RICHARD FEYNMAN: SIMULATING PHYSICS WITH COMPUTERS

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CS294: Reading the Classics

INTRODUCTION

Richard Feynman is known in various circles as a quantum-theoretical physicist, engineer, samba drummer, nobel prize winner, lock picker, radio repairer, and all around curious character. In this paper, we examine a keynote address at the California Institute of Technology in May 1981[1] in which he proposes the idea of a computer that could act as a *quantum mechanical simulator*[**?**]. As we will discuss, this idea was one in a series of key events leading to the idea of a general quantum computing device.

In this paper, we explore Feynman's contribution to the field of quantum computing by examining this keynote address. We begin by outlining Feynman's background with a brief biography and some colorful stories about his life. We continue by examining some of the relevant aspects of quantum mechanics to the problem that Feynman is trying to solve. We then take a look at the history of quantum computing to place this speech in its historical context. Finally, we address the speech and its associated paper in specific, outlining the issues that Feynman raises about the nature of computation as it relates to quantum mechanics.

BIOGRAPHY

Richard P. Feynman was born in New York City on the May 11, 1918. He studied at the Massachusetts Institute of Technology where he obtained his B.Sc. in 1939 and at Princeton University where he obtained his Ph.D. in 1942.

He was a research assistant at Princeton (1940-1941), a member of the highly secret Manhattan project (1941-1945), a professor of Theoretical Physics at Cornell University (1945-1950), a visiting professor and thereafter appointed professor of Theoretical Physics at the California Institute of Technology (1950-1959). He remained at Cal Tech as the Richard Chace Tolman professor of Theoretical Physics until his death in 1988. Like many other scientists involved in the Manhattan project, he suffered from a rare type of cancer for more than 9 years before his death.

Among his major contributions to Physics are his Ph.D work on quantum electrodynamics (QED), Feynman diagrams, and contributions to the theories of superfluidity and quarks. He also proposed, jointly with Murray Gell-Mann, the theory of weak nuclear force. His Ph.D. thesis work was on the probability of a transition of a quantum from one state to some subsequent state.

He invented an entirely new formalism in quantum mechanics, adapted it to the physics of quantum electrodynamics (QED). For this contribution, he was later awarded the Nobel Prize in physics, shared with Schwinger and Tomonaga (1965).

Feynman's books include many outstanding ones which evolved out of his lecture courses. For example, Quantum Electrodynamics (1961), The Theory of Fundamental Processes (1961), The Feynman Lectures on Physics (1963-65) (3 volumes), The Character of Physical Law (1965) and, QED: The Strange Theory of Light and Matter (1985). In particular, Feynman Lectures on Physics have become classic textbooks in physics ever since.

He was among the group who investigated the challenger's accident at 1986 and he explained the cause of the crash by a simple O-ring experiment. Also, in this landmark paper in 1982, he introduces the idea of simulating a quantum mechanical system with a computer, that later on leads to the idea of a universal quantum computer.

There are inumerable anecdotes and curious stories involving Feynman, who loved to tell them himself. He became very popular in the 1980s, with the publishing, by his friend Ralph Leighton, of the two books 'Surely You're Joking, Mr. Feynman' [**?**], and 'What do you care what other people think?' [**?**].

In his childhood he became famouns in his neighborhood for his ability to fix radios, and got progressively better at it. During high school, he would love to take part in mathematical challenges, and in fact, one of his greatest passions throughout life was to solve puzzles and problems. In his blackboard, at the time of his death, it was written 'Know how to solve all problems that have been solved'. He was a percussionist, and loved to play drums. During a 10 month stay in Brazil, while he was teaching physics at the Federal University of Rio de Janeiro, he also took part in an authentic Samba group. His endeavours through art also involved drawing. He was frustrated that artists did not comprehend the beauty of advanced science, and therefore

[1] The keynote was published in 1982 in the International Journal of Physics.

could not portray it, and decided to learn how to do it himself. Apparently, though, he did mostly portraits.

During his time at Los Alamos, he became famous as a a lock pick. He had this as a hobby, but also used it to warn about the lack of security of secret documents. He would open his colleagues locked cabinets and safes, and place notes inside, registering that the documents had been seen. In one instance, after the war, he got access to a file cabinet with the complete plans on how to build the bomb, including all the details on how to separate the Uranium and so forth.

He also deciphered Mayan hieroglyphs, and loved to be in company of beautiful women, among many other things. We lack the space and eloquence to describe these and other amusing tales about Feynman, but his popular books do a good job in that regard. With that said, we move on to the more serious matters now.

QUANTUM MECHANICAL EFFECTS

To fully put Feynman's speech in context requires an examination of several of the more unique aspects of quantum mechanics, namely superposition and entanglement.

Superposition

One of the stranger effects of quantum mechanics is that of an entirely new means of describing the state of an entity. To motivate this discussion, let us examine a set of experiments as shown in Figure 1.

In this experiment, a weak light source is set up to shine at a pair of detectors. These detectors are sensitive enought that they can emit a "click" when an individual photon arrives. This, by the way, is one of the means by which the nature of light acts like a particle. When the light gets dimmer, fewer (not weaker) clicks are observed at the detector.

In the first experiment, a half-silvered mirror is placed in the light beam. Quantum theory predicts (and experiments confirm) that in this case the photons will be detected at one or the other site with equal probability. This is rather strange in and of itself, for how does the photon *decide* which way to go?

In fact, Newton (who, years ahead of his time, was convinced that light was made up of particles, called "corpuscles") observed similar effects when shining light on a semi-reflective surface such as glass. He was unable to explain this phenomenon by any classical explanation.

Now one potential explanation for these effects are that certain photons are predisposed to reflect, while others are predisposed to pass through the mirror. Indeed, more classical explanations could be used for this, in that the different photons may have different trajectories. Yet in the second experiment, more mirrors are used to *recombine* the light beams at a second half-silvered mirror. Now any classical explanation (including the pre-disposition argument) would predict that this recombining would not affect the outcome, and that again the photons would be detected with equal probability at each detector.

However, as is shown in the figure, this is not the case, and instead, *all* the photons are registered at one detector. Now in the third experiment, an opaque object is placed in the path after the first mirror, and it is observed that once again, the photons are detected with equal probability at the two detectors. Furthermore, it is not the case that 50% of the photons are blocked, rather the *all* arrive at the two detectors, just with equal probability.

It should be clear at this point that no classical explanation will suffice for this phenomenon. In trying to apply a traditional explanation, one hits various contradictions, such as requiring that the photon travel backwards in time once it "hits" the blockage.

Quantum theory describes this situation as a *coherent superposition*. In this explanation, once the photon passes the first mirror, one can think of its state as being simultaneously reflected and not reflected, with equal probability. It can exist in this state until it reaches one of the detectors, at which point it is forced into one state or another.

This exposes one of the fundamental differences between quantum and classical mechanics, in that the act of measurement in a quantum system irrevocably changes the system. As we will see, this property is both a blessing and a curse for the design of quantum computers.

Entanglement

Entanglement is another quantum mechanical phenomenon that cannot be explained by classical physics, and in fact challenges the very interpretation of quantum mechanics itself. It led Einstein, Podolski, and Rosen to formulate, in 1935, the EPR Paradox, as a challenge to the completeness of quantum mechanics.

Two or more objects in an entangled state have to be described with reference to one another, even if they are physically separated. For example, two photons may be in an entagled state such that measuring one of them (thus forcing it into one particular state) also forces the result of the measurement of the other photon. This happens even if the two photons are arbitrarily separated, and has been observed experimentally. It seems paradoxical because the two systems seem to instantaneously transmit information about the measurement to the other, even though General Relativity states that no information can be transmitted faster than the speed of light.

The correlation of the measurement results of both photons is found to be higher than probability theory would possibly predict, and, as we see below, it is this fact that is used

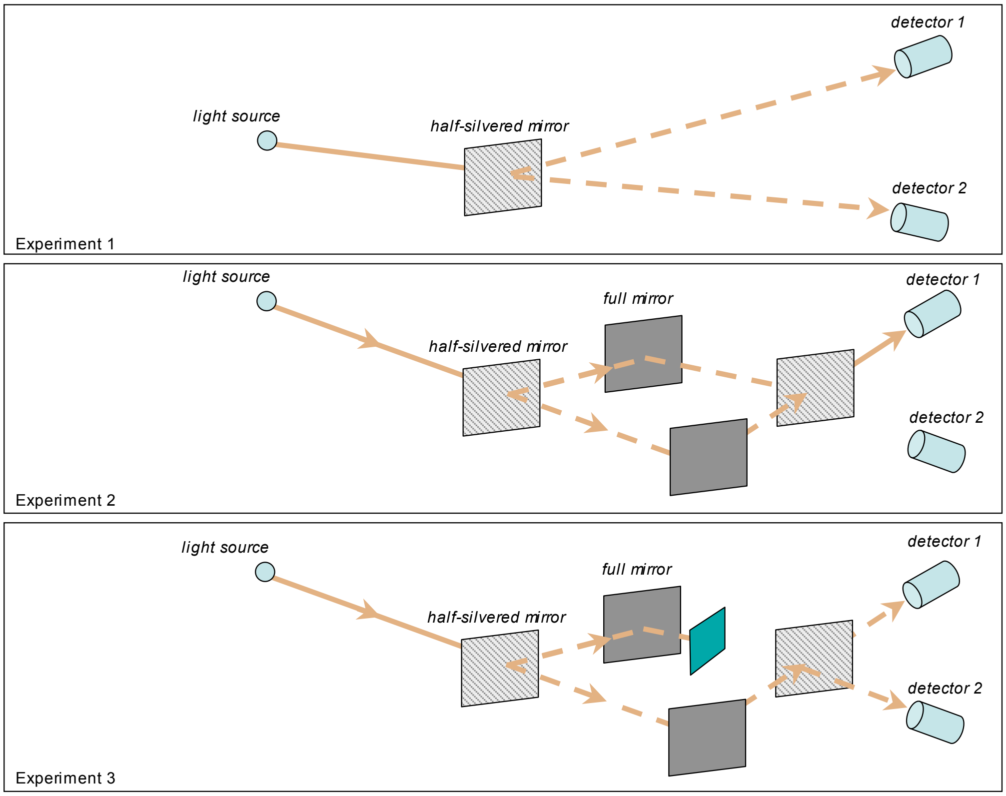

Figure 1. Three experiments demonstrating superposition. In experiment 1, once the photons pass the half-silvered mirror, they have a 50% chance of being detected at each of the two detectors. In experiment 2, once the photon streams are recombined, if the path lengths are equal, then there is a 100% chance they will be detected at the first one. Finally, once one of the streams is blocked, once again, there is a 50% chance of being detected at either.

by Feynman in an example to show how a local probabilistic classical computer cannot simulate quantum mechanics.

HOW QUANTUM COMPUTERS WORK

Qubits

In a quantum computer, the phenomenon of superposition is used as the basic unit of information, called a *qubit*. As in a bit in a classical computer, a qubit stores a binary value, either a one or a zero. However, it is manifested as a two state quantum entity such as the nuclear spin of an atom, an electron that is either spin-up or spin-down, a photon with polarization either horizontal or vertical, etc.

When *measured*, the qubit is found in only one of the two states. In Dirac notation, qubits are represented as a *ket*, where the basic values of 0 and 1 are denoted as $|0\rangle$ or $|1\rangle$.

However, until it is measured, the qubit is in a superposition of 1 and 0, and there is generally a probability distribution on the value. Although these probabilities cannot be measured directly, they can take part in computations.

A bit more formally, a qubit is a unit state vector in a two dimensional Hilbert space where $|0\rangle$ and $|1\rangle$ are orthonormal basis vectors. For each qubit $|x\rangle$, there exist two complex numbers a and b such that

$$|x\rangle = a|0\rangle + b|1\rangle = \begin{pmatrix} a \\ b \end{pmatrix}, \; |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and $|a|^2 + |b|^2 = 1$. Therefore, a and b define the *angle* which the qubit makes with the vertical axis and therefore es the probability that the given bit will be measured as a 0 or as a 1.

Note that there is also the *phase* of the qubit which represents an angle of rotation around the vertical axis. While this phase does not affect the probability of measuring the bit in a certain value, it is crucial for quantum interference effects.

Similar to a classical register, a register of 3 qubits can store $2^3 = 8$ values. Of course, these values are in a superposition, so in effect, the register stores all 8 values at once, with a joint probability distribution across the set of values. However, it is important to note that a qubit contains no more *information* than a classical bit. The reason for this is that once you measure the value, it is forced into one of the two states, and therefore one cannot extract any more information from the qubit than one can in the classical case.

Evolutions

The quantum analog to a classical operation is an *evolution*. An evolution transforms an input qubit or register by some physical process to an output value. Generally, this can be represented as a 2 x 2 matrix that. For example, the rotation operator

$$R_\theta = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix}$$

transforms a qubit state by:

$$|0\rangle \mapsto cos\theta|0\rangle + sin\theta|1\rangle, |1\rangle \mapsto -sin\theta|0\rangle + cos\theta|1\rangle$$

It is important to note that evolutions operate *without* measuring the value of a qubit. Therefore, they can create a new superposition out of the original input state. Essentially, since a qubit register stores a probability distribution across all possible values bits, an evolution performs parallel computation across all these values at once to produce a new superposition.

Furthermore, by the principles of entanglement, measuring one bit can affect another. Consider a two bit system: $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Although the probability that the first bit is $|0\rangle$ is $1/2$, once the second bit is measured, then this probability is either 0 or 1!

It is important to note that not all states are entangled, for example $\frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$. In this state, the value of the first qubit is always 0, while the second bit is evenly distributed among 0 and 1, regardless of the measurement of the first qubit.

Quantum Error Correction

A final note relates to error correction. Turing machines and classical computers are based around the (correct) assumptions that values in registers can be measured and manipulated reliably within the computer. While implementations may require energy input to maintain state, this input is theoretically irrelevant to the computations.

The principles of error correction over a communication channel as introduced by Shannon led to a new field of information theory. However, the application of this field to classical computing are constrained to multi-party communications, and are in general not related to the internal mechanics of a computer.

In the case of quantum computers, it seems likely this will not be the case. As it turns out, physical manifestation of qubits and evolutions turn out to be very sensitive to noise in the environment. Therefore, there is a natural fit for quantum error correction to maintain confidence about the values stored in the computer. Thus unlike in the classical computing context, it seems likely that the fundamental operation of a quantum computer will depend on the reliability predictions coming from information theory, and that a deeper relationship will exist between the two.

HISTORY OF QUANTUM COMPUTERS

A theme leading to the development of quantum computers is found by examining the relationship between computation and thermodynamics. In some senses, the story begins in 1871 with Maxwell's Demon, which violates the second law of thermodynamics by separating out the hot particles from cold particles by opening and closing an imaginary "gate." In 1921, Leo Szilard reduced this problem to that of particle identification. In his work, he showed that the essential action that the demon is performing is to identify the hot from the cold particles. It is interesting to note that Szilard discussed the concept of a 'bit' of information, though the term wasn't coined until later.

After another forty years in 1961, Landauer explored this idea further and determined that it is the *erasure* of information that is dissipative and therefore requires energy to perform. He also notes that erasure of information is irreversable.

In the 1970s, Bennett, Fredkin, Toffoli, and others began to apply the ideas of reversible operations to general computation, and showed that any computation can be made to be reversible, as long as no information is erased. For example, unlike a traditional NAND gate that has two inputs and one output, the reversible NAND gate has two input lines and three outputs, where the first two outputs are identical to the first two inputs (therefore no information is lost), and the third line is the NAND result.

Finally, in 1982, Bennett applied these principles directly to Maxwell's demon, showing that the demon must eventually forget which particles are the hot ones and which are the cold ones, thereby necessarily expending energy to erase the information.

On another tack, in 1935, Einstein, Podolsky, and Rosen describe a *gedanken* experiment in which a single atom produces two particles in opposite directions with identical state (e.g. polarization). Quantum theory predicts *entanglement*, in that the state of one of the particles can be affected by the second.

In 1964, Bell produced a crucial result, in which he showed that no deterministic hidden variable theory can reproduce these quantum theory predictions, and therefore nonlocal interactions can exist. In 1982, Aspect, Dalibard, and Roger support Bell's theorem showing that any traditional interactions must travel faster than the speed of light and therefore are impossible. These results are a boost in the quantum theory camp.

Turning to the actual developments of quantum computers, in 1980, Benioff described a hybrid Turing machine that stores qubits on the tape instead of traditional bits. However, his machine does not use any quantum mechanical effects in computation, as each qubit is measured from the tape at each step.

Feynman's speech and accompanying paper in 1982 is the first work to explicitly discuss the construction of a machine that would operate on quantum mechanical principles. He discusses the idea of a *universal quantum simulator*, i.e. a machine that would use quantum effects to explore other quantum effects and run simulations.

In 1984, Albert described a 'self measuring quantum automaton' that performs tasks no classical computer can simulate, however the machine was largely unspecified. Yet in 1985, Deutsch is credited with the first specified quantum computing system. Although the feasibility of the system is slightly suspect, it did include proscriptions for the gates and operational states of the machine.

In the early 1990s, Bernstein and Vazirani open up the field of quantum computing to the computer science theoretical community and through the 90's, go on to establish quantum complexity theory. Through thir work, the field of quantum algorithms and operations can then be studied in a formal setting, similar to traditional algorithms.

In 1993, Simon described an oracle problem for which quantum computers are exponentially faster than classical ones. Then in 1994, Shor described a quantum algorithm for efficient factorization of large numbers. This result sparked an influx of interest in the field (and a recoil from the cryptography community) since here was an application of computers that could have rippling effects on the overall computer community. Incidentally, in the early 1980's, Weisner and Bennett explored the idea of quantum key exchange, the presence of which would be a comfort to all the security systems compromised by the ability of computationally feasible factorization.

Finally, in 1998, the first functional two qubit nuclear magnetic resonance computer was demonstrated at UC Berkeley. Over the next several years, the systems were improved, and in 2001, a 7-qubit NMR system was demonstrated at IBM Almaden to execute Shor's algorithm and successfully factor the number 15. As noted in the class discussion, it is interesting to note what would happen if they had tried to factor the number 13...

SIMULATING PHYSICS WITH COMPUTERS

In his keynote address, Feynman is concerned with simulating quantum physics with computers. He makes an introduction to this work by referring to the possibilities in computer and also the possibilities in physics. Following his quest for simulating physics with computer comes the question of what type of computer should be used to simaulate physics. He names cellular automata as an option, but he also considers other possible computing systems that could potentially act exactly the same as nature.

He identifies the reversibility of computation results by Bennett [**?**], Fredkin, and Toffoli [**?**, **?**] as an important step toward realizing possibilities in computation. Before he goes into the technical details, he sets the following *rule of simulation:*: the number of computer elements required to simulate a large physical system is only to be proportional to the space-time volume of the physical system. In other words, he refers to simulations with exponential complexity as being against his rule of simulation.

Simulating Time

In order to simulate time, the first assumption that Feynman makes is that the time is discrete. According to him, in cellular automata, time is not simulated, but is rather imitated by being hidden behind the state to state transition. He explores ways to simulate time in cellular automata rather than imitating it. In particular, he shows an example in space-time domain. In his example, the state s_i at the space-time point i is a given function $F_i(s_j, s_k, \dots)$ of the state at the points j, k in some neighborhood of i:

$$S_i = F_i(s_j, s_k, \dots)$$

If F_i is such that it only contain the points previous in time, we can perform the computation in a classical way. However, if F_i is a function of both future and the past, would there be an organized algorithm by which a solution could be computed? Even if the function F_i is known, this task may not be possible. He mentions that classical physics is local, causal and reversible, therefore quite adaptable to computer simulation. Feynman goal is to explore the case of computer simulation for quantum physics.

Simulating Probability

As we turn to quantum physics, we know that we only have the ability to predict probabilities. Feynman denotes that there has always been a great deal of difficulty in understanding the world view that quantum mechanics represents. According to Feynman, there are two ways to simulate a probabilistic theory in a computer: (i) To calculate the probability and then interpret this number to represent nature, and (ii) To simulate by a computer which itself is probabilistic.

If we calculate the probability and then interpret it as a number, there will be a problem with discretizing probability as pointed out by Feynman. If there are only k digits to represent the probability of an event, when the probability of something happening is less than 2^{-k}, it will not happen at all according to the computer. Also, if we have R particles, then we have to describe the probability os a circumstance by giving the probability to find these particles at points $x_1, x_2, x_3, \ldots, x_R$ at the time t. Therefore a k-digit number would be needed for the state of the system, for every arrangement of R values of x. If there are N points in space, we have to have around N^R configurations. This incurs an exponential complexity and thus, is marked as impossible according to the rule of simulation Feynman has set as the assumption.

On the other hand, if we simulate the theory by using a probabilistic computer, the state transition is probabilistic and not predictable. Therefore, even though the probabilistic computer will imitate the nature, it will not be exactly the same as nature, in that nature is unpredictable. Feynman proposes a Monte Carlo approach to alleviate this problem. Basically, if a particular type of experiment is repeated sufficient number of times, the corresponding probability could be found within a statistical accuracy bound. He continues describing the nature of probabilistic computer as being a "local probabilistic" computer, in that the behavior of one region can be determined by disregarding the other regions. According to Feynman, the equations have the following form: At each point $i = 1, 2, \ldots, N$ in space, there is a state s_i chosen from a small state set. The probability to find some configuration $\{s_i\}$ is a number $P(\{s_i\})$. He formulates the state transition from a state to the next as a likelihood calculation:

$$P_{t+1}(\{s_i\}) = \sum_{(s')} [\prod_i m(s_i|s'_j, s'_k,)] P_t(\{s'\})$$

Where $m(s_i|s'_j, s'_k, \ldots)$ is the probability that we move to state s_i at point i when the neighbors have values $s'_j, s'_k, \ldots$ in the neighborhood of i. As j moves far from i, m becomes ever less sensitive to s'_j. At each change, the state at a particular point i will move from what it was to a state s with a probability m that depends only upon the state of the neighborhood. This gives a probability of making a transition. According to Feynman, it is the same as in cellular automata, just that the state transition is probabilistic instead of definite.

Here, Feynman returns to the question of how to simulate a quantum mechanical effect. According to what he described so far, such a system cannot be simulated with a normal computer without having an exponential complexity. The only possible ways to simulate such a system would be to either use a probabilistic computer as he already discussed, or to have a new kind of computer. This new kind of computer should itself be built of quantum mechanical elements that obey quantum mechanical laws.

Quantum Simulators

In the keynote, Feynman only sketches the ideas of what this new kind of quantum mechanical computer would be. He conjectures that if you build a machine that can perform computations using quantum mechanics elements, you could probably simulate with it any quantum mechanical system, "including the physical world", and poses the interesting question of which quantum systems are intersimulatable, or equivalent. This also leads to the question of whether a class of "universal quantum simulators" can be found. He leaves it open if the simple example he shows of linear operators on a two state quantum system can be used as a basis to simulate any quantum system.

Simulating Quantum Systems with Classical Computers

On the other branch, a simple example shows that it is impossible to represent the results of quantum mechanics with a classical universal device. The question is, can you make a Turing machine imitate with the same probability what nature does, output the same probabilities that you observe in quantum mechanical experiments? Again, it would have to be dealing with probabilities, because of the state explosion to store all the numbers.

Feynman demonstrates this with a simple numerical comparison. As was his characteristic, he attempted to simplify things to make it as accessible as possible, and as he put it himself: "I've entertained myself by squeezing the difficulty of quantum mechanics into a smaller and smaller place(...) It seems almost ridiculous that you can squeeze it [the impossibility result we discuss next] to a numerical question that one thing is bigger than another".

So we start with an experiment that verifies the polarization of a photon. In figure **??**,[2] we have a polarized photon go through a calcite crystal. Photons have a polarization in either the x or y axis, as, as we would expect, is actually in a superposition state that is 'decided' when you measure it. When it goes through a calcite, differently polarized photons are separated into either an ordinary ray (O), or an extra-ordinary ray (E), and you can place detectors for each.

[2] These figures are used directly from Feynman's paper [**?**].

Any photon will trigger only one of the detectors, and the probabilities of detector O or E being triggered add to 1.

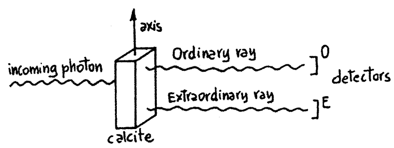

Figure 2. Simple experiment with a polarized photon

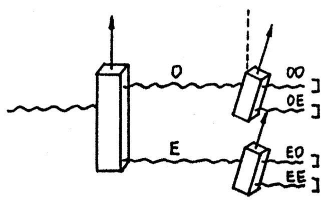

Figure 3. Adding a second calcite after the first one

A similar thing happens if you put a second calcite after each of the ordinary and extra-ordinary rays of the first one, creating 4 possible paths for the photon, as in figure **??**. For each photon, only one of the four detectors will be triggered, and the probabilities all add to one again. So far, so good.

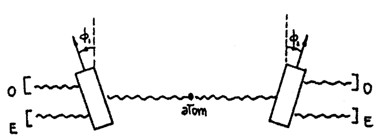

Figure 4. Two "entangled" photons emitted simultaneously by the same atom

However, and this is verified both in experiments and predicted by quantum mechanics, we can get a situation that can't be explained by classical probability (and not simulated by a local probabilistic computer). Figure **??** depicts the situation: an atom simultaneously emits two photons, and there are two separate calcites and detectors. The joint probabilities of getting O and E are given by

$$P_{OO} = P_{EE} = 1/2cos^2(\phi_2 - \phi_1) \quad (1)$$
$$P_{OE} = P_{EO} = 1/2sin^2(\phi_2 - \phi_1). \quad (2)$$

Note that these are only affected by the relative angle between the two calcites, and in particular, if $\phi_1 = \phi_2$, one side can *predict* with certainty what the other side gets, as $P_{OE} = P_{EO} = 0$.

What Feynman then does is show that with regular local probabilities you can't get the same results. Let us try, though.

Let us discuss the results for increments of 30 degrees only. Since it's possible to predict the other side exactly, when a photon comes, the 'answer' for the possible angles must be already determined. This is because my answer at each angle must be the sames as yours. In figure **??**a, where a dark dot represents an ordinary ray, and a white dot, an extraordinary ray. In this diagram, the positions separated by 90 degrees must give opposite answers, so we have 8 possible arrangements. Each pair of photons may give a different set of answers, but the two photons in a given pair will always have the same arrangement.

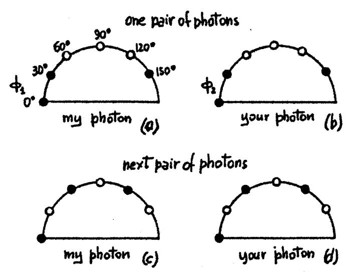

Figure 5. Two possible "configurations" of the photons for the discretized reasoning above.

Let's say now that we agree to measure at 30 degrees apart. Looking at figure **??** what is the probability that we get the same result? It's the number of possible adjacent pairs, and it works out to 2/3 (180 and 150 degrees should also count). If we look at all possible 8 arrangements, it turns out that 2/3 is the largest probability that can be obtained in this case, and this is where the difficulty lies: if we go back to equation **??** and substitute 30°, we obtain 3/4.

Local probabilities cannot explain what is obtained in practice. The two photons are in an entangled state, and measuring one determines the result of measuring the other.

CONCLUSIONS

Richard Feynman was an extraordinary man by many measures, and it was very interesting to read his paper and some of his books. In this work he shows the ability to take new perspective at things that marked many of his accomplishments in life, and helps set the agenda for the research in quantum computing. It is a good example of a negative result that inspired research and progress in an area.

As Christos Papadimitriou mentioned, the discipline of quantum computing is still in its infancy, and its future is not entirely clear. There is the possibility that ever more capable quantum computers will continue to be built, following the pioneering experiments at UC Berkeley and IBM Research. There is also the danger that the progress might be much slower than expected, and that enthusiasm in the field would die down. A third possibility, and this is mentioned by Feynman in his keynote, is that the attempts to develop quantum computing may bring a new understanding to the field of quantum mechanics itself.

REFERENCES

[Ben73] C. H. Bennett. Logical reversibility of computation. *IBM Journal of Research and Development*, 17:525–532, November 1973.

[BESM96] A. Barenco, A. Ekert, A. Sanpera, and C. Machiavello. A short introduction to quantum computing. In *La Recherche*, Nov 1996.

[CP01] Cristian S. Calude and Georghe Paun. *Computing with Cells and Atoms: An Introduction to quantum, DNA, and membrane computing*. Taylor and Francis, New York, 2001.

[Fey81] Richard Feynman. Simulating physics with computers. In *International Journal of Theoretical Physics*, 1981.

[Fey85a] Richard Feynman. *QED: The Strange Theory of Light and Matter*. Princeton University Press, Princeton, NJ, 1985.

[Fey85b] Richard Feynman. *Surely You're Joking Mr. Feynman*. W.W. Norton and Company, New York, 1985.

[FT82] E. Fredkin and T. Toffoli. Conservative logic. *International Journal of Theoretical Physics*, 21:219–253, 1982.

[Ste98] Andrew Steane. Quantum computing. In *Reports on Progress in Physics*, volume 61, pages 117–173, 1998.

[Tof80] T. Toffoli. Reversible computing. Technical memo MIT/LCS/TM 151, MIT Lab for Computer Science, 1980.