Problem 1

- (1) n=8³=512
- (2) Statistically, when n=512 Max k should be between 7 and 8

Maximum Collisions k Relative to Size N

| k | (e/k)^k | n=(k/e)^k/4 |
|----|----------|-------------|
| 6 | 8.65E-03 | 2.89E+01 |
| 7 | 1.33E-03 | 1.88E+02 |
| 8 | 1.78E-04 | 1.41E+03 |
| 9 | 2.09E-05 | 1.20E+04 |
| 10 | 2.20E-06 | 1.13E+05 |
| 11 | 2.10E-07 | 1.19E+06 |
| 12 | 1.83E-08 | 1.37E+07 |
| 13 | 1.46E-09 | 1.71E+08 |
| 14 | 1.08E-10 | 2.31E+09 |
| 15 | 7.47E-12 | 3.35E+10 |
| 16 | 4.82E-13 | 5.19E+11 |
| 17 | 2.92E-14 | 8.56E+12 |

(2) Simulation Result

| | 1st | 2nd | 3rd | Average | % |
|---|-----|-----|-----|---------|------|
| 0 | 229 | 231 | 225 | 228.33 | 45% |
| 1 | 150 | 142 | 146 | 146.00 | 29% |
| 2 | 75 | 78 | 84 | 79.00 | 15% |
| 3 | 32 | 39 | 39 | 36.67 | 7% |
| 4 | 16 | 16 | 14 | 15.33 | 3% |
| 5 | 8 | 4 | 1 | 4.33 | 1% |
| 6 | 2 | 1 | 1 | 1.33 | 0% |
| 7 | 0 | 1 | 2 | 1.00 | 0% |
| | 512 | 512 | 512 | 512 | 100% |

(3) Conclusion : Therefore, this simulation result is within the range of statistical probability.

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02

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#of conflict=4: 16, 3.125 %
#of conflict=5: 8, 1.5625 %
#of conflict=6: 2, 0.390625 %

#of conflict=7:0,0%
#of conflict=8:0,0%
#of conflict=9:0,0%
#of conflict=10:0,0%
#of conflict=11:0,0%
#of conflict=12:0,0%
#of conflict=13:0,0%

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RUN SUCCESSFUL (total time: 797ms)

| 244 | 735 | 212 | 364 | 525 | 34 | 727 | 246 | 225 | 373 | 375 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 314 | 271 | 177 | 376 | 754 | 540 | 15 | 764 | 257 | 665 | 207 |
| 750 | 112 | 560 | 755 | 151 | 374 | 76 | 200 | 620 | 744 | 452 |
| 30 | 370 | 737 | 507 | 773 | 306 | 370 | 121 | 42 | 551 | 663 |
| 504 | 167 | 261 | 224 | 101 | 510 | 645 | 71 | 265 | 26 | 247 |
| 500 | 712 | 31 | 227 | 375 | 445 | 774 | 6 | 724 | 547 | 753 |
| 300 | 573 | 33 | 725 | 276 | 356 | 717 | 567 | 755 | 736 | 773 |
| 724 | 443 | 250 | 314 | 177 | 202 | 676 | 737 | 502 | 734 | 101 |
| 500 | 561 | 607 | 601 | 634 | 612 | 574 | 750 | 342 | 154 | 72 |
| 463 | 345 | 200 | 433 | 126 | 745 | 211 | 152 | 14 | 675 | 421 |
| 435 | 137 | 767 | 723 | 305 | 54 | 317 | 730 | 775 | 25 | 154 |
| 552 | 245 | 575 | 176 | 147 | 655 | 620 | 17 | 43 | 144 | 115 |
| 542 | 365 | 476 | 40 | 750 | 524 | 257 | 226 | 537 | 677 | 377 |
| 55 | 12 | 52 | 112 | 632 | 546 | 152 | 347 | 755 | 51 | 354 |
| 756 | 353 | 46 | 103 | 333 | 501 | 571 | 735 | 747 | 364 | 715 |
| 452 | 377 | 261 | 750 | 201 | 771 | 130 | 657 | 372 | 367 | 45 |
| 550 | 216 | 750 | 543 | 400 | 763 | 235 | 65 | 711 | 772 | 710 |
| 403 | 770 | 166 | 703 | 455 | 22 | 704 | 565 | 553 | 177 | 442 |
| 740 | 40 | 122 | 277 | 51 | 337 | 673 | 574 | 501 | 176 | 500 |
| 254 | 312 | 357 | 414 | 327 | 672 | 472 | 426 | 770 | 473 | 541 |
| 766 | 402 | 157 | 751 | 115 | 357 | 565 | 23 | 272 | 61 | 721 |
| 671 | 750 | 777 | 154 | 176 | 144 | 73 | 73 | 710 | 453 | 52 |
| 115 | 434 | 415 | 77 | 444 | 6 | 417 | 271 | 463 | 172 | 415 |
| 544 | 333 | 335 | 252 | 703 | 126 | 261 | 451 | 335 | 406 | 5 |
| 422 | 761 | 577 | 617 | 433 | 762 | 442 | 156 | 555 | 453 | 70 |
| 440 | 241 | 450 | 345 | 554 | 171 | 647 | 277 | 567 | 764 | 454 |
| 457 | 744 | 565 | 623 | 602 | 521 | 724 | 357 | 717 | 737 | 717 |
| 626 | 250 | 303 | 656 | 200 | 465 | 202 | 704 | 275 | 245 | 254 |
| 410 | 3 | 40 | 731 | 326 | 546 | 221 | 75 | 556 | 755 | 776 |
| 641 | 627 | 750 | 662 | 567 | 306 | 51 | 203 | 744 | 606 | 666 |
| 573 | 760 | 600 | 1 | 271 | 724 | 651 | 555 | 717 | 167 | 235 |
| 307 | 170 | 376 | 502 | 253 | 645 | 131 | 60 | 150 | 712 | 155 |
| 120 | 620 | 474 | 410 | 664 | 554 | 475 | 273 | 650 | 20 | 446 |
| 202 | 103 | 344 | 246 | 344 | 172 | 567 | 433 | 732 | 530 | 303 |
| 332 | 624 | 732 | 600 | 207 | 377 | 732 | 713 | 333 | 537 | 265 |
| 174 | 772 | 656 | 306 | 376 | 206 | 252 | 755 | 351 | 725 | 44 |
| 42 | 212 | 644 | 5 | 132 | 170 | 22 | 234 | 735 | 644 | 765 |
| 671 | 116 | 375 | 33 | 656 | 320 | 465 | 170 | 507 | 162 | 72 |
| 542 | 666 | 77 | 343 | 526 | 706 | 170 | 153 | 31 | 301 | 557 |
| 271 | 256 | 65 | 703 | 527 | 541 | 460 | 351 | 625 | 764 | 432 |
| 100 | 536 | 62 | 236 | 17 | 255 | 746 | 307 | 411 | 530 | 260 |
| 115 | 653 | 732 | 516 | 647 | 366 | 675 | 741 | 101 | 41 | 257 |
| 675 | 173 | 310 | 76 | 770 | 174 | 533 | 702 | 344 | 100 | 342 |
| 765 | 734 | 412 | 734 | 747 | 51 | 312 | 567 | 725 | 745 | 606 |
| 405 | 574 | 225 | 640 | 445 | 450 | 102 | 152 | 244 | 127 | 773 |

| 145 752 | 247 701 | 665 757 | 451 235 | 751 202 | 670 42 | 437 | 754 | 627 | 213 | 531 |
|--------------------|------------|------------|------------|------------|-----------|-----|-----|-----|-----|-----|
| 1212200 | | 737 | 233 | 202 | 72 | | | | | |
| 1521201 | | | | | | | | | | |
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| 0021211 | | | | | | | | | | |
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|--|-------------------------|--------|
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| 2 1 | | |
| #of confli | 225 | 43.95% |
| #of confli | 146 | 28.52% |
| #of confli | 84 | 16.41% |
| #of confli | 39 | 7.62% |
| #of confli | 14 | 2.73% |
| #of confli | 1 | 0.20% |
| #of confli | 1 | 0.20% |
| #of confli | 2 | 0.39% |
| #of confli | 0 | 0% |

RUN SUC(756ms)