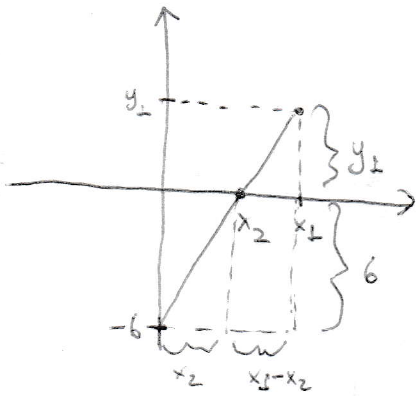
Ged - Questions - Assignment - 06 (1)

Task 1:



Given is a point (x_1, y_1).
After projection our point will be $(x_2, 0)$.
With the similar triangles, we get the equation:

$$\frac{6}{y_1 + 6} = \frac{x_2}{x_1} \iff \frac{6x_1}{y_1 + 6} = x_2$$

With this we get the following transformation matrix M:

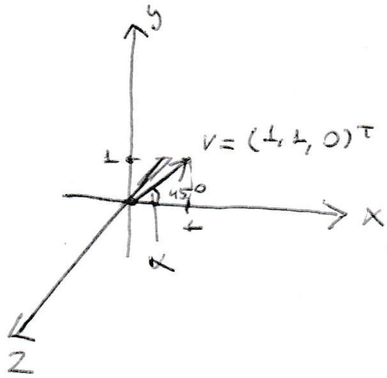
$$M = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 6 \end{pmatrix}$$

, because for a point (x_1, y_1) we have:

$$M \cdot \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 6 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6x_1 \\ 0 \\ y_1 + 6 \end{pmatrix}$$

, and after homogenous division we get the point $\left(\frac{6x_1}{y_1+6}, 0\right)$ which is $(x_2, 0)$. $\square$

Task 2:



We solve this problem with three steps.

1. Rotate α degrees around z-axis, so the vector is equivalent to x-axis
2. Rotate point 90° degrees around x-axis
3. Reverse rotate around z-axis so the vector comes back to its original position.

Because of isosceles Triangle with 90° degrees in the x-y-Plane with Vector $v = (1, 1, 0)^T$, we get $\alpha = 40^\circ$. Thus we have the transformation matrix: (See next page)

All rotations are counter-clockwise (To get to the step 1 for example, calculate $R_z(-\alpha)$)

$$\cancel{M = R_z(-45^\circ) \cdot R_x(90^\circ) \cdot R_z(45^\circ) = \begin{pmatrix} \cos -45^\circ & -\sin -45^\circ & 0 \\ \sin 45^\circ & \cos -45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 90^\circ & -\sin 90^\circ \\ 0 & \sin 90^\circ & \cos 90^\circ \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ \\ 0 & \sin 45^\circ & \cos 45^\circ \end{pmatrix}}$$

$$M = R_z(45°) \cdot R_x(90°) \cdot R_z(-45°)$$

$$= \begin{pmatrix} \cos(45°) & -\sin(45°) & 0 \\ \sin(45°) & \cos(45°) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(90°) & -\sin(90°) \\ 0 & \sin(90°) & \cos(90°) \end{pmatrix} \cdot \begin{pmatrix} \cos(-45°) & -\sin(-45°) & 0 \\ \sin(-45°) & \cos(-45°) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & -1 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{pmatrix}.$$

$\square$