1. **Sierpiński gasket**
   Reproduce the construction of the Sierpiński gasket and the empirical determination of its dimension as shown in the lecture. Use the *box-counting algorithm*.

2. **"How long is the coast of Britain?"**
   is the title of a classic paper by Benoît Mandelbrot. The length of the coast depends on the scale of measurement, the shorter the "ruler", the longer the coast. The coast is obviously a fractal and we can measure the length with the *box-counting algorithm*: we cover a map of England with $n^2 = 1, 4, 16, \cdots$ boxes and study the percentage of filled boxes. For a two-dimensional object, this ratio will converge to a constant, for a one-dimensional object (like the border of Colorado), the ratio will fall with $1/n$, i.e. the number of boxes needed will not be proportional to n^2 but to n. In the case of the length of the coast, we expect that the ratio falls with $(n)^{d_f-2}$ with fractal dimension d_f between 1 and 2.

   (a) Implement the box counting algorithm. Make a doubly logarithmic plot of the ratio of filled and total boxes against the total number of boxes per dimension (or just use i on the x-axis if $n = 2^i$). Find the range where the resulting curve is linear and determine the fractal dimension from the slope.

   (b) Try to use a very large number of boxes per dimension, e.g. $n = 8192$. Can you explain the behaviour of the percentage of filled boxes?

   (c) What about Norway? Did Slartibartfast's hard work pay off?

   You can use the image of the English coast and the Python skeleton from the code section of the homepage.

3. **Self-similar objects**
   Construct a self-similar object with the following procedure:

   1. start at $(x, y) = (0.5, 0)$;
   2. for the next step, replace (x, y) with

$(0.05x, 0.6y)$	10%	
$(0.05x, -0.5y + 1)$	10%	
$(0.46x - 0.15y, 0.39x + 0.38y + 0.6)$	20%	
$(0.47x - 0.15y, 0.17x + 0.42y + 1.1)$	20%	
$(0.43x + 0.28y, -0.25x + 0.45y + 1.0)$	20%	
$(0.42x + 0.26y, -0.35x + 0.31y + 0.7)$	20%	probability,

   and repeat 10000 times. Plot the resulting figure. Change some of the coefficients and study the effect.

4. **Ballistic deposition**
   In this exercise we consider a simple, one-dimensional model for the deposition of particles at some surface. Particles are falling from the top to random positions on a line of length L. The line is divided into 256 deposition sites. The height of the pile of particles at deposition site i is called h_i. When a particle falls into site i, h_i is increased by one. Additionally, neighbouring particles can prevent holes from being filled. This is ensured by applying the rule

   $$h_i = \begin{cases} \max(h_{i-1}, h_{i+1}) & \text{for } h_i < h_{i-1} \text{ and } h_i < h_{i+1} \\ h_i + 1 & \text{otherwise} \end{cases}, \tag{1}$$

   using periodic boundary conditions.

   (a) Examine the change of the average height with time.

   (b) Determine the fractal dimension of the surface line.