# State University of New York at Buffalo

## CSE 534 Spring 2017 Homework Set #2

Assignment Date: Thursday March 9, 2017; Due: ***Thursday March 16, 2016 at 11:00am***

Name: ______________________________ Student Number: _______________________

**NOTE**: You are required to work on your own. You may discuss with your fellow students. However, copying of answers from your fellow students will be unacceptable. Anyone who is found violating the academic integrity policy will be subject to severe consequence!

Problem (1) **(Digital Audio Coding – Masking tone)**

Given that the level of a *masking tone* at the 8th band is 60 dB, and 10 msec after it stops, the masking effect to the 9th band is 25 dB.

(a) What would MP3 do if the original signal at the 9th band is at 40 dB?

(b) What if the original signal is at 20 dB?

(c) How many bits should be allocated to the 9th band in (a) and (b) above?

Problem (2) **(Information Theory – Entropy Coding)**

(a) What is the entropy of the image below, where numbers (0, 20, 50, 99) denote the gray level intensities?

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 99 | 99 | 99 | 99 | 99 | 99 | 99 | 99 |
| 20 | 20 | 20 | 20 | 20 | 20 | 20 | 20 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 50 | 50 | 50 | 50 | 0 | 0 |
| 0 | 0 | 50 | 50 | 50 | 50 | 0 | 0 |
| 0 | 0 | 50 | 50 | 50 | 50 | 0 | 0 |
| 0 | 0 | 50 | 50 | 50 | 50 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

(b) Show step by step how to construct the Huffman tree to encode the above four intensity values in this image. Show the resulting code for each intensity value.

(c) What is the average number of bits needed for each pixel, using your Huffman code? How does it compare to the entropy computed in (a)?

Problem (3) **(Information Theory - Quantization)**

Suppose the input source is Gaussian-distributed with zero mean and unit variance – that is, the probability density function is defined as:

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

We wish to find a four-level Lloyd-Max quantizer. Let $\boldsymbol{y}_i = [y_i^0, \dots, y_i^3]$ and $\boldsymbol{b}_i = [b_i^0, \dots, b_i^3]$. The initial reconstruction levels are set to $\boldsymbol{y}_0 = [-2, -1, \ 1, \ 2]$. This source is unbounded, so the outer two boundaries are $+\infty$ and $-\infty$.

Follow the Lloyd-Max algorithm: the other boundary values are calculated as the mid-points of the reconstruction values. We now have $\boldsymbol{b}_0 = [-\infty,\ -1.5,\ 0,\ 1.5,\ \infty]$. Continue another iteration for $i = 1$, using the Eq. (2) defined below and find $y_1^0$, $y_1^1$, $y_1^2$, $y_1^3$, using integration shown in Equation (2). Also calculate the squared error $\boldsymbol{d}_1$: the difference between $\boldsymbol{y}_1$ and $\boldsymbol{y}_0$. Repeat such calculations for three times and output the following parameters: $\boldsymbol{b}_3$, $\boldsymbol{y}_3$, and $\boldsymbol{d}_3$.

Equation (1): $$b_i^j = \frac{y_i^{j+1} + y_i^j}{2}$$

Equation (2): $$y_i^j = \frac{\int_{b_{i-1}^{j-1}}^{b_{i-1}^j} x f_X(x)dx}{\int_{b_{i-1}^{j-1}}^{b_{i-1}^j} f_X(x)dx}$$

**Reference 1:** Lloyd-Max Quantizer Algorithm (in depth coverage) [Stuart P. Lloyd, "Least Squares Quantization in PCM", *IEEE Trans. on Information Theory*, Mar. 1982]
**Reference 2:** Book Chapter: Scalar Quantization (compact coverage); Uploaded to UBlearns.