoulos in the early 1980s and are closely linked to Clarke's development of the generalized gradient of a locally Lipschitz functional. Like variational inequalities, the initial interest in hemivariational inequalities emerged from mechanical problems. These inequalities effectively handle problems involving nonmonotone, nonsmooth, and multivalued constitutive laws, forces, and boundary conditions, both theoretically and numerically. Over the past three decades, hemivariational inequalities have proven useful across various fields, leading to numerous problems modeled mathematically in terms of these inequalities.

- **Fractional Differential Equations**

It is worth noting that most existing research works focused on integer order dynamics. However, in the real world, it has been shown that many natural phenomena cannot be effectively interpreted by the integer order dynamics, such as chemotaxis behavior and food searching of germs. Nevertheless, fractional order dynamics possess excellent memory and hereditary properties, resulting in superior performance and stronger robustness than standard integer order dynamic systems. It has been proved that some cases, such as the macromolecule fluids, lateral inhibition of biological vision systems, and automobiles running on the road's surface containing viscoelastic materials, can be more accurately described by fractional-order dynamic systems. Researchers pointed out that many physical systems are unsuitable to be characterized by integer order dynamics, such as high-speed aircraft traveling on rainy days or snowy days and vehicles moving on top of sand or muddy road.

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## Brief summary of Research Work

- Differential inclusions, which naturally extend the concept of ordinary differential equations, have become integral part to various scientific fields due to their wide-ranging applications, forming a significant branch within the general theory of differential equations. In addressing the existence of mild solutions and the controllability of differential inclusions, both topological properties (such as continuity, measurability, and compactness) and geometric properties (such as convexity) of the associated multivalued maps are crucial. Notably, the results differ substantially between convex and nonconvex valued multimaps, as do the continuity properties, including upper semicontinuity, lower semicontinuity, and Hausdorff continuity. Additionally, the characteristics of the state and control spaces are pivotal in determining the existence and controllability of solutions. Research conducted in Banach spaces presents more significant challenges compared to Hilbert spaces. With these considerations in mind, we focused on the controllability of systems governed by semilinear differential inclusions and hemivariational inequalities, specifically examining exact, approximate, and null controllability problems.
- We observe that many models in mechanics, engineering, chemistry, and fluid flow have recently been successfully modeled by fractional hemivariational inequalities. The constitutive equation for a quasistatic frictionless contact problem of a viscoelastic body is modeled with the fractional Kelvin-Voigt law, and the contact condition is described by the Clarke subdifferential of a nonconvex and nonsmooth functional. The variational formulation of this problem is provided in the form of a fractional hemivariational inequality. Therefore, we study the exact and approximate controllability of systems governed by evolution fractional hemivariational inequalities involving Caputo fractional derivatives. We also established the approximate controllability and optimal controllability of systems involving  $\psi$ -Hilfer fractional derivatives.

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## Honors & Awards

**CSIR JRF (December 2017)**, Got 98th rank in the NET exam conducted by CSIR  
**CSIR JRF (June 2018)**, Got 98th rank in the NET exam conducted by CSIR

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## Teaching Assistant

Spring 2022 **Mathematical Methods**, B. Tech. 1st Year, IIT Roorkee

- **Subject Taught:** Ordinary Differential Equations, Partial Differential Equations, Laplace Transform, Z-Transform, Fourier Series, Fourier Transforms

- **Instructor:** Prof. Saikat Saha
- Autumn 2021 **Mathematics I**, B.Tech. 1st Year, IIT Roorkee
  - **Subject Taught:** Matrix Algebra, Differential Calculus, Integral Calculus, Vector Calculus
  - **Instructor:** Prof. Ankik Kumar Giri
- Spring 2021 **Mathematical Methods**, B.Tech 1st Year, IIT Roorkee
  - **Instructor:** Prof. Ram Jiware
- Autumn 2020 **Mathematics I**, 1st year B. Tech., IIT Roorkee
  - **Instructor:** Prof. S. K. Gupta
- Spring 2020 **Mathematics I**, B. Tech. 1st Year, IIT Roorkee
  - **Instructor:** Prof. Rama Bhargava

## Workshops

- NCMW - Control Theory for Differential Equations (2022)  
Date: 28 Nov 2022 to 10 Dec 2022  
Venue: IISER Kolkata
- International Workshop on Fractional Derivatives: Theory and Computations with Applications (FDTCA 2021)  
Date: November 12-14, 2021  
Venue: Department of Mathematical Sciences Indian Institute of Technology (BHU) Varanasi
- Control theory meets the theory of Homogenization  
Date: February 28- March 3, 2023  
Venue: Ramanujan Hall, Department of Mathematics, IIT Bombay
- “Fundamentals of Differential Equations with Applications to Real World Problems  
Date: July 8- July 13, 2024  
Venue: Department of Mathematics, Jaypee University of Information Technology, Wahnaghat (Shimla Hills), Himachal Pradesh, INDIA

## Conference

- 87<sup>th</sup> Annual Conference of the Indian Mathematical Society  
Date: December 4-7, 2021  
Venue: Online  
Organized by: GM University, Maharashtra
- International Conference on Dynamical systems, Control and their Applications  
Date: July 1-3, 2022  
Venue: IIT Roorkee
- SIAM Conference on Control Theory and Its Applications (SIAM CT 2023)  
Date: July 24-26, 2023, Philadelphia, Pennsylvania, USA
- The International Congress on Industrial and Applied Mathematics (ICIAM) 2023 (online)  
Date: August 20-25, 2023, Organized by: Waseda University, Tokyo, Japan

## Languages:

Bengali (Native), Hindi, English

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## References

Bholanath Kumbhakar

Research Scholar

Department of Mathematics, Indian Institute of Technology Roorkee



Personal Webpage



Researchgate



Mathscinet



Google Scholar

**Declaration:** I hereby declare that all the statements made herein are true to my best of knowledge and belief.

Place: Roorkee

Bholanath Kumbhakar