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# Laser Technology – Exercise Sheet 1

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## 1 Photons in a laser cavity

From an optical laser resonator (depicted in Fig. 1) of length $L$, light is outcoupled through the right mirror with a power reflection coefficient $R$. The outcoupled light has a power $P_{\text{out}}$ and a wavelength $\lambda$. The gain medium inside the cavity has a refractive index $n$. The reflection coefficient $R$ is formed by the interface between the semiconductor and air ($n$ = 1), and thus obtained as $R = (n-1)^2 / (n+1)^2$.

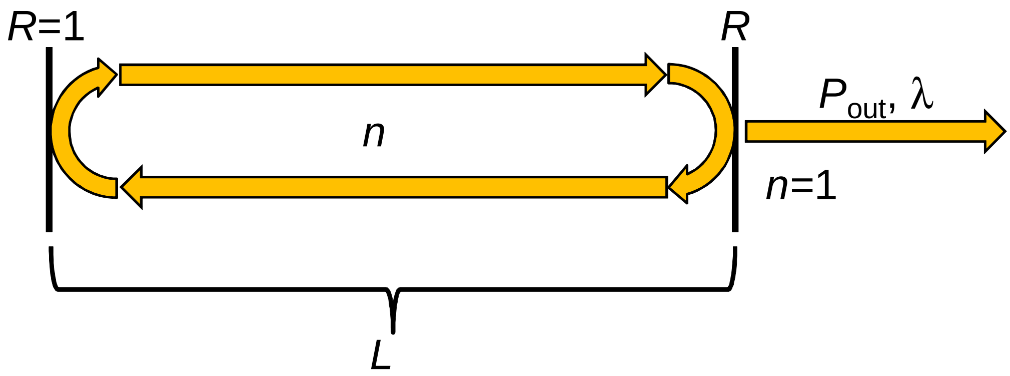


**Figure 1** Schematic representation of the laser cavity.

a) How many photons per second are outcoupled from the cavity?

   The number of emitted light quanta (photons) in a second is the fraction of optical power and photon energy $\dot{N}_{ph} = P_{\text{out}}\lambda/hc$.

b) How many photons are in the cavity? Assume for the calculation that $(1 - R) \ll 1$, and thus the power within the cavity can be assumed to be constant at different positions in the cavity.

   The power inside the cavity is $P = P_{\text{out}}/(1 - R)$. The calculation of the photon number is based on the calculation of the energy, which is $E = P \cdot T_r = P \cdot 2Ln/c$. The number of photons is then $N = E/(hf) = E\lambda/(hc) = 2Ln\lambda P_{\text{out}}/\left[(1-R)hc^2\right]$.

c) Calculate these quantities for typical values of a semiconductor laser, $L$ = 1 mm, $\lambda$ = 670 nm, $P_{\text{out}}$ = 5 mW, $n$ = 3.3.

   $\dot{N}$ = 1.68 × 10$^{16}$ s$^{-1}$, $N$ = 519350 for R = 0.2861 ($N$ = 37.1 × 10$^{6}$ for R = 0.99)

## 2 Gain saturation

The power dependent roundtrip gain is often modeled by $G = G_0/(1 + P/P_{\text{sat}})$, where $G_0$ is the small signal gain, $P$ is the intracavity power and $P_{\text{sat}}$ is the saturation power. The laser setup is as shown in

Fig. 1, i.e., with outcoupling at the right side with a reflection coefficient $R$, and given $G_0$ and $P_{\mathrm{sat}}$. The outcoupled power is $P_{\mathrm{out}}$.

a) How big is $P$ and $P_{\mathrm{out}}$ as a function of $R$? What is the maximum obtainable $P_{\mathrm{out}}$, and for which $R$ is it obtained? What is the maximum obtainable $P$, and for which $R$ is it obtained?

   The internal cavity power from the above formula $P = P_{\mathrm{sat}}\left(\frac{G_0}{G} - 1\right)$. We consider a lossless waveguide, therefore $G = 1/R$ and $P = P_{sat}\,(RG_0 - 1)$. The outcoupled power $P_{out} = (1-R)P = (1-R) \times (RG_0 - 1)\,P_{sat}$.

   The internal cavity power is maximized if $R$ = 1. In this case, substitution yields $P = P_{sat}\,(G_0 - 1)$.

   The $R$ with the maximum power is obtained from the root of the derivative of $P_{\mathrm{out}}$

$$\partial_R P_{\mathrm{out}} = -\,(RG_0 - 1)\,P_{sat} + (1 - R)\,G_0 P_{\mathrm{out}} = 0 \Rightarrow R = (1 + G_0)/2G_0. \tag{1}$$

   By substitution, the maximum output power is $P_{out} = (G_0 - 1)^2\,P_{sat}/4G_0$.

b) Plot $P_{\mathrm{out}}$ as a function of $R$ for typical values of a semiconductor laser, $G_0$ = 100, $P_{\mathrm{sat}}$ = 2 mW.