

Verifying imperative programs with Dafny

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Abstract

The goal of this work is to show how to formally reason about imperative programming constructs such as assignments, loops, arrays and especially about dynamically allocated objects. To be able to achieve that, basic Dafny constructs will be shown, such as functions, methods, pre- and postconditions. The discussion thereafter will move towards recursion and termination as well as inductive datatypes. Having learned from those chapters, loop invariants and their usage will be explored. Because analyzing objects in the heap is more challenging, searching and modifying arrays will be covered. The final chapter will include a detailed discussion about Dafny's dynamic frames and their significance.

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1 Introduction

1.1 Why Dafny?

When software engineers encounter the field of formal methods and verification, this usually happens in an academic setting, where proof techniques are learned and done by hand. Moreover, actually taking advantage of those methods in practice involves a steep learning curve and lots of time. This unfortunately leads to less acceptance of verification techniques in an industry where fast time to market is essential Reid et al. (2020). Dafny promises to solve those problems. As a programming language designed to support specifications and proofs, it comes with an automated verifier that integrates seamlessly into most modern IDEs ¹ making rigorous verification part of the software development process, thus reducing costly late-stage bugs that may be missed by testing. The language was designed by Rustan Leino in 2009 and its current version at the time of this writing is 4.9.0 from 31. October 2024. Dafny is heavily used at Amazon Web Services to develop critical components of their access management, storage, and cryptography infrastructures Chakarov et al. (2022).

1.2 Dafny's build system

The main idea in such verification-aware programming languages is that code is divided into two parts - the specification part and the implementation part Leino (2023). The built-in verifier in Dafny acts as an extended type checker and constantly proves that the provided implementation actually meets the behavior stated in the specification part of the given function, method or class. This is done by transforming the code into an intermediary that a tool called Boogie can understand. The correctness of the Boogie program implies the correctness of the Dafny program. Boogie then generates first-order verification rules that are passed to the Z3 SMT solver. Any violations of these conditions are passed back as verification errors Herbert et al. (2012). This process is visualized in Figure 1.

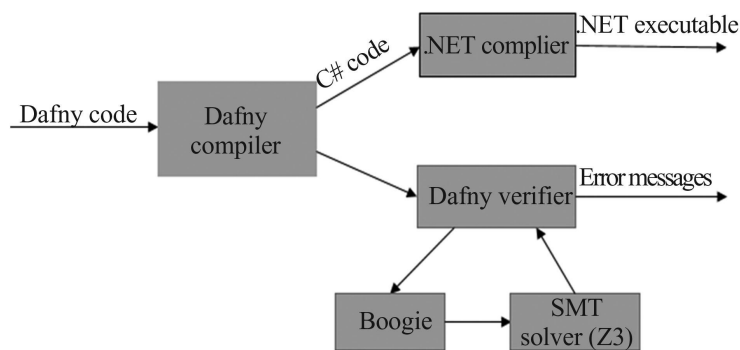


Figure 1: The Dafny build system as shown in Herbert et al. (2012)

¹Integrated Development Environments (IDEs)

2 Building Blocks of Dafny

This work assumes the reader is familiarized with the fundamentals of program-semantics such as Floyd logic and Hoare triples, as they build the reasoning framework of Dafny and help to understand it more effectively.

2.1 Methods

As in other programming languages, a *method* is a block of code that prescribes some behavior. For instance, Listing 1 shows the declaration of a method called `Triple`.

```

1 method Triple(x: int) returns (r: int)
2   ensures Average(x) == 3 * x {
3     var y := 3 * x;
4     r := x + y;
5   }
```

Listing 1: A simple method in Dafny.

This method takes an *in-parameter* `x` of type integer and returns an *out-parameter* `r`, also of type integer. The **ensures** keyword is used to specify the method’s *postcondition*. A postcondition expresses a property that must hold after every invocation of the method through all possible return points. Postconditions are part of the method’s specification and appear before the body block Herbert et al. (2012). The body of a method is a list of statements that give the method’s implementation. In Dafny, methods can have any number of in- and out-parameters. Inside the method, the out-parameters are like local variables and can be assigned and read. When the method ends, whatever values were assigned to the out-parameters will be returned to the caller. The in-parameters however cannot be re-assigned Leino (2023).

2.2 Functions

Functions in Dafny are very mathematical in nature, because they are deterministic. Any two invocations of a function with the same arguments will result in the same value. Listing 2 shows how a simple function named `Average` looks like.

```

1 function Average(a: int, b: int): int
2   requires 0 <= a && 0 <= b {
3     (a + b) / 2
4   }
```

Listing 2: A simple function in Dafny.

Whereas a method is declared to have some number of out-parameters, a function instead declares a result type, and while a method body is a statement, the body of a function is an expression Leino (2023). Another important difference between functions and methods is, that functions are *transparent*. This means that their implementation is directly visible to the caller. That’s the reason why functions can also be used to describe pre- and postconditions like the precondition on line 2 in Listing 2. Methods on the other hand are *opaque*, so callers fully rely on the postcondition to reason about their return values.

2.3 Ghost constructs

Sometimes when reasoning about a program more information is needed than what the compiler has at runtime. A declaration, variable, statement, etc., that is used only for specification purposes is called a *ghost*. The verifier takes all ghosts into account, whereas the compiler erases all ghosts when it produces executable code. Other ghost constructs include pre- and postconditions (declared by **requires** and **ensures** clauses). They are used solely to specify the behavior of the program and establish a contract between callers and implementations Leino (2023).

```

1 method InvalidAssignment() returns (y: int) {
2   ghost var x := 10;
3   y := 2 * x; // cannot assign to compiled variable using a ghost
4 }

```

Listing 3: Invalid assignment to ghost.

To make sure ghost constructs can be successfully deleted after compilation, Dafny makes sure the compiled code doesn't rely on any ghost constructs. Listing 3 demonstrates a violation of this rule, which results in a verification error.

3 Recursion and termination

A recursive function is one that calls itself directly or indirectly. To be able to terminate, all recursive functions need a base case - a condition where the function stops calling itself and returns a result. An example is shown with a function called `SeqSum` in Listing 4, which computes the sum of all integers in a sequence. On line 4, if the base case isn't met, the function invokes itself with an incremented value for `lo`.

```

1 function SeqSum(s: seq<int>, lo: int, hi: int): int
2   requires 0 <= lo <= hi <= |s|
3   decreases hi - lo {
4     if lo == hi then 0 else s[lo] + SeqSum(s, lo + 1, hi)
5 }

```

Listing 4: A function computing the sum of all integers in a sequence recursively.

3.1 The decreases clause

Dafny can prove termination by using the **decreases** clause. If we can label each function invocation with a natural number and make sure that successive invocations strictly decrease that label, then it follows that at run time the recursive calls can only execute a finite number of times, and that is all the information needed to prove that the recursion eventually terminates Herbert et al. (2012). As shown on line 3 in Listing 4, the label of each invocation of `SeqSum` decreases gradually, because with each invocation the difference between `hi` and `lo` is smaller. When it reaches zero, the recursion terminates.

3.2 Well-founded relations

Termination metrics are not restricted to natural numbers. Termination can also be proven when two labels, which represent successive recursive invocations, are

in a relation that is *well-founded* Leino (2023). This relation, denoted as $a \succ b$, signifies that a reduces to b . For it to be *well-founded*, the following three conditions must be true:

- \succ is *reflexive*: The relation never relates an element to itself. $a \succ a$ never holds.
- \succ is *transitive*: Whenever $a \succ b$ and $b \succ c$ hold, then so does $a \succ c$.
- \succ satisfies the *descending chain condition*: the relation has no *infinite descending chain*.

From these conditions it follows that a well-founded relation is a strict partial order that additionally satisfies the descending chain condition. Dafny pre-defines \succ for each type, and in some cases also for values between different types (but in most cases, values from different types are not related in this partial order) Leino (2023).

4 Inductive datatypes

If you want to refer to you own chapters, figures, tables or the like, you can make use of the `ref{}` command, for example:

- section ?? on page ??

4.1 Subsection within Foundations

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5 Methodology

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6 Results

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7 Conclusion

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