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SHABANI TEAM #1: Quantumtroopers

Wavefunction Generation as Noise Evaluation

Motivation

Feynman remarked that the true power of quantum computers is in simulating quantum-mechanical systems. This is believed to be intractable classically (in the class **NP**); however, we know that quantum simulation is in **BQP⊇P**, meaning that it is efficiently realizable on a quantum computer.

Preparing wavefunctions is a crucial first step in simulating a system; once the desired wavefunction is created, it can be evolved according to the laws of quantum mechanics.

Despite our increasing capability to control entanglement, we are still in the NISQ era, meaning such highly-entangled preparations are subject to large amounts of noise. It is the focus of this project to characterize the effects of this noise on the Gaussian wavefunction, and we propose this as a metric for evaluating the noise on quantum hardware.

Klco and Savage, arXiv:1904.10440 [quant-ph]

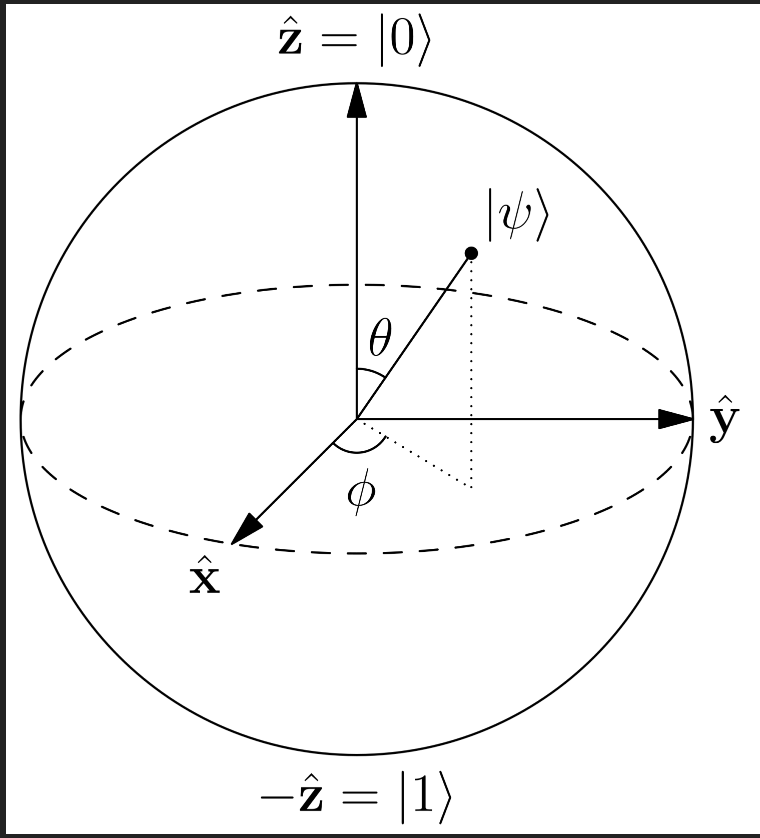
Noise in Quantum Information

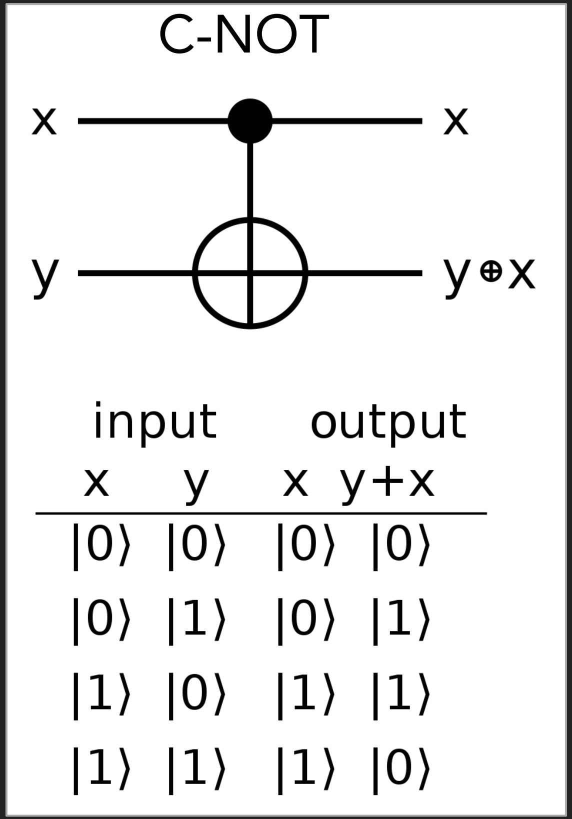
A single qubit state can be represented by a point on a sphere with the north and south poles as the $|0\rangle$ and $|1\rangle$ state.

Gates perform rotations between two quantum states.

Noise for a state in Quantum information takes the form of unintended unitary transforms (rotations on the bloch sphere for a single qubit).

- Individual qubit error types
  - Bit flip error (z-rotation, θ)
  - Phase error (xy-rotation, ϕ)
- Gate errors are defined by deviations from landing in the target state.
- CNOT gates are large sources of error and notoriously hard to implement.





Kitaev-Webb Algorithm

- The algorithm stated here prepares a Gaussian wavefunction on N qubits.
- Define a state via recursive description
- Implements an algorithm that iteratively prepares a discrete 1D Gaussian wave function on N qubits, with 2^N probabilities.

Lemma 1. The state of the periodic discrete Gaussian can be expressed recursively by

$$|\xi_{\mu,\sigma,N}\rangle = |\xi_{\frac{\mu}{2},\frac{\sigma}{2},N-1}\rangle \otimes \cos(\alpha)|0\rangle + |\xi_{\frac{\mu-1}{2},\frac{\sigma}{2},N-1}\rangle \otimes \sin(\alpha)|1\rangle$$

where the angle α is given by

$$\alpha = \cos^{-1}\left(\sqrt{\frac{g(\mu/2,\sigma/2)}{g(\mu,\sigma)}}\right)$$

Algorithm 1 Kitaev-Webb

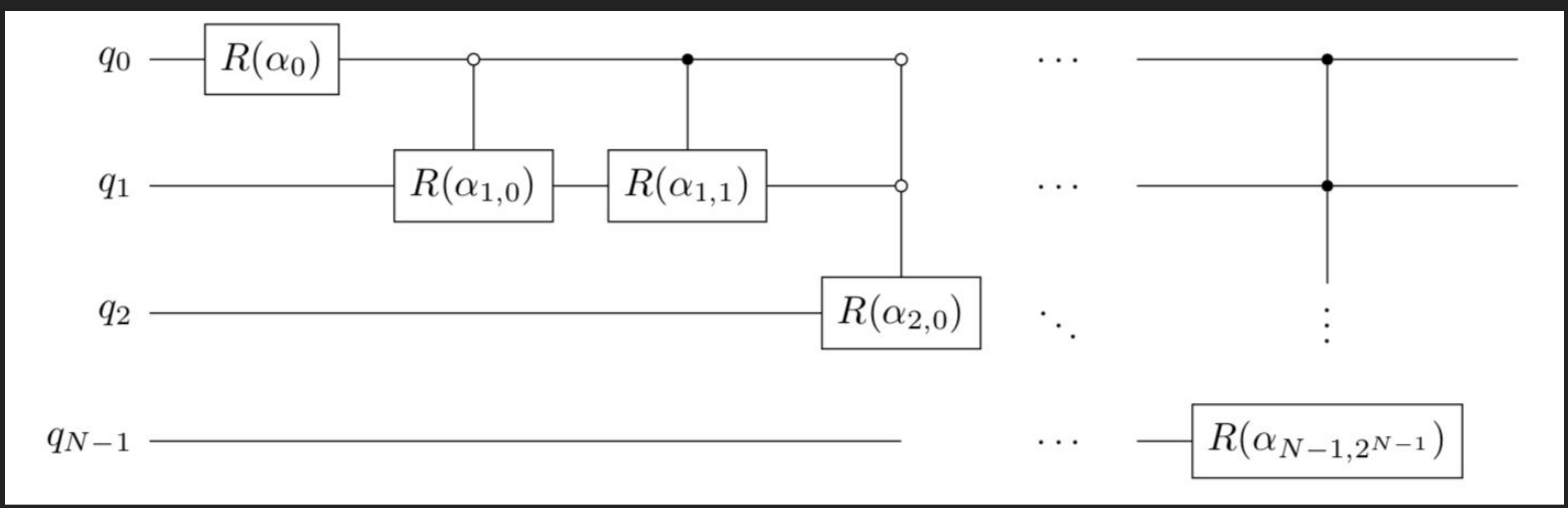
Input: The parameters $\mu, \sigma \in \mathbb{R}^+$, $k \in \mathbb{N}$, and $N \in \mathbb{N}$.
Output: A discrete approximation of the periodic state $|\xi_{\mu,\sigma,N}\rangle$.

1. **Initial State:** $|0^k\rangle|\mu,\sigma,N\rangle|0^N\rangle$;
2. **Compute** α in the first register, yielding the state $|\alpha\rangle|\mu,\sigma,N\rangle|0^N\rangle$;
3. **Apply the rotation** α **on** q_0, yielding the superposition state $|\alpha\rangle|\mu,\sigma,N\rangle|0^{N-1}\rangle \otimes (\cos(\alpha)|0\rangle + \sin(\alpha)|1\rangle)$
4. **Uncompute** α from the register containing it, obtaining $|0^k\rangle|\mu,\sigma,N\rangle|0^{N-1}\rangle \otimes (\cos(\alpha)|0\rangle + \sin(\alpha)|1\rangle)$
5. **Execute a change of parameters controlled by** q_0, given by $\cos(\alpha)|0^k\rangle|\frac{\mu}{2},\frac{\sigma}{2},N-1\rangle|0^{N-1}\rangle \otimes |0\rangle + \sin(\alpha)|0^k\rangle|\frac{\mu-1}{2},\frac{\sigma}{2},N-1\rangle|0^{N-1}\rangle \otimes |1\rangle$
6. **Whenever** $N > 1$**, Apply steps 2-5 on all qubits except the last**, giving $\cos(\alpha)|0^k\rangle|\frac{\mu}{2},\frac{\sigma}{2},N-1\rangle|\xi_{\frac{\mu}{2},\frac{\sigma}{2},N-1}\rangle \otimes |0\rangle + \sin(\alpha)|0^k\rangle|\frac{\mu-1}{2},\frac{\sigma}{2},N-1\rangle|\xi_{\frac{\mu}{2},\frac{\sigma}{2},N-1}\rangle \otimes |1\rangle$
7. **Reverse the parameter change**, yielding
$$|0^k\rangle|\mu,\sigma,N\rangle \otimes \left(\cos(\alpha)|\xi_{\frac{\mu}{2},\frac{\sigma}{2},N-1}\rangle|0\rangle + \sin(\alpha)|\xi_{\frac{\mu}{2},\frac{\sigma}{2},N-1}\rangle \otimes |1\rangle\right)$$
$$= |0^k\rangle|\mu,\sigma,N\rangle|\xi_{\mu,\sigma,N}\rangle$$
as desired.

Kitaev and Webb, arXiv:0801.0342

Kitaev-Webb Algorithm

- Adapting this to be more Qiskit-friendly
- Some hybridization with the more general Grover-Rudolph state preparation algorithm
- Notice how highly-entangled the circuit is. Because of this, goodness-of-fit to a Gaussian is a robust measure of how well one's hardware handles compounded noise.

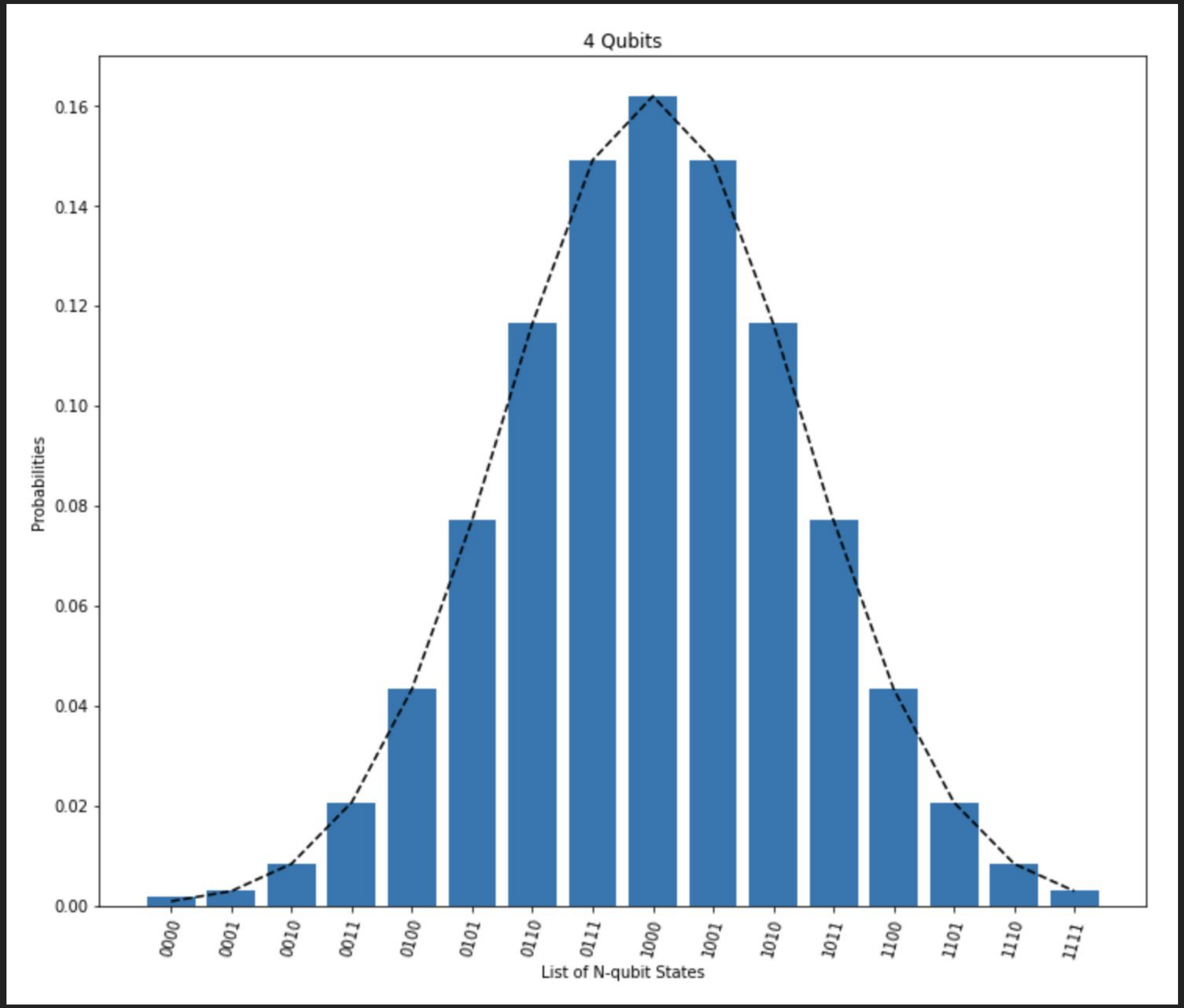


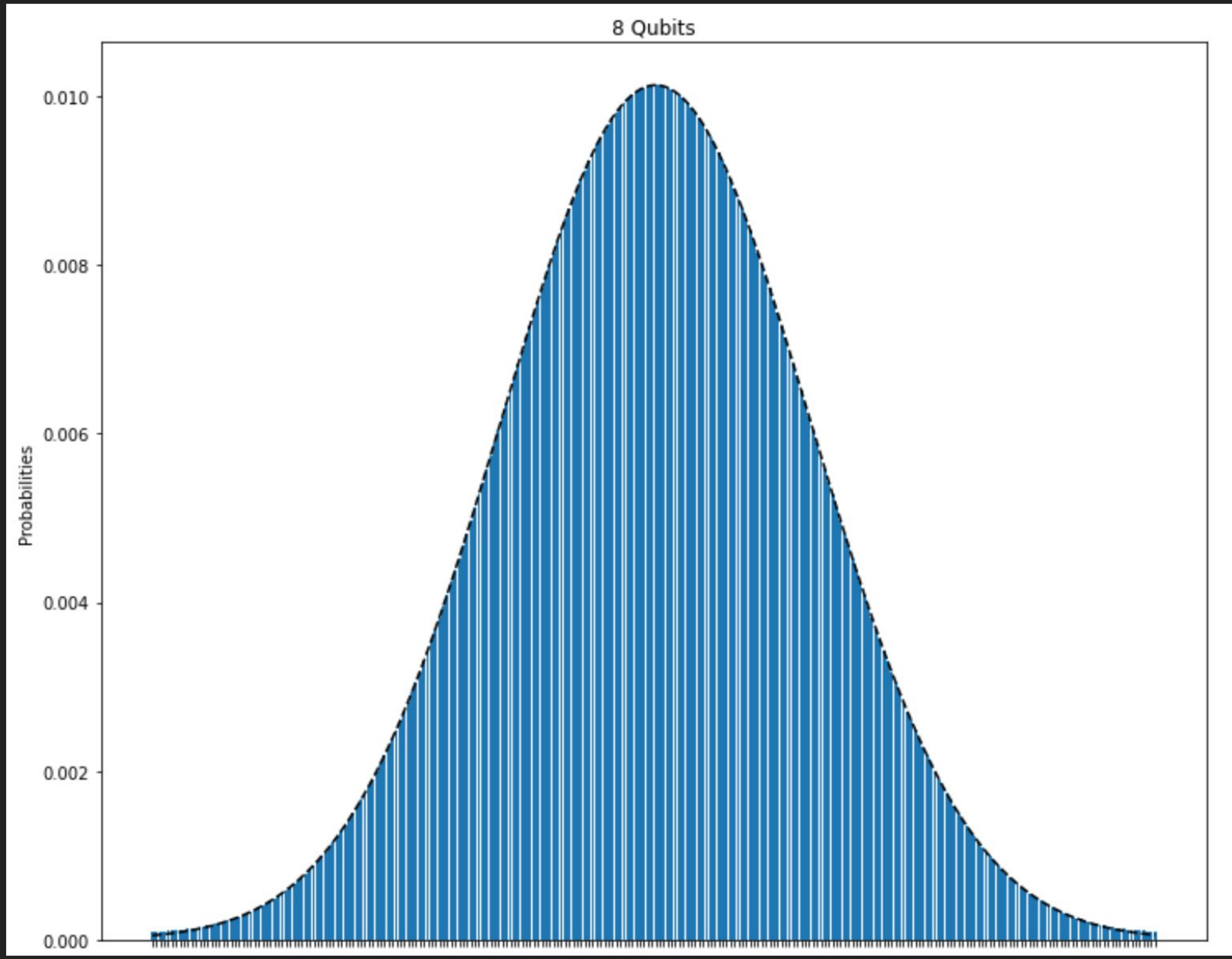
Kitaev and Webb, arXiv:0801.0342
Grover and Rudolph, arXiv:quant-ph/0208112

Noiseless Qiskit Simulations for 4 and 8 qubits

- Pure simulation where the blue histogram shows binning of 100,000 simulated measurements against the dashed line of fitted gaussian

- However, the state created is highly entangled and more subject to noisy deformations

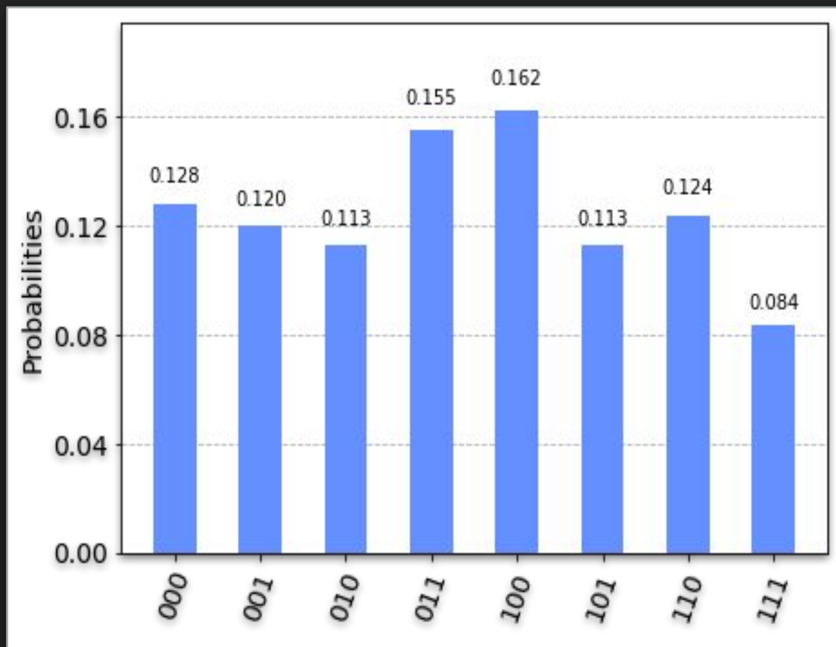




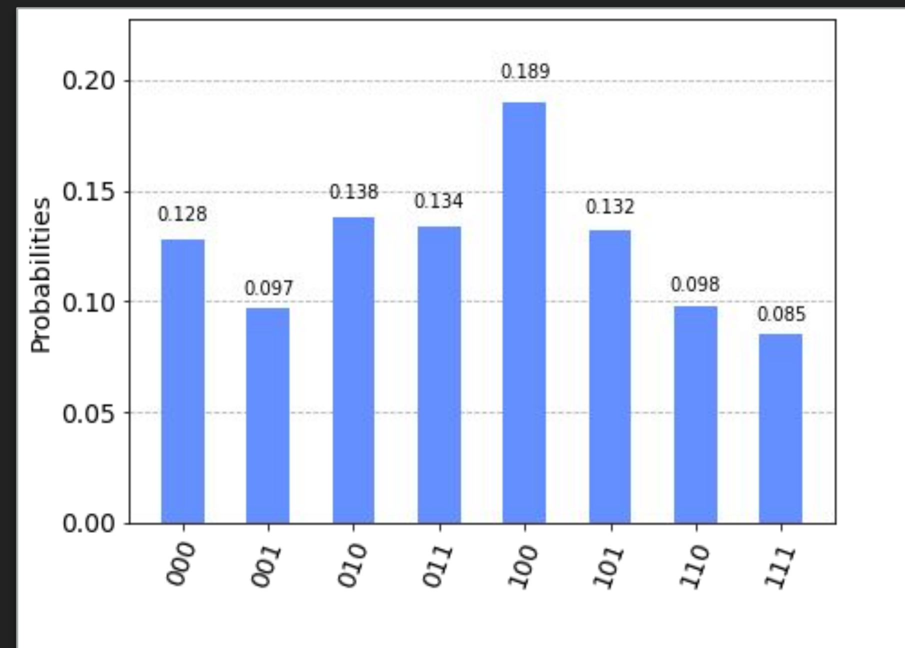
Effect of noise on real devices

- In order to understand the deviation from expected results when run on ibmq_london and other hardware, we simulated the same computation process.
- qasm_simulator extracts noise models from the individual quantum devices.
- When we removed coupling between the qubits the gaussian computed looked better, or at least more symmetric.
- The more qubits we used, the <u>worse</u> the actual and simulated results were; this is because there are more CNOTs used in higher-qubit calculations, so the results were noisier.

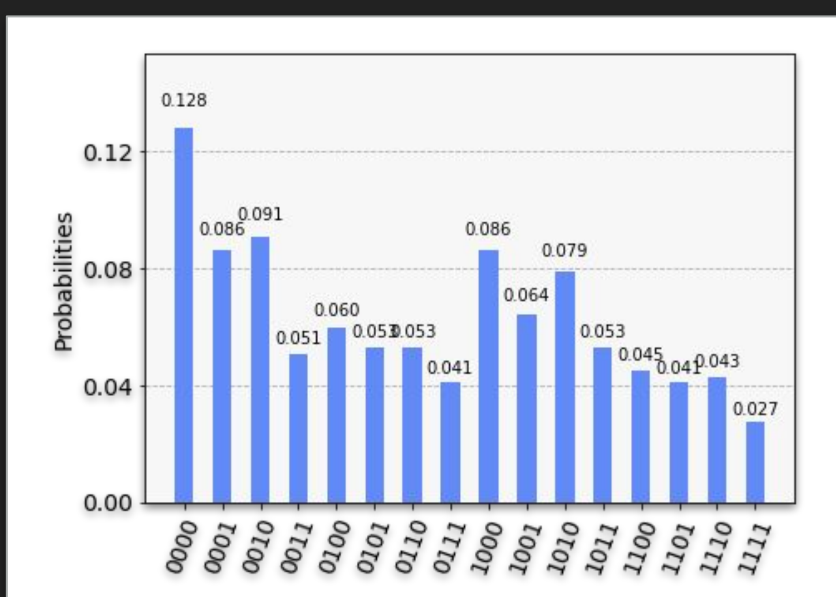
Creating Gaussian on Real Devices (3 & 4 qubits)

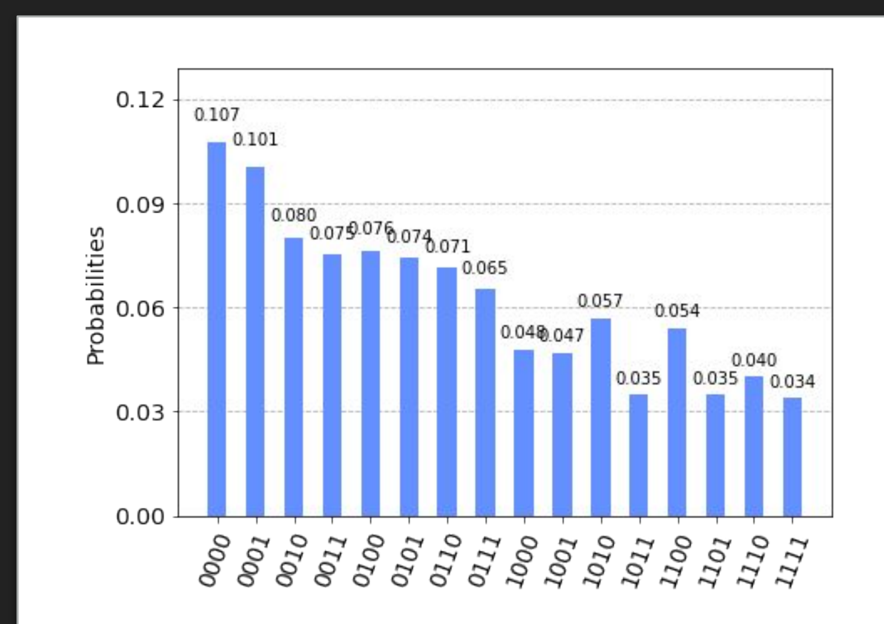


Run on ibmq_london



Simulated with ibm_london noise model on QASM





- The top row is done with 3 qubits, the bottom with 4.
- The left is run on the actual hardware, the right is simulated to perform like that hardware.
- More qubits make the shape worse, due to the larger amount of CNOT gates.

Computation Against Theory

- Comparing to the expected gaussian calculated purely from theory

$$f(x,\sigma) = Ae^{\frac{-(x-x_0)^2}{2\sigma^2}}$$

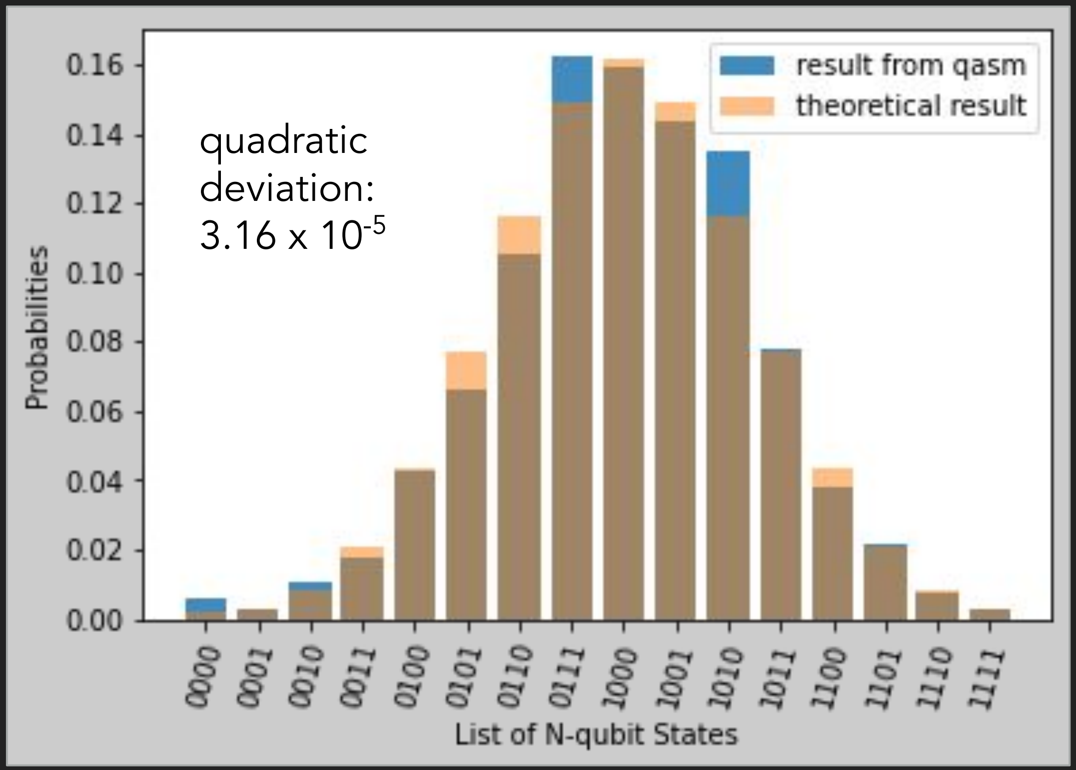
  - Where x is the qubit state, $|x_0\rangle$ is the center state $|100\ldots\rangle$, A is a normalization factor, and sigma controls the width

- To the right we have a simulation on QASM
- Below we have two runs on real hardware against the expected gaussian showing a skew towards the $|00\ldots\rangle$ state

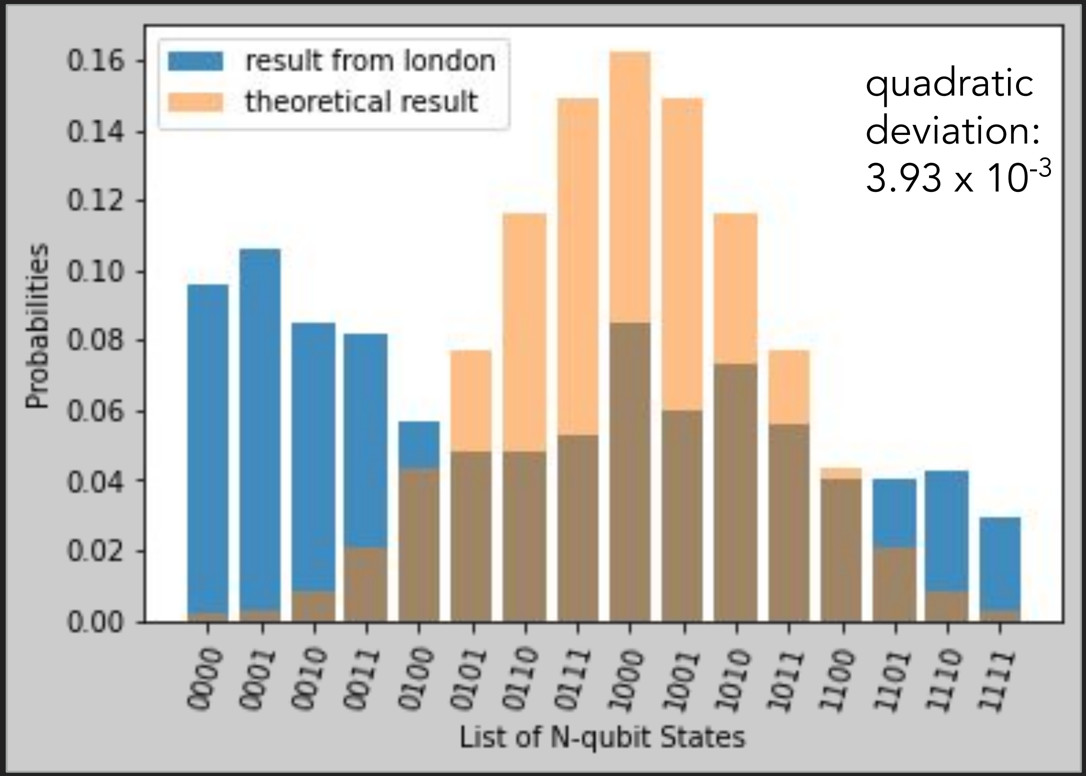
$$\text{quadratic deviation} = \sum_{i=1}^{N} \frac{(x_i - y_i)^2}{N}$$

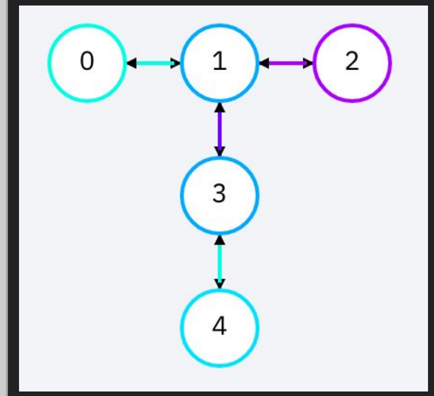
  - Where x_i/y_i are the data/fit

- 4 Qubit QASM simulation (1024 shots, noiseless)

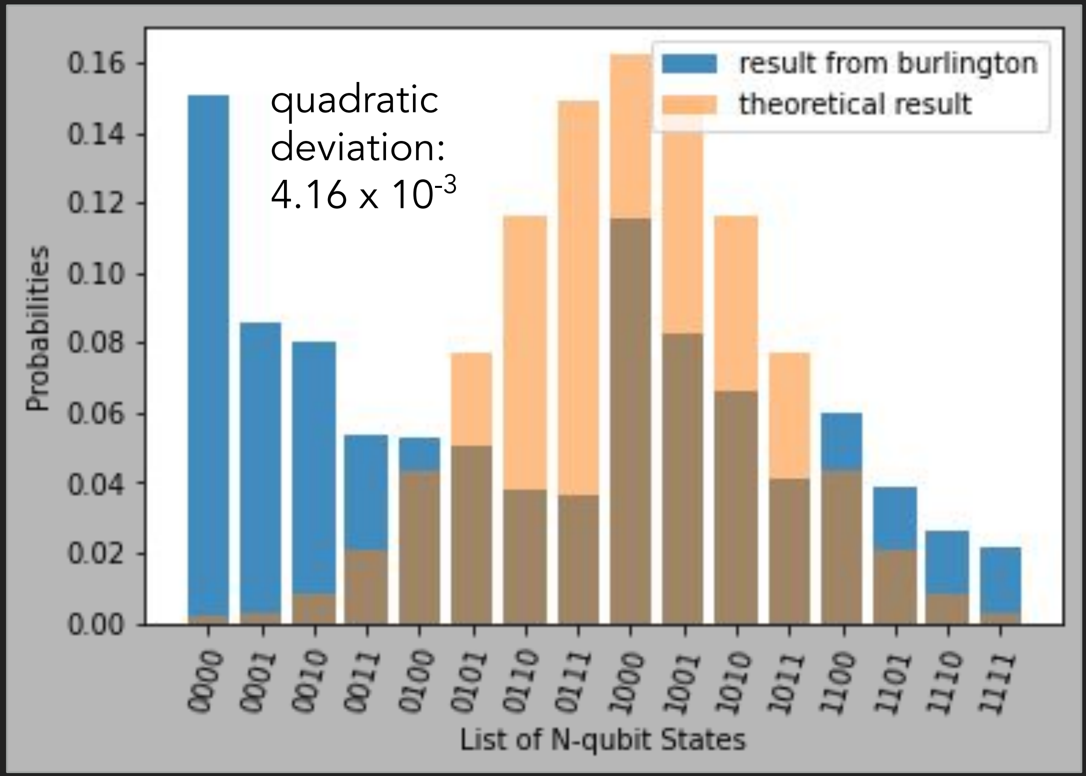


- ibmq_london (1024 shots)





- ibmq_burlington (1024 shots)



Conclusions About Noise in State Generation

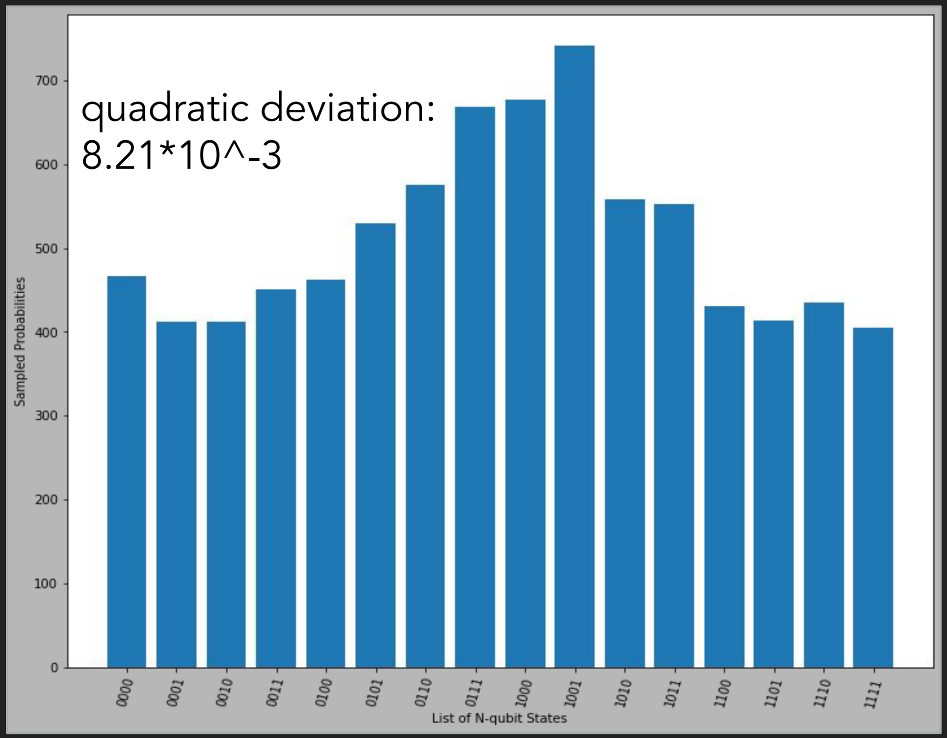
- We implemented a function to generate a highly entangled gaussian wave function on qiskit hardware

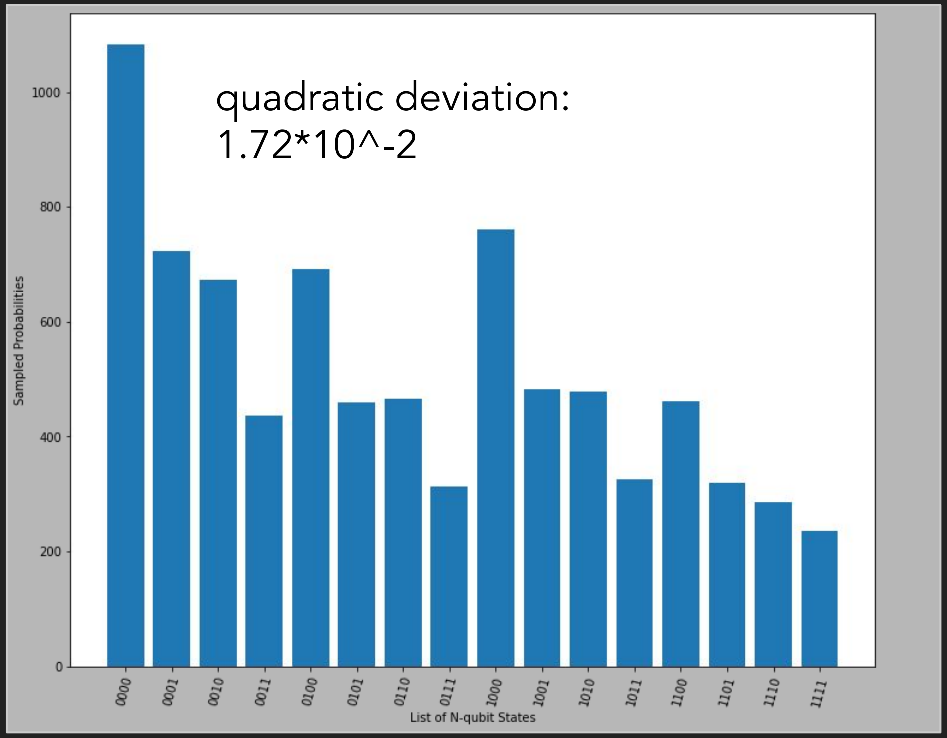
- Noisy simulations and chips result in skewed distributions, which seem configuration-dependent, as well as affected by N.
  - N > 3 is when the asymmetry becomes manifestly compounded by the number of CNOTs in the circuit

- We began isolating <u>how certain types of noise distort the Gaussian.</u>

- For the top right graph, this is a custom QASM simulation with only depolarizing error. We observe this error resulting in a broadened gaussian.

- For the bottom right graph, this is a custom QASM simulation with only bit-reset errors. We observe a leftward skew in the simulation with higher probability to default to |0⟩.





Next Steps

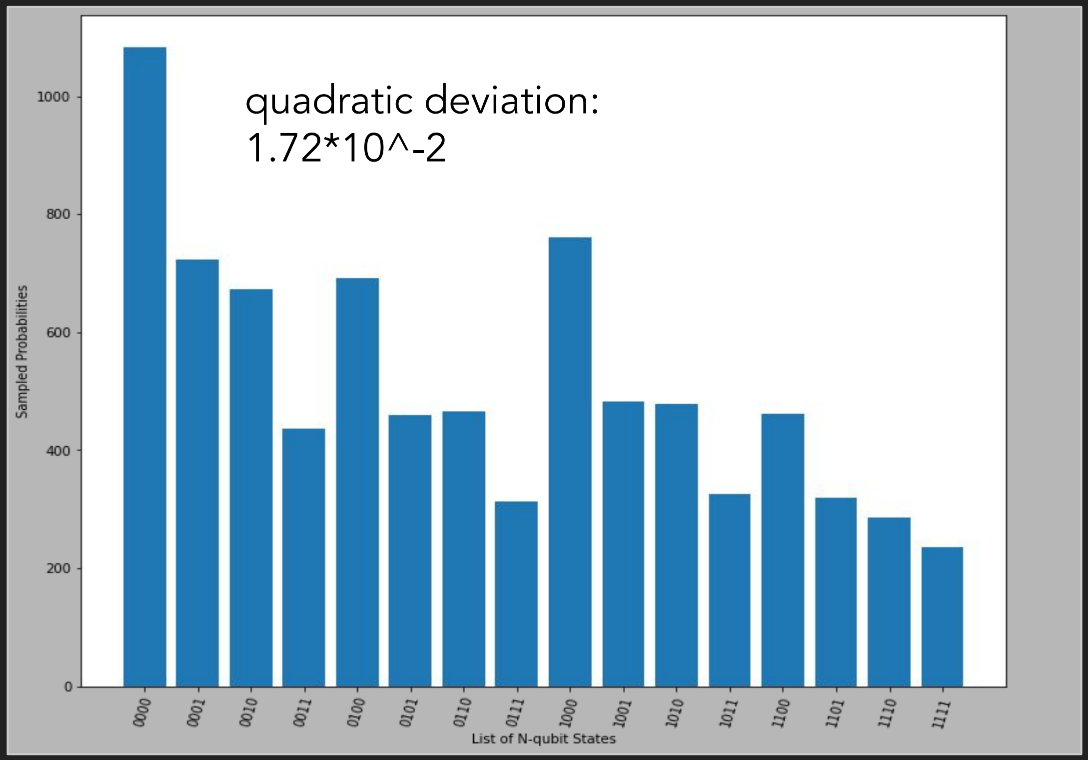
- The behavior of the QASM simulation under bit-reset errors seems to be more dominant than the broadening seen from depolarization in the results we find for ibm_burlington for example.

- <u>If we know the sources of error, we can obtain better calculations [1] and extend the power of noisy real hardware.</u>

- We began to explore custom graph connection configurations and would have that as a next question in this analysis.
  - That is, <u>which configuration of qubits minimizes error?</u>

[1] Kandala, A., Temme, K., Córcoles, A.D. et al. Error mitigation extends the computational reach of a noisy quantum processor. Nature 567, 491–495 (2019). https://doi.org/10.1038/s41586-019-1040-7

- QASM bit-reset simulation



- ibm_burlington