

# Medusa

## Mutant Equivalence Detection Using SAT Analysis



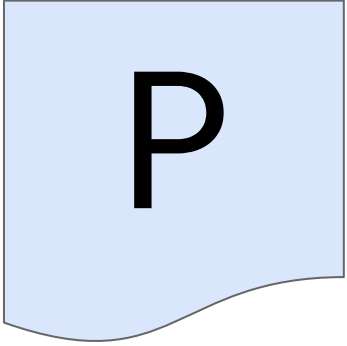
**Benjamin Kushigian<sup>1,2</sup>, Amit Rawat<sup>1</sup>, René Just<sup>2</sup>**

<sup>1</sup>UMass Amherst

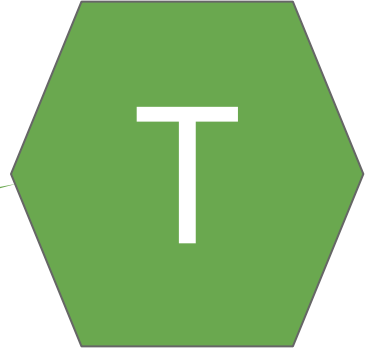
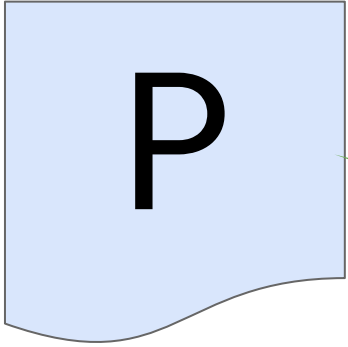
<sup>2</sup>University of Washington



**Question: Is our program correct?**



**Let's test its correctness**



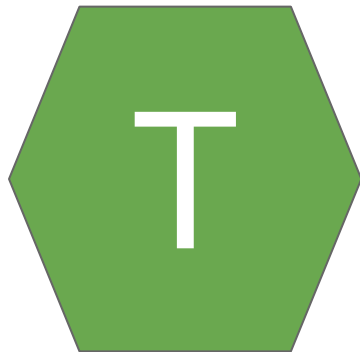
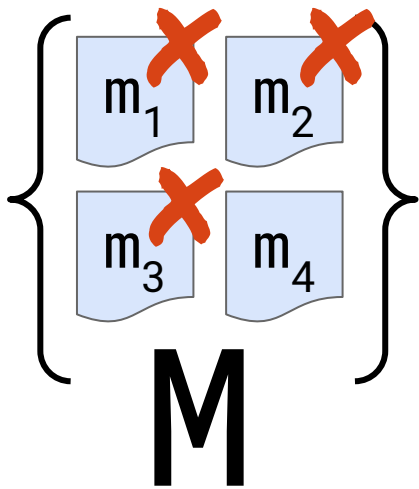
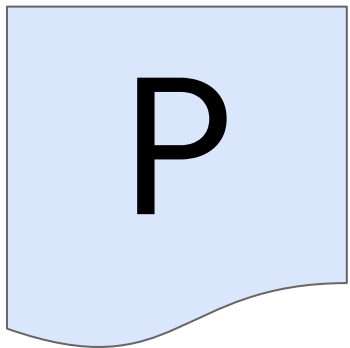
*Great,  
T passes on P!*

**Question: how good is our test suite?**



**Let's analyze  $T$ 's effectiveness**

# Let's analyze $T$ 's effectiveness



## Mutation Analysis

- Generate mutant set  $M$
- Run  $T$  on mutants in  $M$
- Each mutant that fails a test in  $T$  is *killed*

# Mutation Score

For test suite  $T$  and mutant set  $M$ , the ***mutation score*** of  $T$  is

$$\mu(T) = \frac{\left| \left\{ \text{Killed mutants in } M \right\} \right|}{\left| \left\{ \text{Nonequivalent mutants in } M \right\} \right|}$$

# Mutation Score

For test suite  $T$  and mutant set  $M$ , the ***mutation score*** of  $T$  is

$$\mu(T) = \frac{\left| \left\{ \text{Killed mutants in } M \right\} \right|}{\left| \left\{ \text{Nonequivalent mutants in } M \right\} \right|}$$

↑  
**UNDECIDABLE**

# A common proxy: Mutation Kill Ratio

For test suite  $T$  and mutant set  $M$ , the ***mutant kill ratio*** of  $T$  is

$$\rho(T) = \frac{\left| \left\{ \text{Killed mutants in } M \right\} \right|}{\left| \left\{ \text{Nonequivalent mutants in } M \right\} \right|}$$

*Mutants in  $M$  not known to be equivalent*

**DECIDABLE**





# Mutation Kill Ratio vs. Mutation Score

- $\rho(T)$  approximates  $\mu(T)$

- **Undetected Equivalent**

- **Mutants Skew Analysis**

- **Results**

**Say we have an equivalent mutant**



- All tests in  $T$  need to be run on  $m_4$ 
  - $m_4$  is equivalent: it passes all tests
  - This wastes computer resources
- A human developer tries to kill  $m_4$ 
  - This wastes the developer's time

# **Detecting Equivalent Mutants**

# Detecting Equivalent Mutants

There are a number of ways to detect equivalent mutants

1. Hand proof
2. Compiler techniques
3. **Automated reasoning tools**

# Detecting Equivalent Mutants

## Two Main Challenges

- **Applicability:** What programs can a system reason about?
- **Efficiency:** How many resources does a system need?

# Applicability: What can be reasoned about?

## Applicability challenges:

- Loops and recursion
  - Loop unrolling can witness non-equivalence but cannot prove equivalence in general

# Applicability: What can be reasoned about?

## Applicability challenges:

- Loops and recursion
  - Loop unrolling can witness non-equivalence but cannot prove equivalence in general
- Heap space

# Applicability: What can be reasoned about?

## Applicability challenges:

- Loops and recursion
  - Loop unrolling can witness non-equivalence but cannot prove equivalence in general
- Heap space

## Approaches to increase applicability (future work described in the paper):

- Middle Out Constraint Generation
- Exception Abstraction
- Foldability: is **(fold f xs acc)** equivalent to **(fold f' xs acc)**?



# Efficiency: How many resources are needed?

## Efficiency challenge:

- Solving NP-hard problems such as SAT

Medusa improves efficiency with **constraint forking**

- Decreases problem size
- Reuses work done by SMT solver

# **Proving Program Equivalence**

# Question: are these equivalent?

2

=

2

```
int squarePos(int x) {  
    if (x > 0) x = x * x;  
    return x;  
}
```

```
int squarePos(int x) {  
    if (x <= 0) x = x * x;  
    return x;  
}
```

4

≠

2

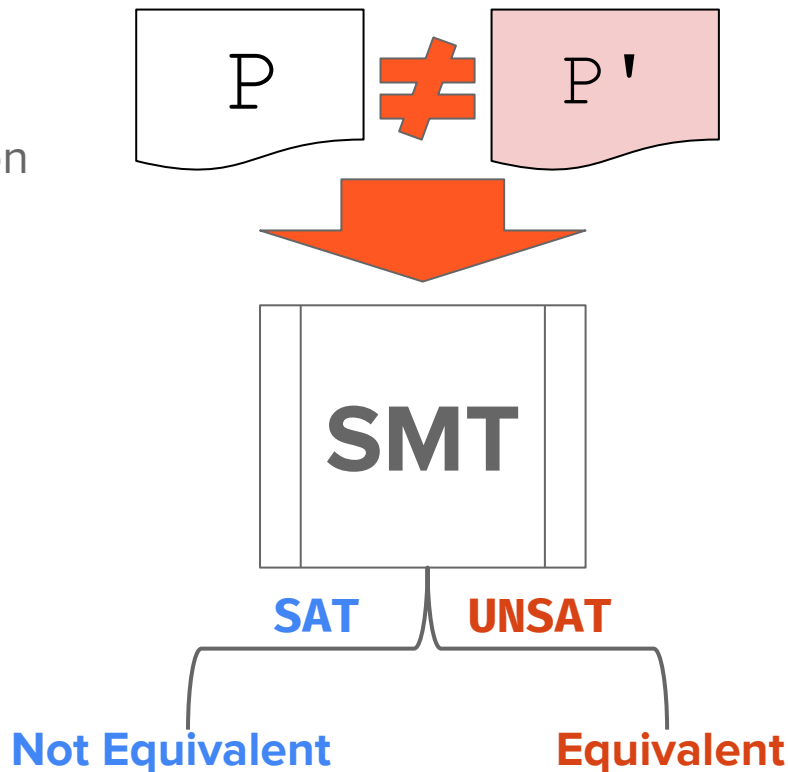
ORIGINAL

NOT EQUIVALENT

MUTANT

# Question: How to (dis)prove equivalence?

- Transform query to SMT *constraints*
  - Constrain both programs' execution
  - Assert inputs are equal
  - Assert outputs are different
- Ask SMT if constraints are satisfiable
  - **SAT** => **Not Equivalent**
  - **UNSAT** => **Equivalent**
  - May also be **UNKNOWN**



# Question: How to constrain Java?

## Observation

- Java is complicated
- No formal semantics
- Semantics defined by Javac/JVM

## We Should:

- Compile program
- Constrain bytecode

# Question: How to constrain bytecode?

Two types of bytecode instructions

## **Straight Line**

- **Operand Stack Instructions**  
iadd, imul, push, pop...
- **Variables Table Instructions**  
istore, iload, ...

## **Branching**

- **Control Flow Instructions**  
ifgt, iflt, if\_cmpgt, ...



***Distinction captured by  
Control Flow Graphs***

# High Level Overview

Given a program and a mutant:

```
int squarePos(int x) {  
    if (x > 0) x = x * x;  
    return x;  
}
```

ORIGINAL

```
int squarePos(int x) {  
    if (x <= 0) x = x * x;  
    return x;  
}
```

MUTANT

# High Level Overview

Given a program and a mutant:

## 1. Compile to bytecode

```
0: iload_1  
1: ifle 8  
4: iload_1  
5: iload_1  
6: imul  
7: istore_1  
8: iload_1  
9: ireturn
```

ORIGINAL

```
0: iload_1  
1: ifgt 8  
4: iload_1  
5: iload_1  
6: imul  
7: istore_1  
8: iload_1  
9: ireturn
```

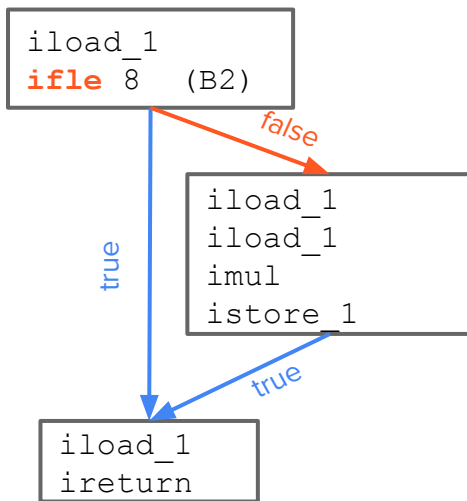
MUTANT



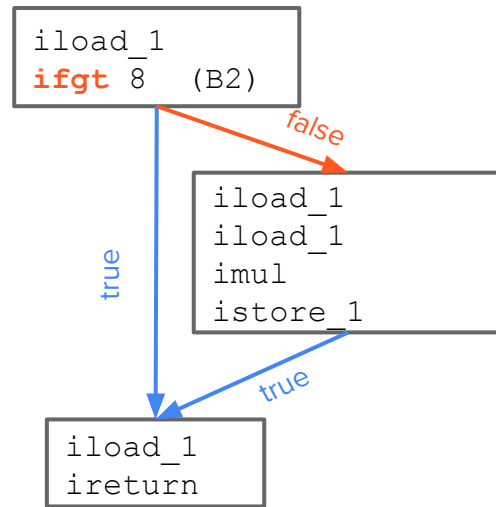
# High Level Overview

Given a program and a mutant:

1. Compile to bytecode
2. **Construct CFG**



**ORIGINAL**

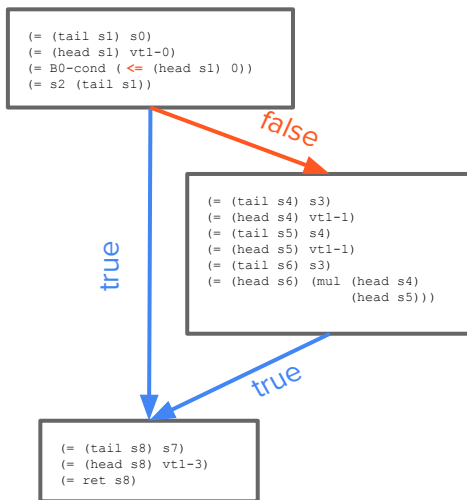


**MUTANT**

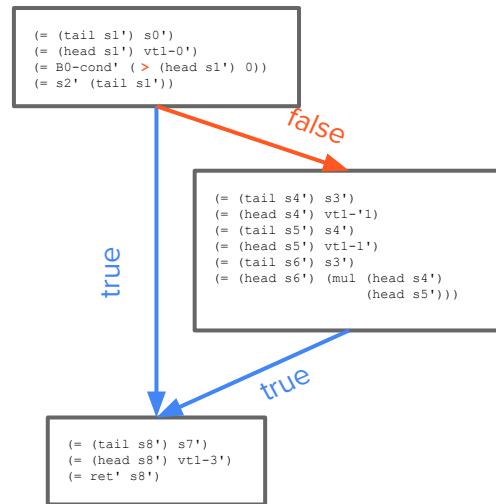
# High Level Overview

Given a program and a mutant:

1. Compile to bytecode
2. Construct CFG
3. **Generate constraints**



ORIGINAL

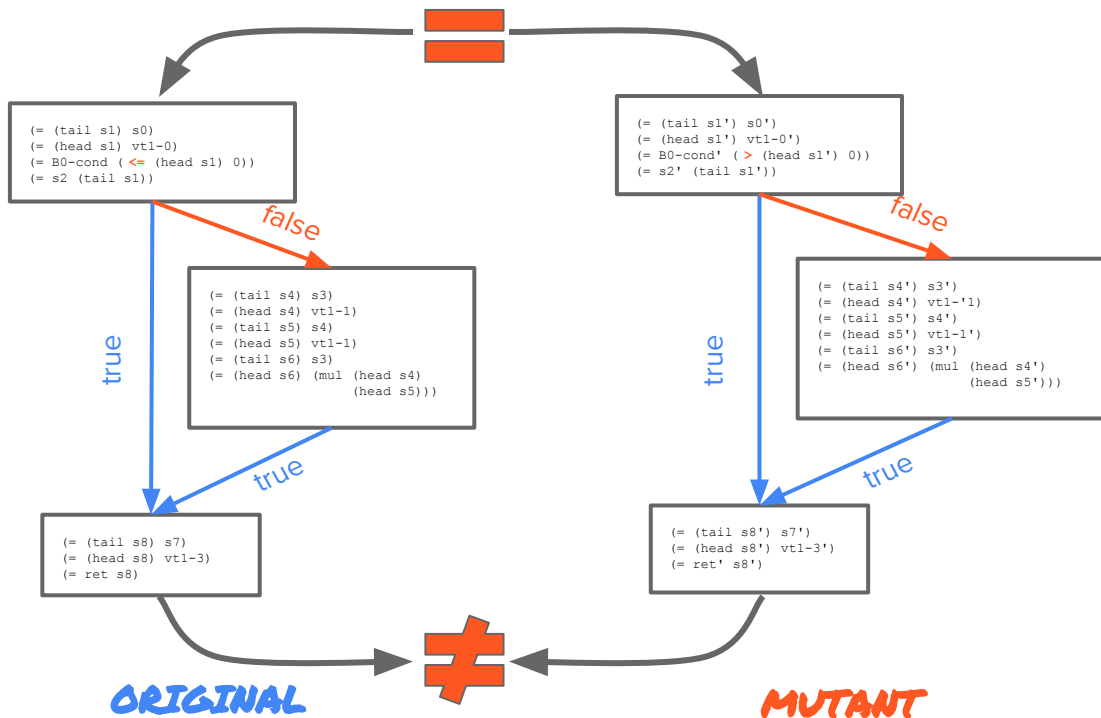


MUTANT

# High Level Overview

Given a program and a mutant:

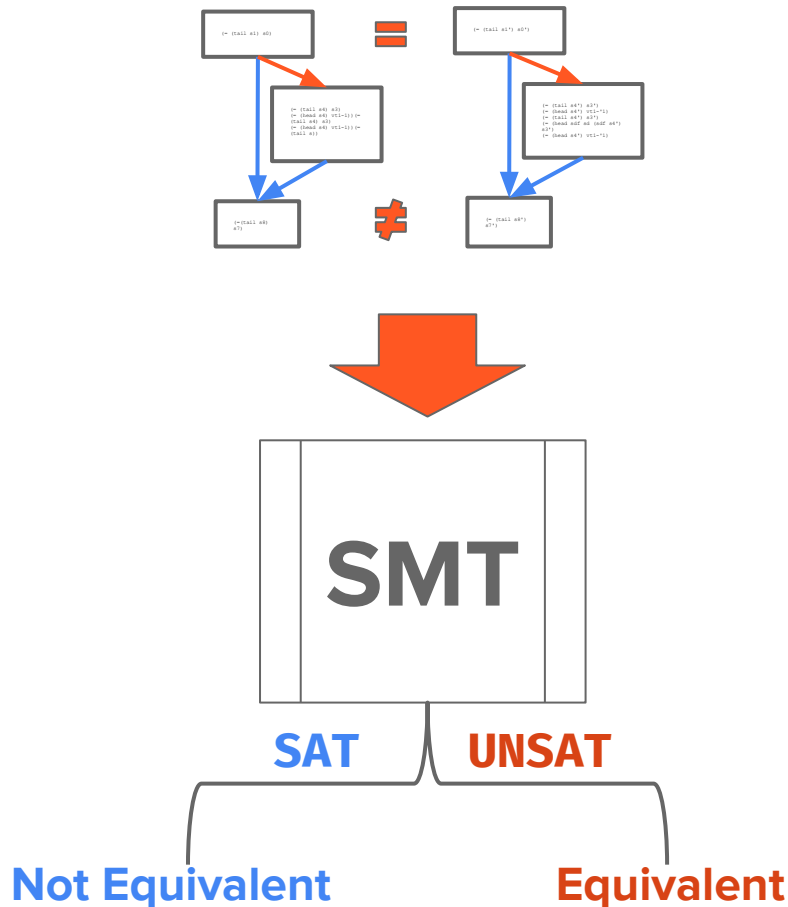
1. Compile to bytecode
2. Construct CFG
3. Generate constraints
4. **Equivalence Query**
  - Assert inputs are equal
  - Assert outputs aren't equal



# High Level Overview

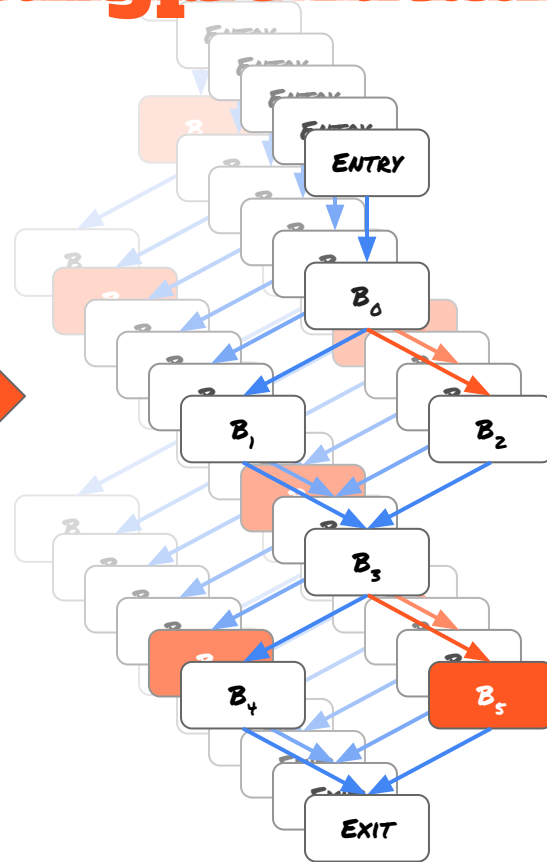
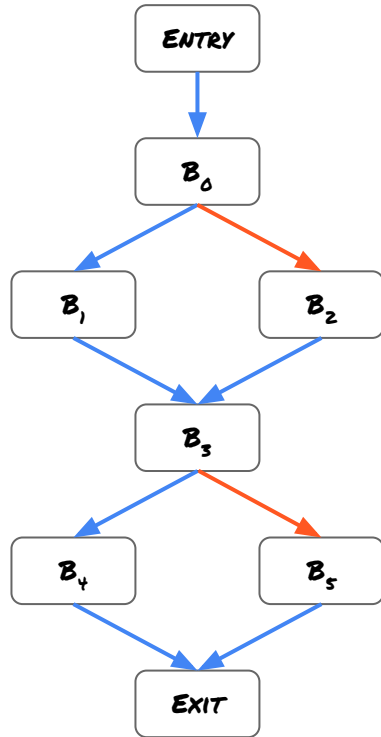
Given a program and a mutant:

1. Compile to bytecode
2. Construct CFG
3. Generate constraints
4. Equivalence Query
  - Assert inputs are equal
  - Assert outputs aren't equal
5. **Ask SMT if equivalence query is satisfiable**

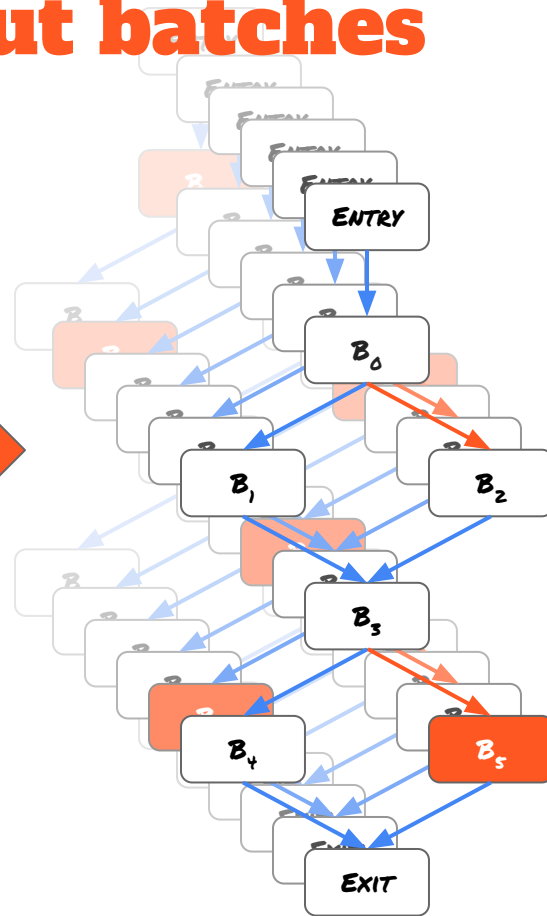
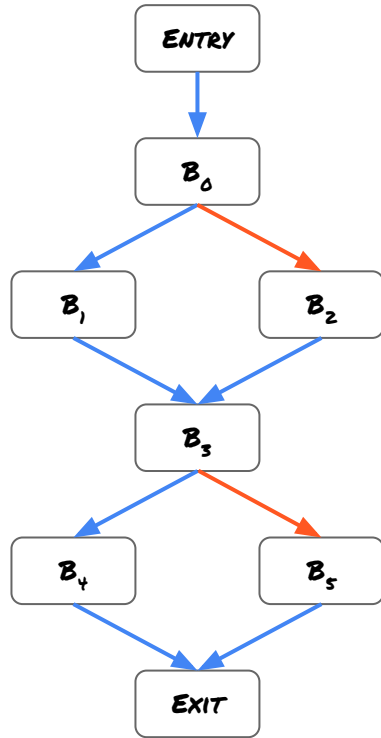


# Scaling Up

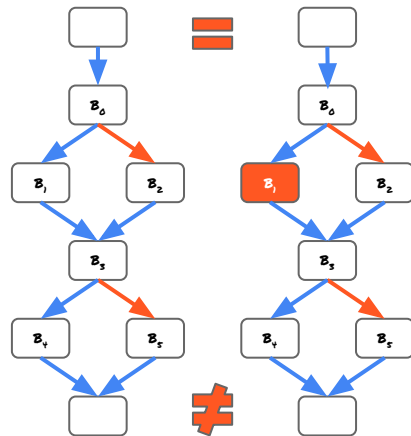
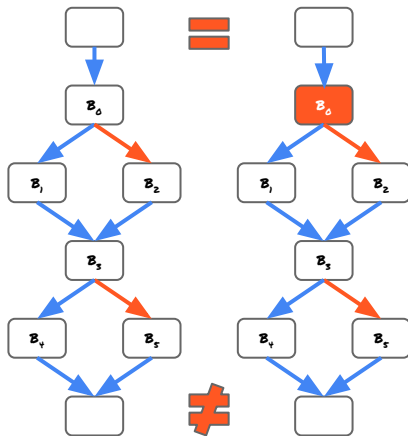
# Can We Handle Multiple Mutants?



# Strategies: Reason about batches

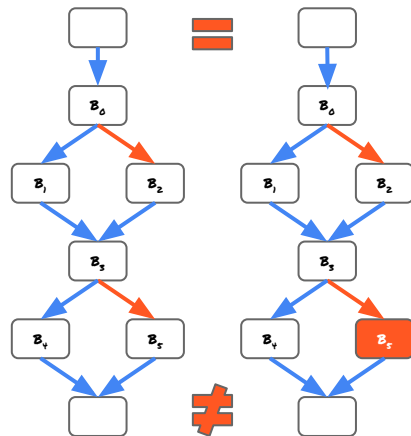
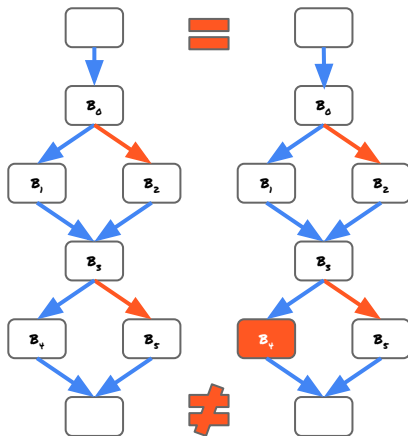


# Naive Strategy



## Naive Strategy

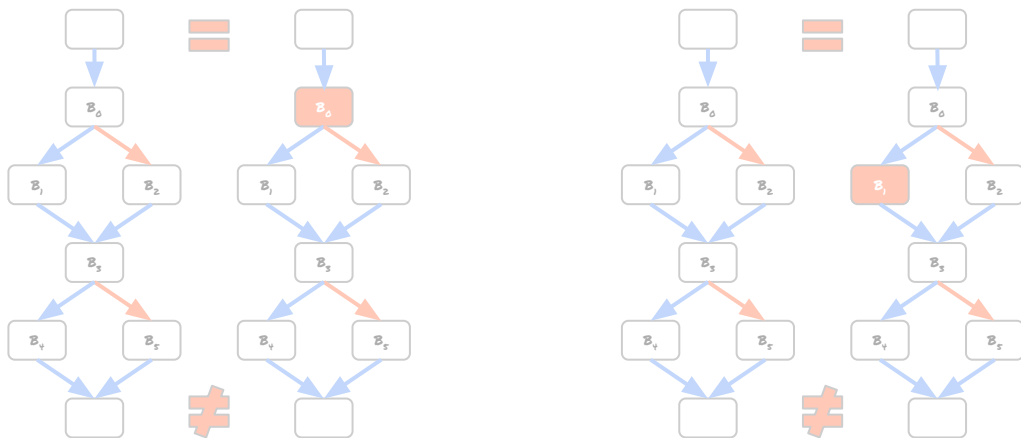
for each mutant  $m$  of program  $P$ :  
constrain  $P$   
constrain  $m$   
assert inputs are equal  
assert outputs are not equal  
check sat





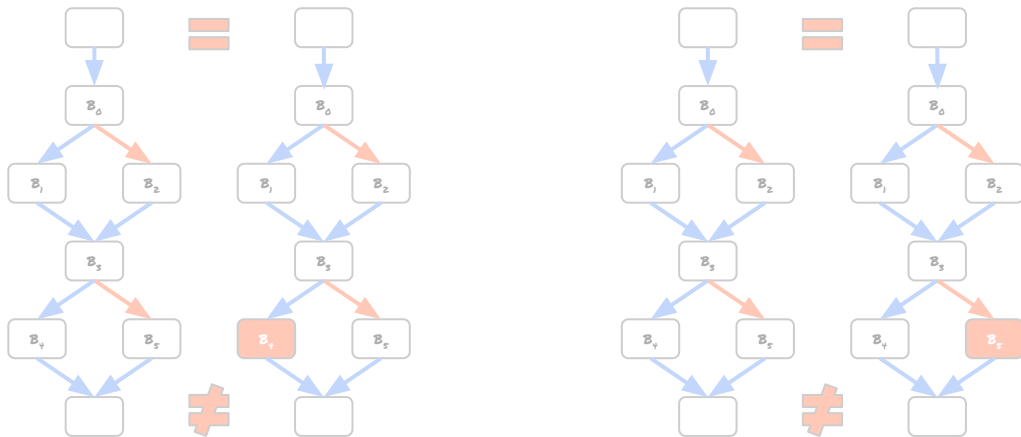
# Naive Strategy

**Observation:** naive recomputes  $P$ 's constraints every time



## Naive Strategy **SCOPES TO THE RESCUE!**

for each mutant  $m$  of program  $P$ :  
**constrain  $P$**   
constrain  $m$   
assert inputs are equal  
assert outputs are not equal  
check sat



# SMT Scopes

Scopes allow SMT to cache computations

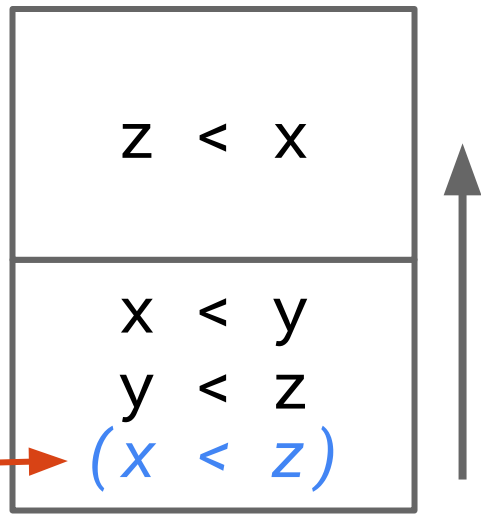
- Pushed and popped to a stack

```
(assert (< x y))  
(assert (< y z))  
(check-sat)      ; SAT
```

```
(push)           ; New scope  
(assert (< z x))  
(check-sat)      ; UNSAT
```

```
(pop)  
(check-sat)      ; SAT
```

Doesn't need  
to be  
intermediate  
computation



scope stack

# Naive Strategy

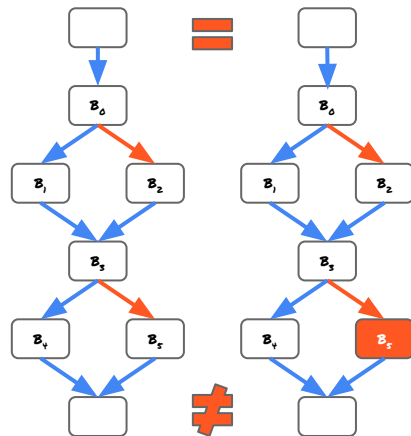
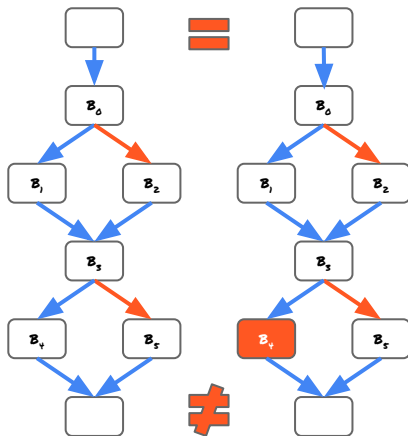
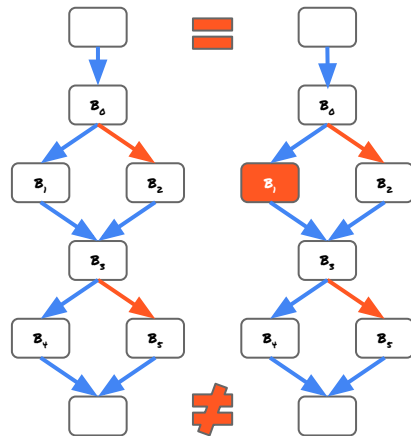
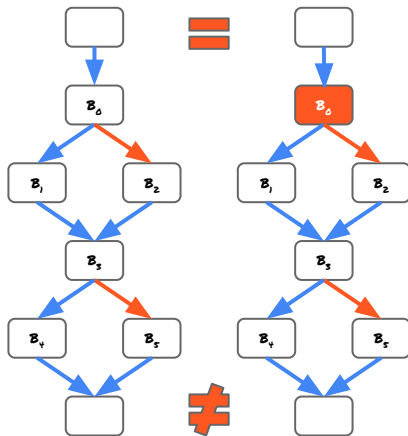
**Observation:** naive recomputes  $P$ 's constraints every iteration

**Solution:** use scopes!

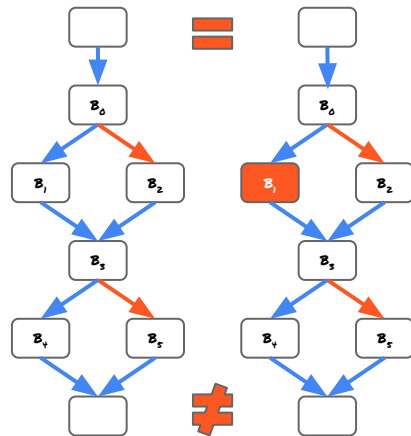
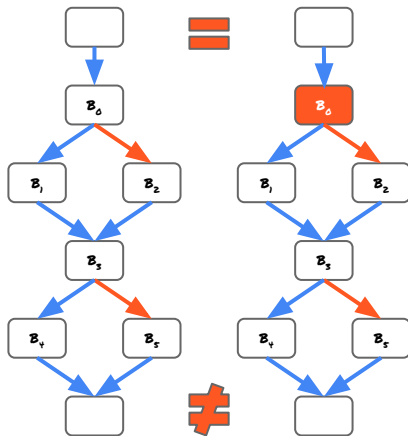
## Naive Strategy

for each mutant  $m$  of program  $P$ :

- constrain  $P$**
- constrain  $m$
- assert inputs are equal
- assert outputs are not equal
- check sat



# Cache Strategy



## Cache Strategy

### constrain P

for each mutant m of program P:

#### push scope

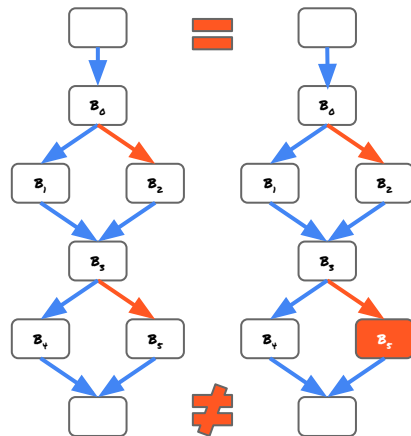
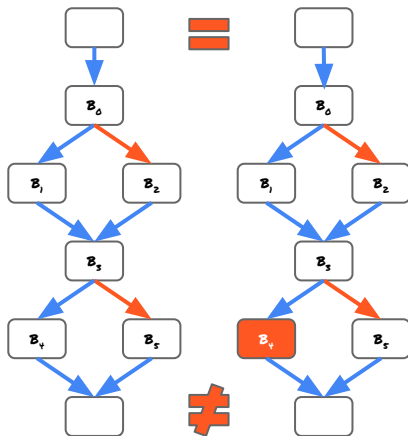
constrain m

assert inputs are equal

assert outputs are not equal

check sat

#### pop scope



# Cache Strategy

**Observation:** mutant's basic blocks are reconstrained each iteration

## Cache Strategy

### constrain P

for each mutant m of program P:

**push scope**

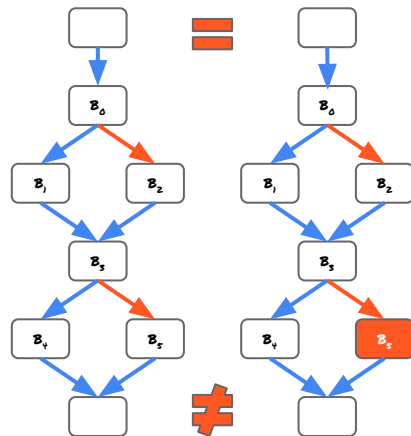
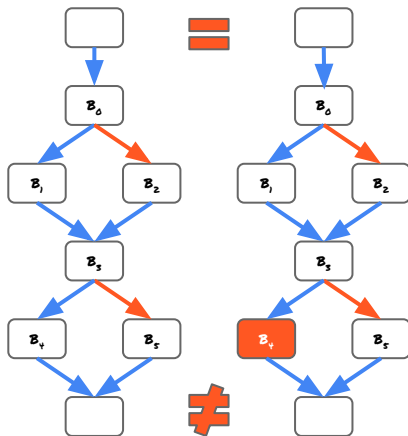
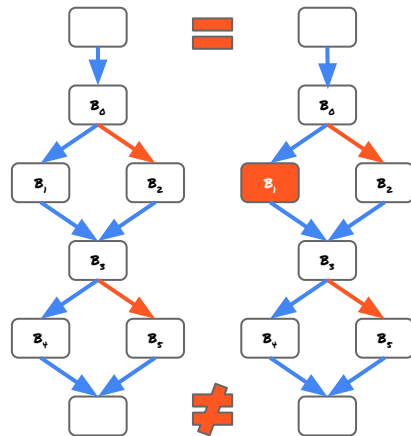
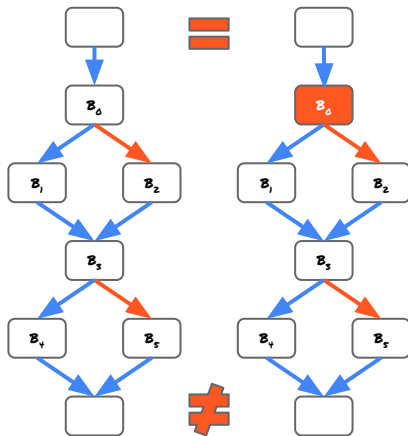
constrain m

assert inputs are equal

assert outputs are not equal

check sat

**pop scope**



# Cache Strategy

**Observation:** mutant's basic blocks are reconstrained each iteration

## Cache Strategy

### constrain P

for each mutant m of program P:

**push scope**

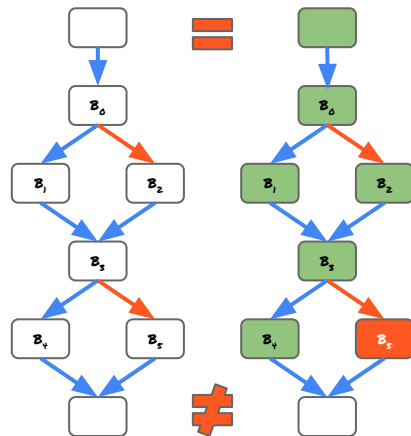
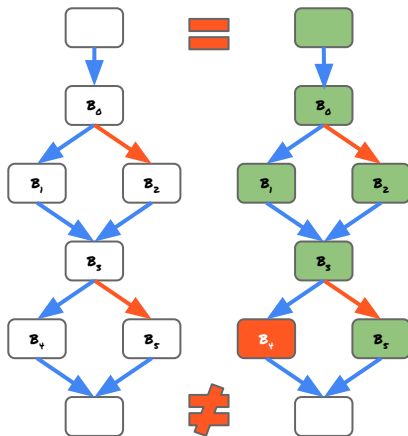
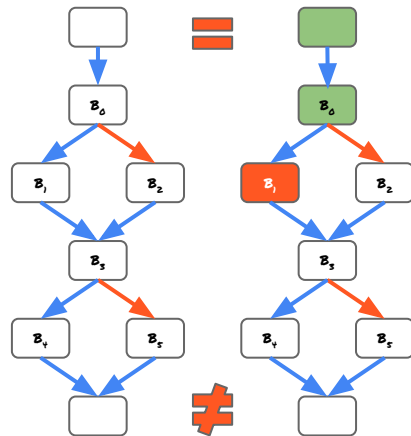
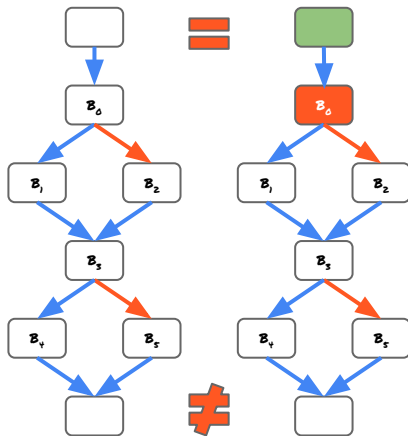
constrain m

assert inputs are equal

assert outputs are not equal

check sat

**pop scope**



# Atomic CFG Mutants

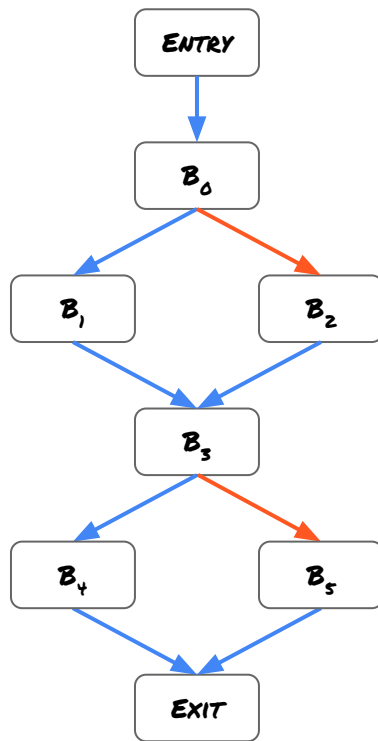
## A Special Case

**Atomic CFG Mutants** are mutants that

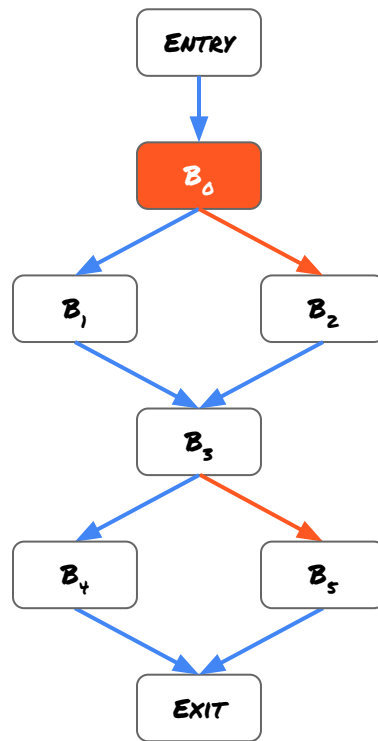
1. have the same CFG structure as the original program
2. mutate at most one basic block

Medusa uses this structural information to perform **constraint forking**

*252 of 488 (52%) of studied mutants were atomic CFG mutants*



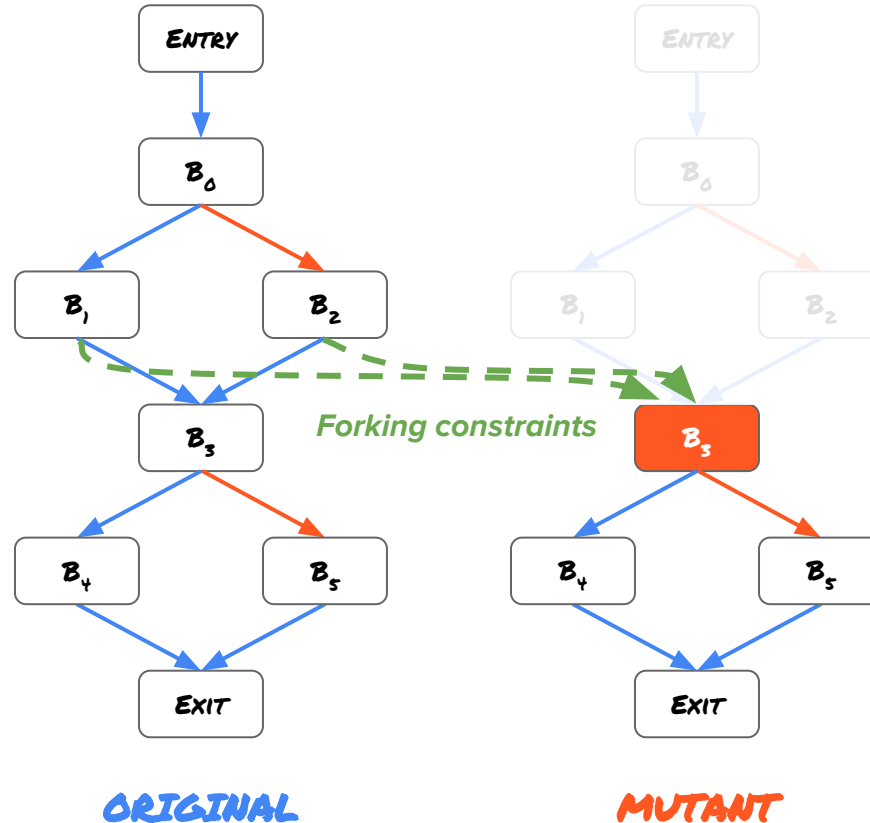
**ORIGINAL**



**MUTANT**

# Fork Strategy

- Execution starts the same
- Use original program's constraints until mutated block is reached
- This
  - Reuses P's constraints for the beginning of execution
  - Makes formulas smaller

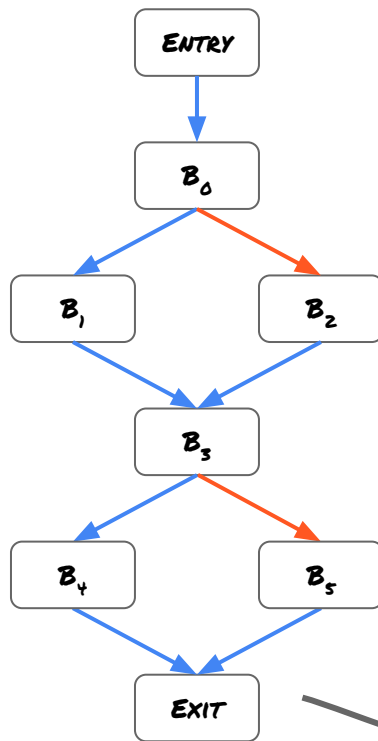




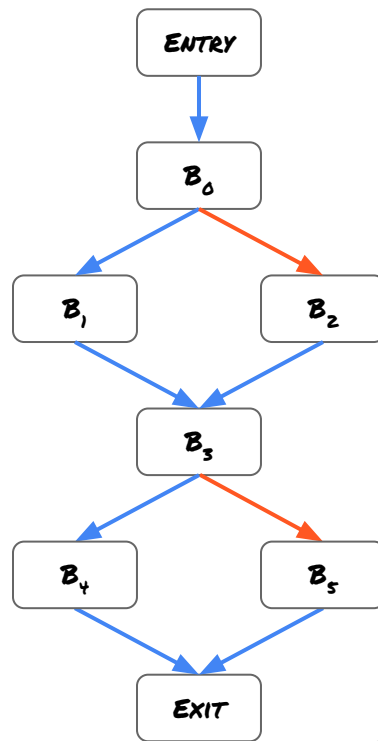
# Fork Strategy

## Setup

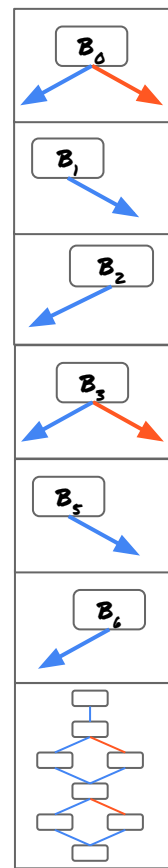
1. Order basic blocks  $B_0, B_1, \dots, B_n$ :  
each block comes before its children
2. Make two copies of the original constraints
3. Push a scope, and assert all of the first copy of the constraints
4. Bottom up, assert each block constraint and its outgoing edges into new scopes



ORIGINAL



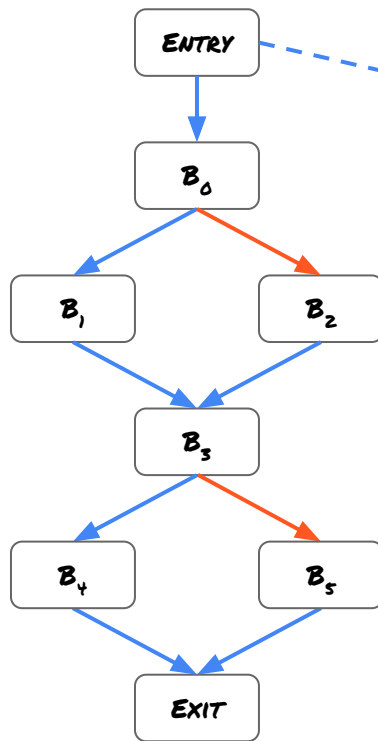
MUTANT



scope stack

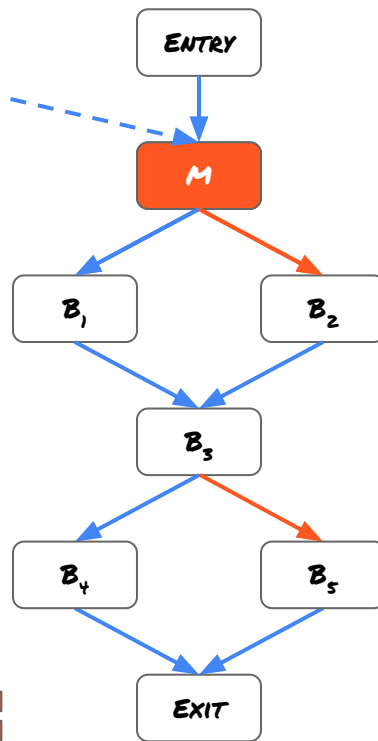
# Fork Strategy

- Pop top scope,, clearing  $B_0$ 's constraints.
- For each mutated version  $M$  of  $B_0$ :
  - Push a new scope; assert  $M$ 's block/edge constraints
  - Fork constraints
  - Check satisfiability
  - Pop the scope

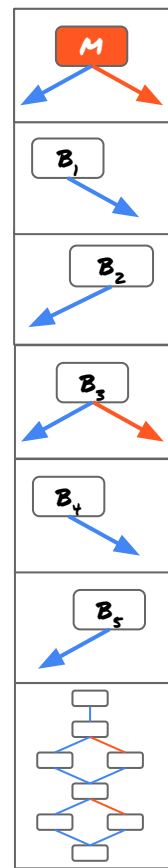


ORIGINAL

≠



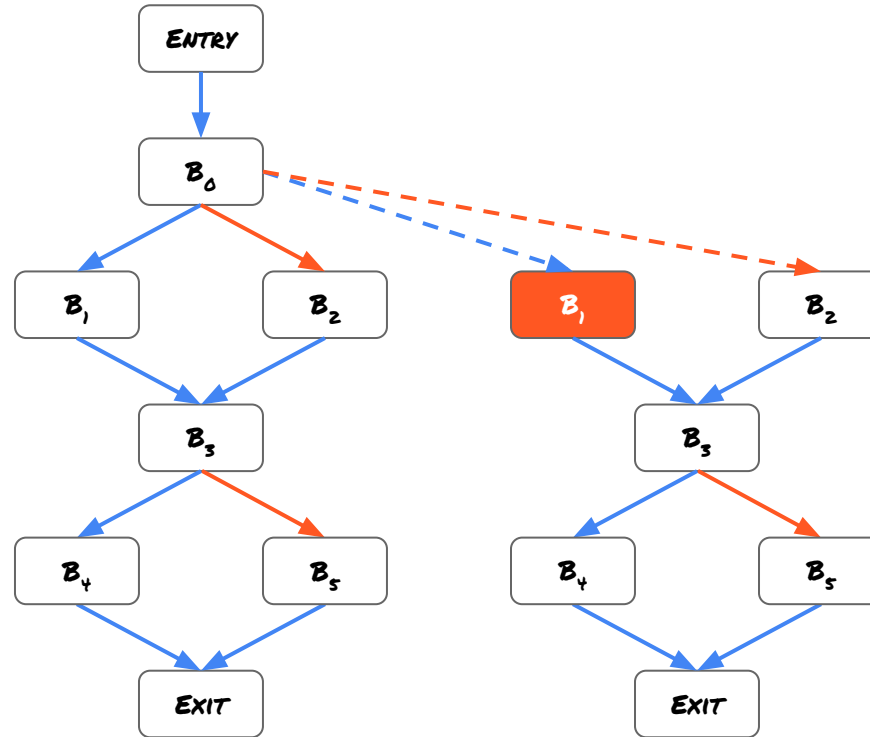
MUTANT



scope stack

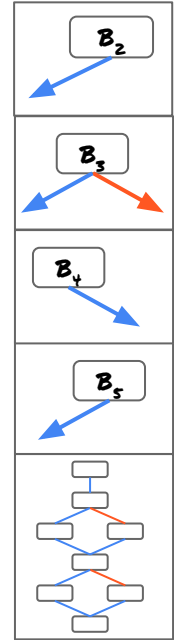
# Fork Strategy

Continue until all the blocks have been processed...



ORIGINAL

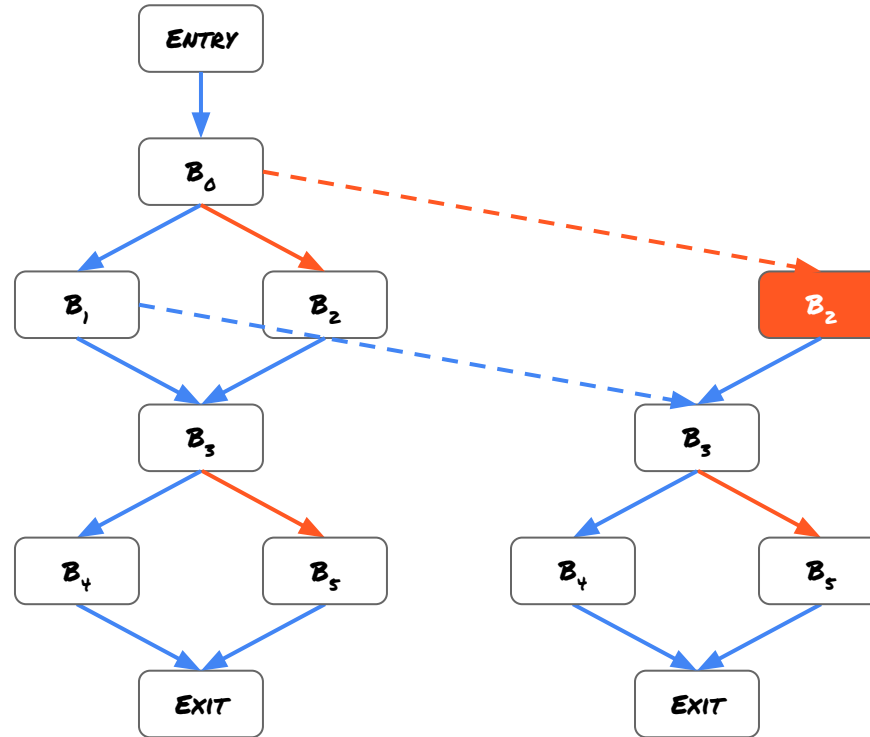
MUTANT



scope stack

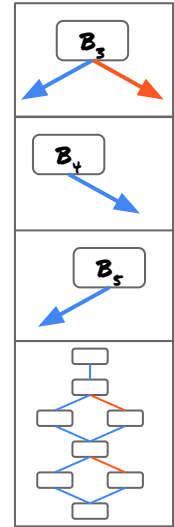
# Fork Strategy

Continue until all the blocks have been processed...



ORIGINAL

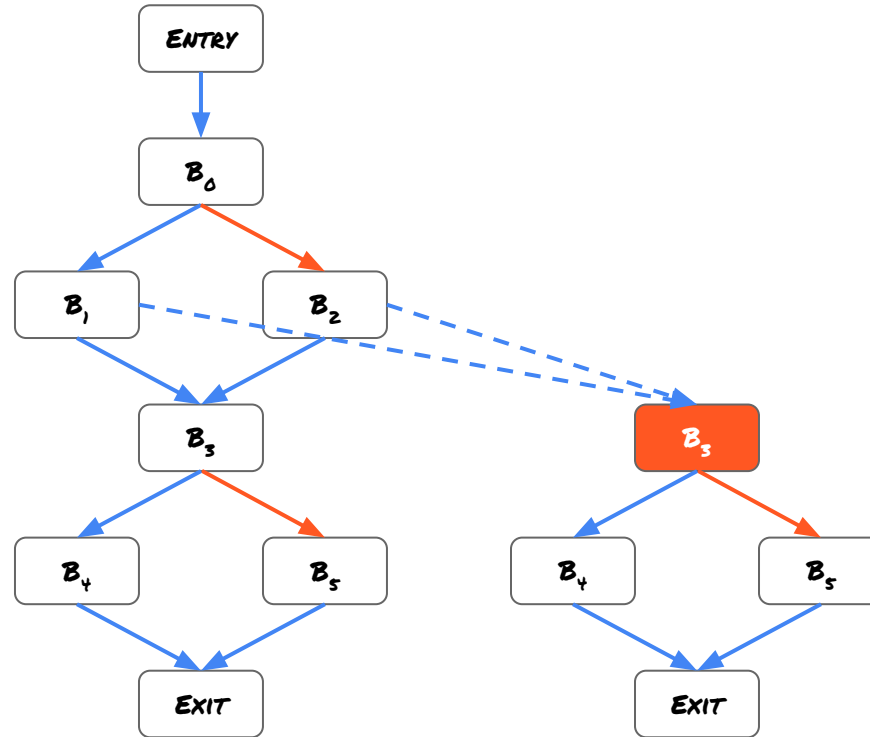
MUTANT



scope stack

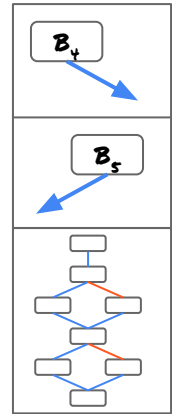
# Fork Strategy

Continue until all the blocks have been processed...



ORIGINAL

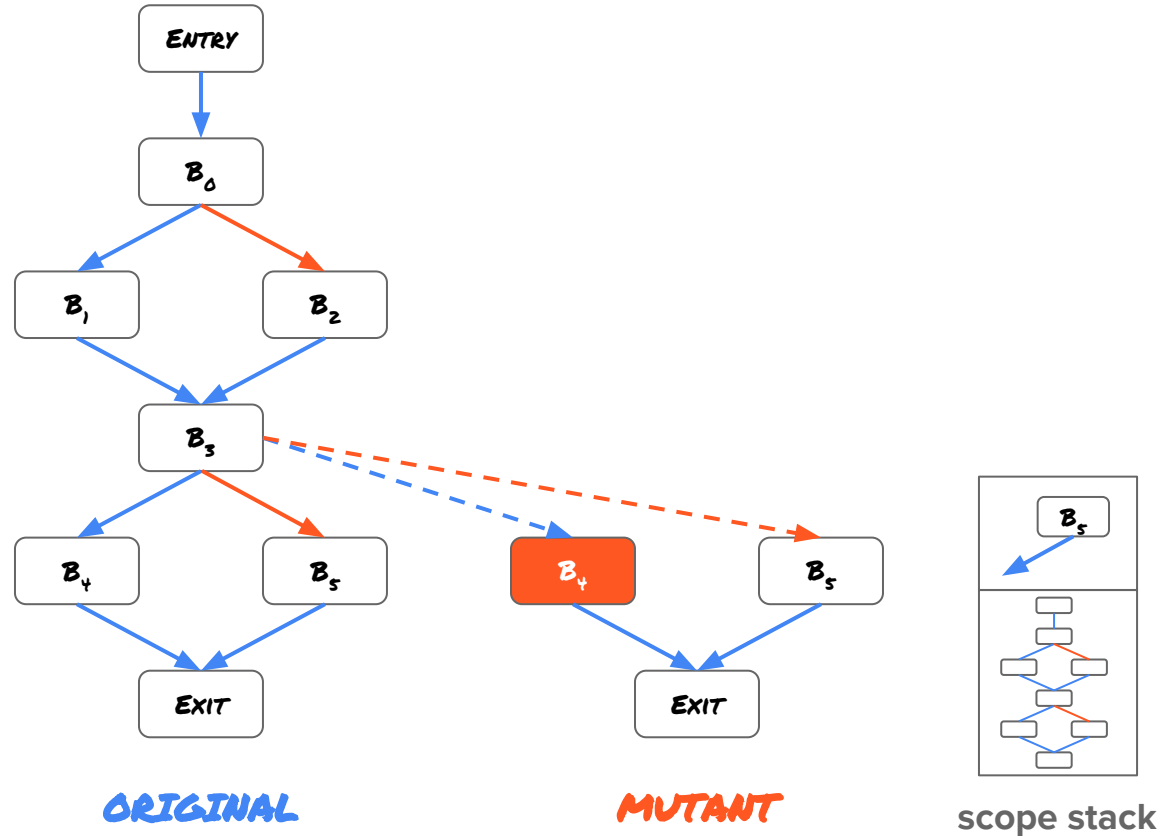
MUTANT



scope stack

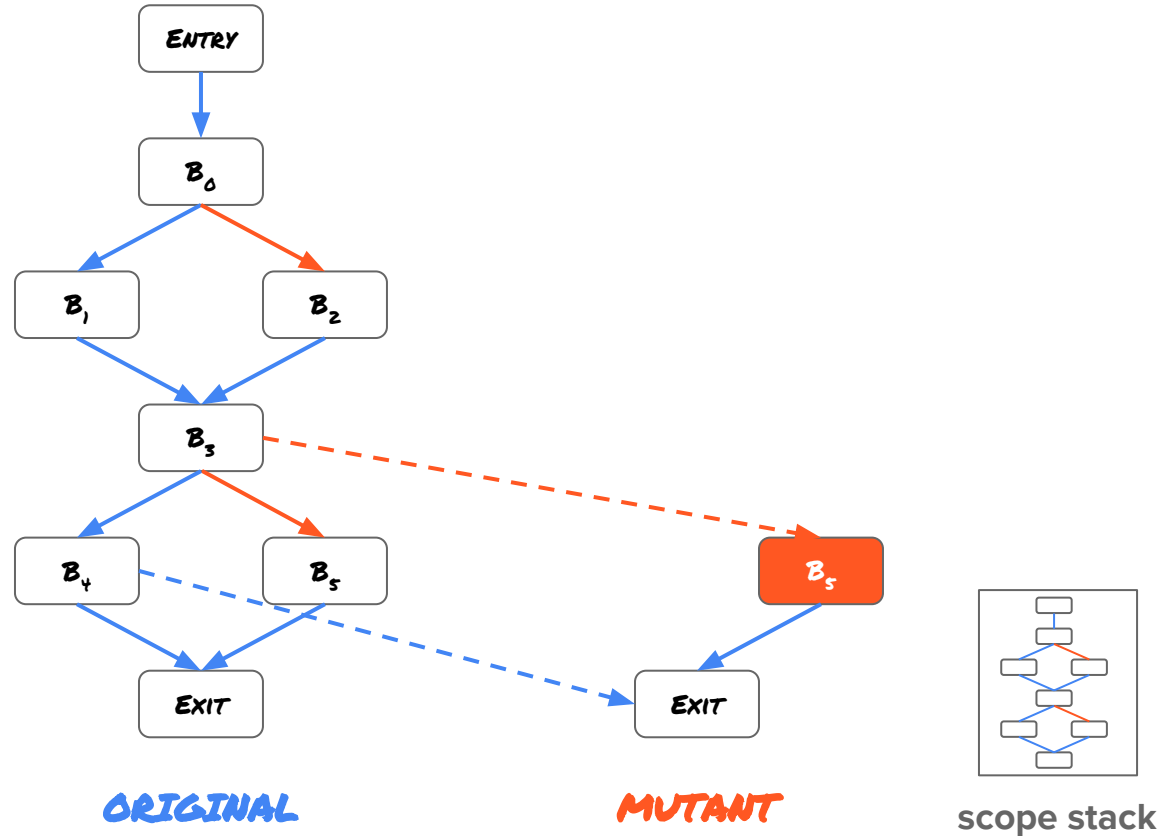
# Fork Strategy

Continue until all the blocks have been processed...



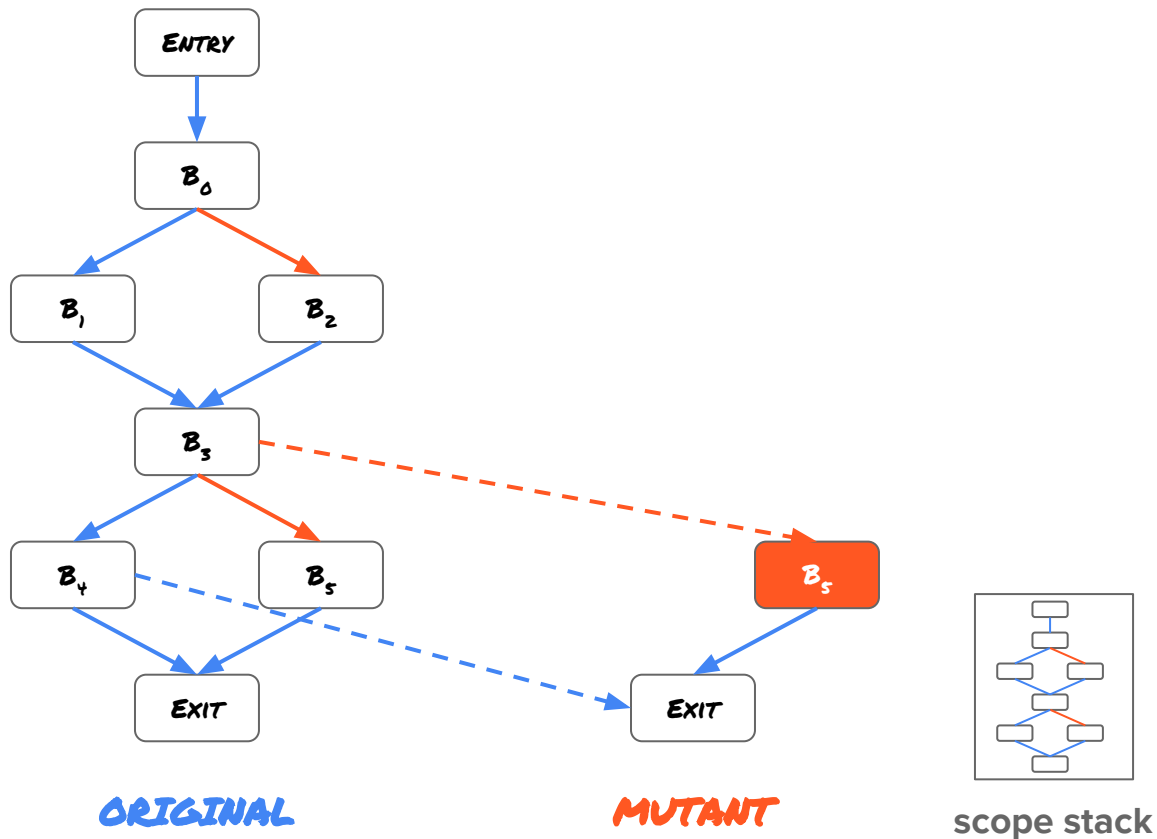
# Fork Strategy

Continue until all the blocks have been processed...



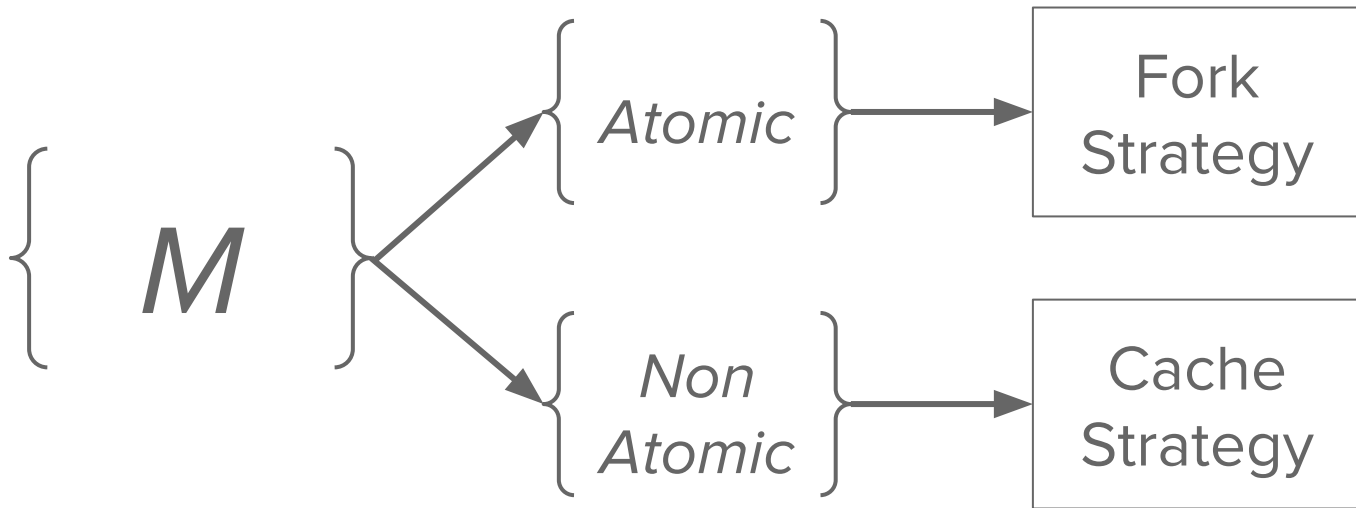
# Fork Strategy

- Fork Strategy is only applicable to atomic CFG mutants
- The strategy can be generalized (future work)
- Medusa currently uses a **hybrid strategy**





# Hybrid Strategy



# **Preliminary Results**

# Three Test Subjects

1. **Tax** — computes single-payer tax amount for a given income.
2. **TicTacToe** — checks the win condition for the game of Tic Tac Toe, including bounds checking on inputs.
3. **Triangle** — classifies a triangle as equilateral, isosceles, scalene, or invalid

# Mutants Generated by MAJOR

MAJOR generated **488** mutants

- Ground truth calculated by hand
- 37 equivalent (7.6%)
- Medusa correctly classified all mutants
- 252 atomic (52%)
  - Atomic mutants are common

| Subjects  |       | All | Atomic |
|-----------|-------|-----|--------|
| Tax       | total | 99  | 84     |
|           | equiv | 10  | 10     |
| TicTacToe | total | 267 | 98     |
|           | equiv | 23  | 19     |
| Triangle  | total | 122 | 70     |
|           | equiv | 4   | 3      |
| All       | total | 488 | 252    |
|           | equiv | 37  | 32     |

# Mutants Generated by MAJOR

MAJOR generated **488** mutants

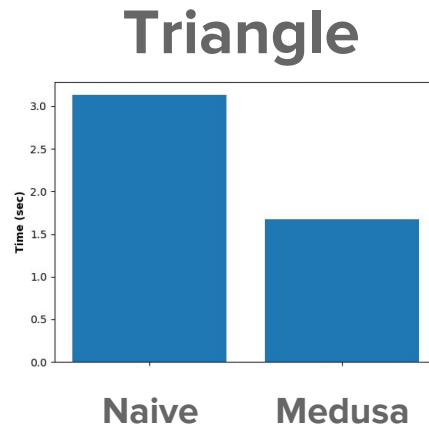
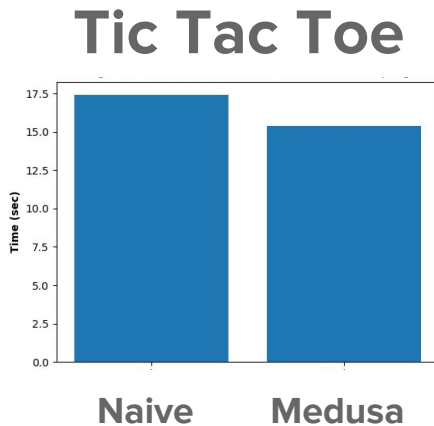
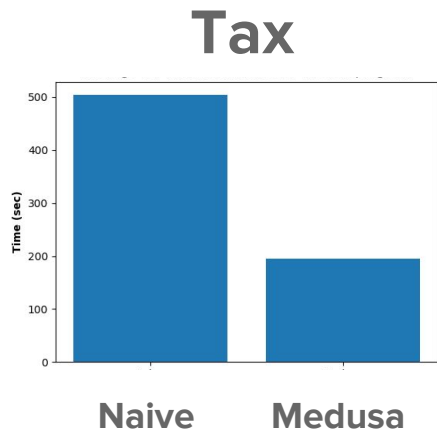
- Ground truth calculated by hand
- 37 equivalent (7.6%)
- Medusa correctly classified all mutants
- 252 atomic (52%)
  - Atomic mutants are common

**TicTacToe** has lower ratio of atomic mutants (large number of short circuiting boolean operators from bounds checking)

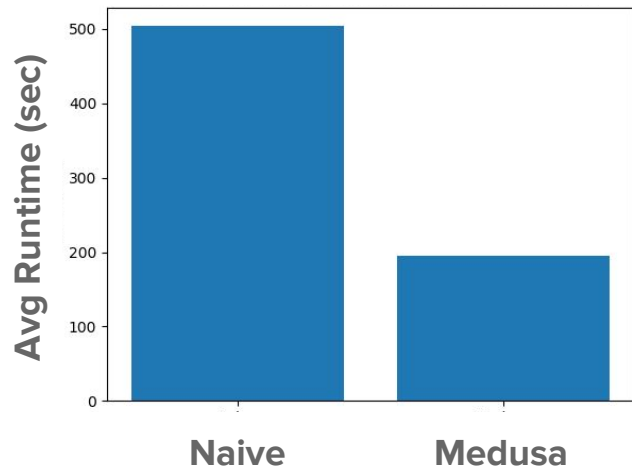
| Subjects  |       | All | Atomic |
|-----------|-------|-----|--------|
| Tax       | total | 99  | 84     |
|           | equiv | 10  | 10     |
| TicTacToe | total | 267 | 98     |
|           | equiv | 23  | 19     |
| Triangle  | total | 122 | 70     |
|           | equiv | 4   | 3      |
| All       | total | 488 | 252    |
|           | equiv | 37  | 32     |

# Summary of Medusa's performance

- **Medusa** was run 5 times on each subject and mutant (run times are averaged)
- **Tax** was long-running due to floating point computations.



# Runtime results: Tax



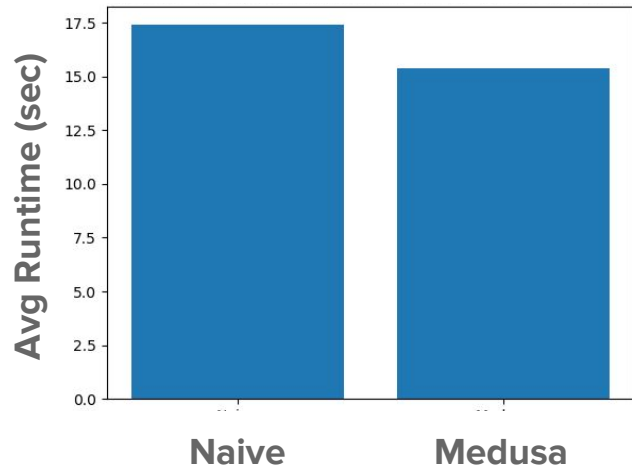
|            | #  | Naive | Cache (Improve) | Fork (Improve) |
|------------|----|-------|-----------------|----------------|
| Atomic     | 84 | 521   | 258 (50.5%)     | 197 (62.2%)    |
| Non Atomic | 15 | 410   | 185 (54.9%)     | N/A            |
| All        | 99 | 504   | 247 (51.0%)     | 195 (61.3%)    |

**HYBRID**

## Results

- **Atomic** mutants tend to be **longer running** than **non atomic** mutants
- **Cache** has a **51% improvement** over naive
- **Hybrid** has a **61% improvement** over naive

# Runtime results: Tic Tac Toe



|            | #   | Naive | Cache (Improve) | Fork (Improve) |
|------------|-----|-------|-----------------|----------------|
| Atomic     | 98  | 17.9  | 16.5 (7.8%)     | 13.9 (22.5%)   |
| Non Atomic | 169 | 17.0  | 16.3 (4.1%)     | N/A            |
| All        | 267 | 17.4  | 16.4 (6.3%)     | 15.4 (11.5%)   |

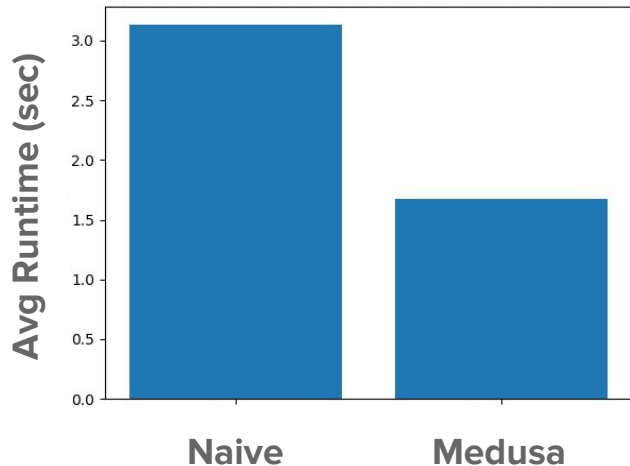
**HYBRID**

## Results

- **Atomic** mutants tend to be **longer running** than **non atomic** mutants
- **Cache** has a **6% improvement** over naive
- **Hybrid** has a **12% improvement** over naive



# Runtime results: Triangle



|            | #   | Naive | Cache (Improve) | Fork (Improve) |
|------------|-----|-------|-----------------|----------------|
| Atomic     | 70  | 3.72  | 3.26 (12.4%)    | 1.3 (65.1%)    |
| Non Atomic | 52  | 2.34  | 2.16 (7.7%)     | N/A            |
| All        | 122 | 3.13  | 2.79 (10.9%)    | 1.67 (46.6%)   |

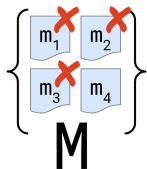
**HYBRID**

## Results

- **Atomic** mutants tend to be **longer running** than **non atomic** mutants
- **Cache** has a **11% improvement** over naive
- **Hybrid** has a **47% improvement** over naive

# Medusa: Mutant Equivalence Detection Using SAT Analysis

## Let's analyze T's effectiveness



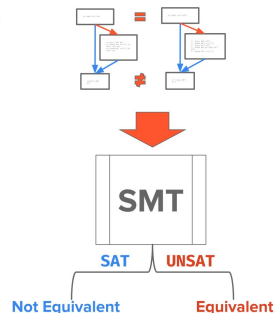
### Mutation Analysis

- Generate mutant set  $M$
- Run  $T$  on mutants in  $M$
- Each mutant that fails a test in  $T$  is *killed*

## High Level Overview

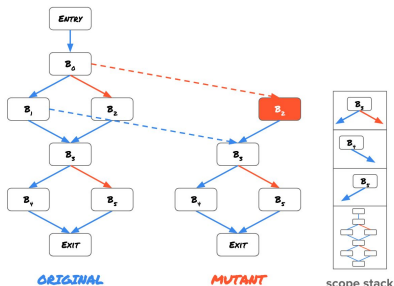
Given a program and a mutant:

1. Compile to bytecode
2. Construct CFG
3. Generate constraints
4. Equivalence Query
  - Assert inputs are equal
  - Assert outputs aren't equal
5. Ask SMT if equivalence query is satisfiable



## Fork Strategy

Continue until all the blocks have been processed...



## Summary of Medusa's performance

- Medusa was run 5 times on each subject and mutant (run times are averaged)
- Tax was long-running due to floating point computations.

