1. **Wavelets**

(a) Show that the Daub4 coefficients c_0–c_3 are (up to sign/index flips)

$$c_0 = 0.482963\,,\ c_1 = 0.836516\,,\ c_2 = 0.224144\,,\ c_3 = -0.12941\,, \tag{1}$$

when demanding orthogonality ($AA^T = 1$) of the filter matrix

$$A = \begin{pmatrix} c_0 & c_1 & c_2 & c_3 \\ c_3 & -c_2 & c_1 & -c_0 \\ c_2 & c_3 & c_0 & c_1 \\ c_1 & -c_0 & c_3 & -c_2 \end{pmatrix}\,, \tag{2}$$

and vanishing results for the application of the detail filter (2nd row of A)

$$A \cdot y = \begin{pmatrix} * \\ 0 \\ * \\ * \end{pmatrix} \tag{3}$$

to smooth $y = (0, 1, 2, 3)$ and constant $y = (1, 1, 1, 1)$ signals.

(b) Implement A for signals with more than $n = 4$ samples (n even).

(c) Find the Daub4 wavelet transform for the ("chirp") signal $y(t) = \sin\big(60t^2\big)$ with 4, 8, 16, 32, 64, 128, 256, 512, and 1024 samples. Plot the smooth and detail coefficients for each step of the transform. Can you explain the magnitude and shape of the smooth and detail coefficients in each step?

(d) Implement the inverse transform and verify that the coefficients obtained in the previous step reconstruct the original signal.

(e) Plot the Daub4 e5 base wavelet by calculating the inverse transformation of a sample Y with 1 at the fifth entry and 0 at all other entries. Do this for 16, 64, and 512 samples. Does the shape and position of the wavelet depend on the total number of samples?