GED – Assignment – 5 – Questions

Question 1

The vertices in the vertex buffer:
$v_0 = (2,2,0)$, $v_1 = (1,1,0)$, $v_2 = (2,1,0)$, $v_3 = (3,1,0)$, $v_4 = (1,0,0)$
$v_5 = (3,0,0)$

We start with triangle_0, suggested in the first constraint.
It uses the vertices: v_0, v_1, v_2. The corresponding order in the index buffer starts with v_0, because of second constraint.
We check the last constraint: For $\lambda_1 = v_0$, $\lambda_2 = v_1$, $\lambda_3 = v_2$
(λ_i is the order in Index buffer)

$$c = (\lambda_2 - \lambda_1) \times (\lambda_3 - \lambda_1) = (v_1 - v_0) \times (v_2 - v_0) = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

z-Component is positive, trying other possibility: $\lambda_1 = v_0$, $\lambda_2 = v_2$, $\lambda_3 = v_1$:

$$c = (v_2 - v_0) \times (v_1 - v_0) = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}, \text{ z-Component negativ,}$$

with this we have the order: $0, 2, 1$:
Just like above we compute triangle_1:

$\lambda_1 = v_0$, $\lambda_2 = v_2$, $\lambda_3 = v_3$, $c = (v_2 - v_0) \times (v_3 - v_0) = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\lambda_1 = v_0$, $\lambda_2 = v_3$, $\lambda_3 = v_2$, $c = (v_3 - v_0) \times (v_2 - v_0) = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

Then the order is now: $0, 2, 1, 0, 3, 2$:
Compute triangle_2:

$\lambda_1 = v_2$, $\lambda_2 = v_3$, $\lambda_3 = v_5$, $c = (v_3 - v_2) \times (v_5 - v_2) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

The order is now: $0, 2, 1, 0, 3, 2, 2, 3, 5$:
Compute triangle_3:

$\lambda_1 = v_2$, $\lambda_2 = v_4$, $\lambda_3 = v_5$, $c = (v_4 - v_2) \times (v_5 - v_2) = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1+1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

$\lambda_1 = v_2$, $\lambda_2 = v_5$, $\lambda_3 = v_4$, $c = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

The order is now: $0, 2, 1, 0, 3, 2, 2, 3, 5, 2, 5, 4$

Compute triangle_4: $\lambda_1 = v_1, \lambda_2 = v_2, \lambda_3 = v_4$

$$c = (v_2 - v_1) \times (v_4 - v_1) = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}0\\-1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\-1\end{pmatrix}$$

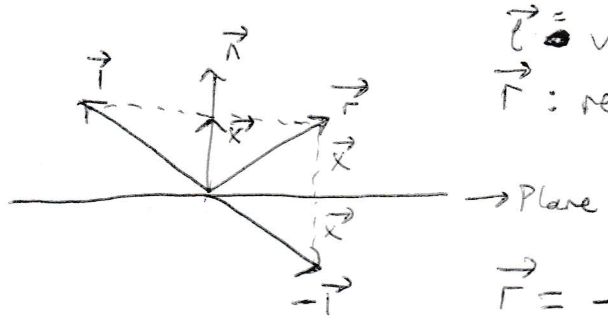
The order is now: 0, 2, 1, 0, 3, 2, 2, 3, 5, 2, 5, 4, 1, 2, 4

Altogether we have the following index buffer:

triangle_0			triangle_1			triangle_2			triangle_3			triangle_4		
0	2	1	0	3	2	2	3	5	2	5	4	1	2	4

Question 2

Look at the model:



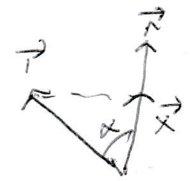
$\vec{n}$: normal vector

$\vec{l}$: vector from ~~source~~ point to light source

$\vec{r}$: reflected ~~light~~ vector

$$\vec{r} = -\vec{l} + 2\vec{x}$$

Assume $\vec{n}_0$ and $\vec{l}_0$ unit length:



$$\vec{x} = \cos(\alpha) \cdot \vec{n}_0 \quad , \quad \cos(\alpha) = \vec{n} \cdot \vec{l}_0$$

With the model above: $\vec{r} = 2(\vec{n} \cdot \vec{l}) \cdot \vec{n} - \vec{l}$ (For $\vec{x}$, $\vec{x}_0$ means normalized $\vec{x}$)

$w_i = (-10, 0, 0)$, $p = (0,0,0)$ $\vec{n} = \begin{pmatrix}1\\2\\0\end{pmatrix}$, because of the definition of the normal (our plane satisfies $(x,y,z) \cdot (1,2,0) = 0$)

$\vec{l} = \cancel{w_i - p} \; w_i = \begin{pmatrix}-10\\0\\0\end{pmatrix} \qquad \|\vec{l}\| = 10, \quad \vec{l}_0 = \frac{1}{10}\begin{pmatrix}-10\\0\\0\end{pmatrix} = \begin{pmatrix}-1\\0\\0\end{pmatrix}$

$\vec{n} = \begin{pmatrix}1\\2\\0\end{pmatrix}, \quad \|\vec{n}\| = \sqrt{1^2 + 2^2} = \sqrt{5} \; ; \; \vec{n}_0 = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix}$

$$\vec{r}_0 = 2 \cdot (\vec{n}_0 \cdot \vec{l}_0) \cdot \vec{n}_0 - \vec{l}_0 = -2 \cdot \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix} - \begin{pmatrix}-1\\0\\0\end{pmatrix} = -\frac{2}{5}\begin{pmatrix}1\\2\\0\end{pmatrix} - \begin{pmatrix}-1\\0\\0\end{pmatrix} = \begin{pmatrix}+\frac{3}{5}\\-\frac{4}{5}\\0\end{pmatrix}$$

$$= \begin{pmatrix}+0{,}6\\-0{,}8\\0\end{pmatrix}$$

(We normalized $\vec{n}$ and $\vec{l}$, because wanted the normalized reflection vector $\vec{r}_0$.)

$$= \begin{pmatrix}0{,}6\\-0{,}8\\0\end{pmatrix}$$