

Orthogonalitate

$u, v \in \mathbb{R}^{n \times n}$ dc. $u^T v = 0$

$$\sum_{i=1}^n u_{ij} v_{ji} = 0$$

$Q = [u \ v \ z] \in \mathbb{R}^3$ (ortogonal între ei)

↳ matrice ortogonală

$$R = Q^T Q = \begin{bmatrix} \dots & \bar{q}_1^T \bar{q}_1 \\ \dots & \bar{q}_2^T \bar{q}_2 \\ \dots & \bar{q}_m^T \bar{q}_m \end{bmatrix}^{\text{ort}} = I_n \quad (Q \text{ ortonormal})$$

Ex:

$$\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\|Qx\| = \|x\|$, Q ortogonală

$$Q^T Q = I_n = Q^{-1} Q$$

Deci dc. Q ortogonală $Q^{-1} = Q^T$

$$\boxed{Qx = b} \Rightarrow x = Q^{-1}b = Q^T b$$

$$m \left\{ \begin{bmatrix} \vdots \\ Q \end{bmatrix} \right. \quad Q^T Q = I_n$$

$$\|Qx\|_2 = \sqrt{x^T Q^T Q x} = \sqrt{x^T x} = \|x\|_2$$

$$m \left\{ \begin{bmatrix} \vdots \\ Q \end{bmatrix} \right. \quad Q Q^T = I_n$$

$$\boxed{Ax = b} \quad m \left\{ \begin{bmatrix} \vdots \\ A \end{bmatrix} \begin{bmatrix} \vdots \\ x \end{bmatrix} = \begin{bmatrix} \vdots \\ b \end{bmatrix} \right. \quad m \Leftrightarrow \min_{x \in \mathbb{R}^n} \|Ax - b\|_2^2$$

$x_{\text{CMMP}}^* \sim \text{Cond ord I (Format)}$

$$\tilde{V} f(x^*) = A^T (Ax^* - b)$$

$$A^T (Ax^* - b) = 0$$

$$\underbrace{A^T A}_{n \times n} \underbrace{x_{\text{CMMP}}^*}_{m \times n} = A^T b \quad (\text{normal})$$

$$x_{\text{CMMP}}^* = (A^T A)^{-1} A^T b$$

$$\boxed{A} \begin{bmatrix} \vdots \\ x \end{bmatrix} = \boxed{b}$$

↳ O infinitate de sol. (subspatiu de solutii)

$$x_{\text{CMMP}}^* \rightarrow \text{dist min fata de origine}$$

$\min \|x\|_2$

$$Ax = b$$

$$x_{\text{CMMP}}^* = A^T z^*$$

$$A^T A z^* = b \Rightarrow z^* = (A^T A)^{-1} b$$

$$x_{\text{CMMP}}^* = \arg \min_x \|Ax - b\|_2$$

$$| \quad Ax \approx b$$

$$| \quad UAx \approx Ub$$

↓

Triunghi ortogonal

$$R \cdot x \approx b$$

Reflector elementar Householder

$$U = I_m - 2 \mu \mu^T$$

$$= I_m - \frac{1}{\beta} \mu \mu^T, \quad \beta = \frac{\|\mu\|^2}{2} \quad (\text{dc. } \mu \text{ nu e normal})$$

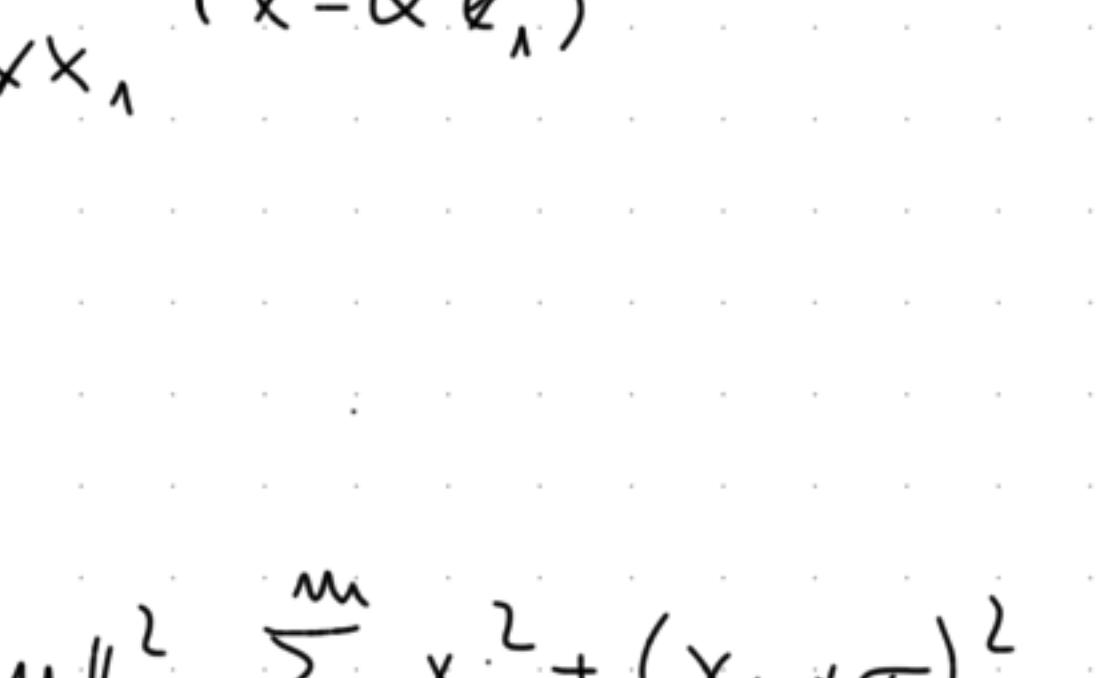
$$U^T U = I_m$$

Ex:

$$x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \mu = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Ux = x - \frac{(\mu^T x)}{\beta} \mu =$$

$$= \begin{bmatrix} 3 \\ 4 \end{bmatrix} - 2 \cdot 3 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$



$\mu \rightarrow$ dictarea axa de reflexie

$$\mu' = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow U'x = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\boxed{Ux = \left(I_m - \frac{\mu \mu^T}{\beta} \right) x = \begin{bmatrix} x \\ 0 \\ \vdots \\ 0 \end{bmatrix}}$$

$$\|x\| = \|Ux\| = \|\alpha e_1\| = |\alpha| \Rightarrow \alpha = \pm \|x\|$$

$$\alpha^2 = \|x\|^2$$

$$Ux = \alpha e_1 = x - (\mu^T x) \mu$$

$$\alpha \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ \vdots \\ 0 \end{bmatrix} - \boxed{\mu^T x} \begin{bmatrix} \mu \\ \vdots \\ 0 \end{bmatrix} \quad | \quad x^T \Rightarrow \alpha x_1 = (\alpha x_1)^2 - (\mu^T x)^2 \Rightarrow$$

$$\Rightarrow \mu^T x = \|x\|^2 - \alpha x_1 \Rightarrow$$

$$\Rightarrow \mu = \frac{1}{\sqrt{\|x\|^2 - \alpha x_1}} (x - \alpha e_1)$$

$$\text{Ese } \sigma_k = \sqrt{\sum_{i=1}^m x_i^2}$$

↳ reflector care anuleaza de la $k+1$

$$\mu = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ x_{k+1} + \sigma \\ x_m \end{bmatrix} \Rightarrow Ux = \begin{bmatrix} x_1 \\ \vdots \\ x_{k-1} \\ -\sigma \\ 0 \end{bmatrix}$$

$$\frac{\|\mu\|^2}{2} = \frac{\sum_{i=k+1}^m x_i^2 + (x_{k+1} + \sigma)^2}{2} = \sum_{i=k+1}^m x_i^2 + x_{k+1}^2 + 2\sigma x_{k+1} = \sigma x_{k+1}$$

$$\min_x \|R' d - d'\|_2^2 + \|d''\|_2^2 = \|d''\|_2^2 > 0$$

$$R' x_{\text{CMMP}}^* = d'$$

$$R' x = d'$$

$$Q^T b = d = \begin{bmatrix} d' \\ d'' \end{bmatrix}$$

$$Q^T A = \begin{bmatrix} R' \\ 0 \end{bmatrix}$$

$$R' x = d'$$