Pearson Type IV curve fit to the frequency domain

Adam Lu

March 22, 2018

Let's read in the data

```r
# Load the readxl library
library(readxl)

# Read in data as a data frame
fsp <- read_xlsx("17217_frequency_spectrum.xlsx")
fsp
```

```
## # A tibble: 149,970 x 2
##        freq      amp
##       <dbl>    <dbl>
##  1 1.033330 0.250259
##  2 1.066663 0.482672
##  3 1.099996 0.856958
##  4 1.133330 0.170223
##  5 1.166663 0.054996
##  6 1.199996 0.109106
##  7 1.233329 0.624864
##  8 1.266662 0.463502
##  9 1.299996 0.468146
## 10 1.333329 0.901121
## # ... with 149,960 more rows
```

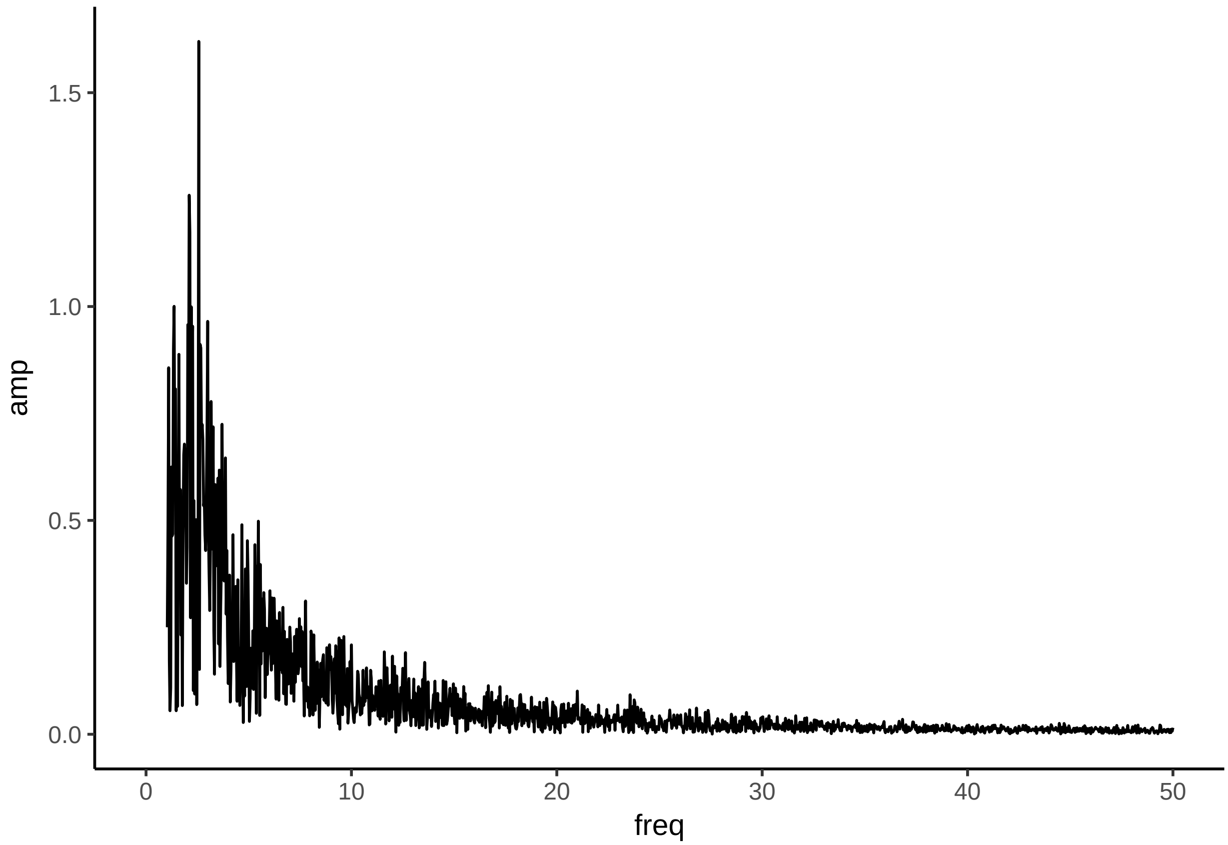
Let's plot the data

```r
# Load the dplyr and ggplot2 libraries
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
library(ggplot2)

# Plot the data as a line
ggplot(aes(freq, amp), data = fsp) +
    geom_line() + xlim(0, 50) + theme_classic()
```

```
## Warning: Removed 148500 rows containing missing values (geom_path).
```



To fit to a density function, we first need to calculate the **area under the curve**

```r
# Load the DescTools library
library(DescTools)

# Calculate the area under the curve
area <- AUC(x = fsp$freq, y = fsp$amp)
area
```

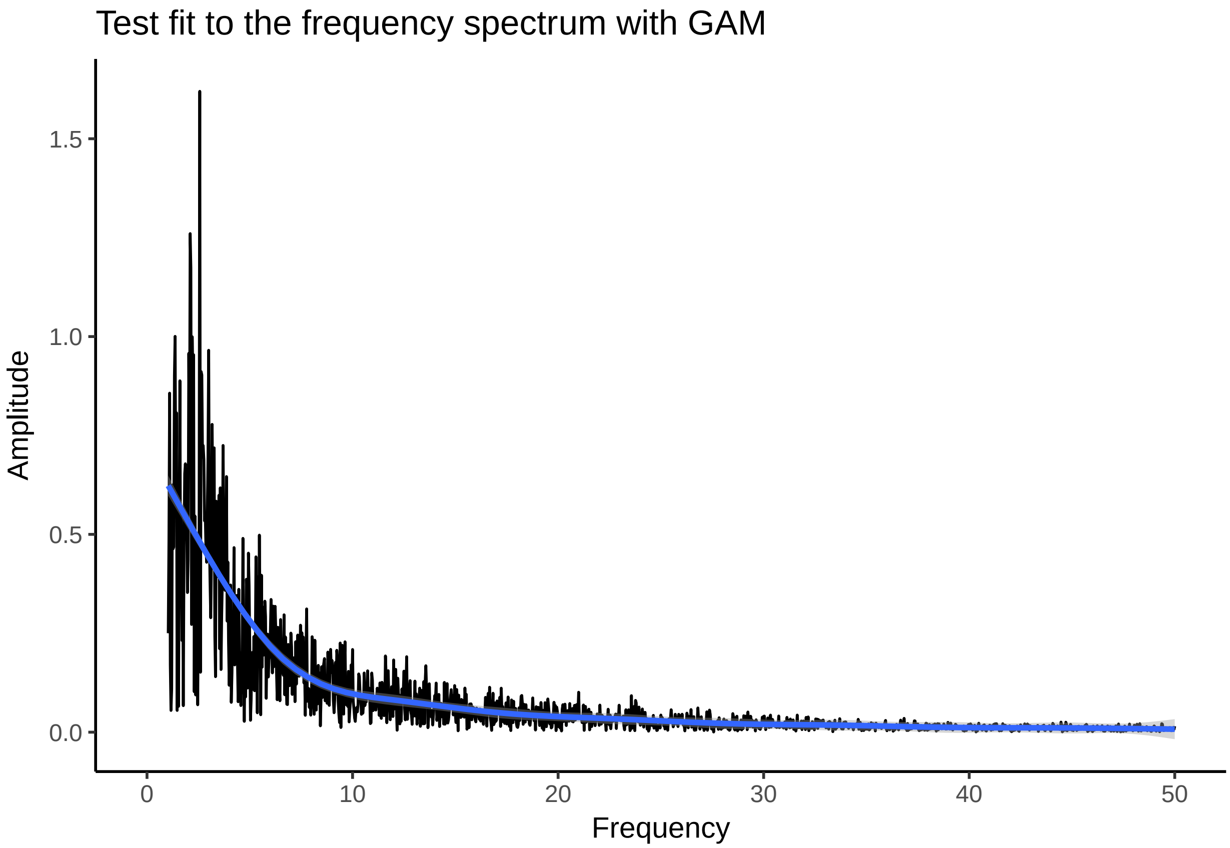
```
## [1] 4.915767
```

We want to fit to a **Pearson Type IV density function**. Let's first try to use `geom_smooth()` (which uses the generalized additive model (gam) method):

```r
# Load required libraries
library(gsl)
library(PearsonDS)

# Define the formula for curve fitting
pearson4Curve <- amp ~ area * dpearsonIV(freq, m, nu, location, scale)

# Plot the data with a fitted curve
ggplot(aes(freq, amp), data = fsp) +
    geom_line() + xlim(0, 50) + theme_classic() +
    geom_smooth(formula = pearson4Curve) +
    ggtitle("Test fit to the frequency spectrum with GAM") +
    xlab("Frequency") + ylab("Amplitude")
```

```
## `geom_smooth()` using method = 'gam'
```

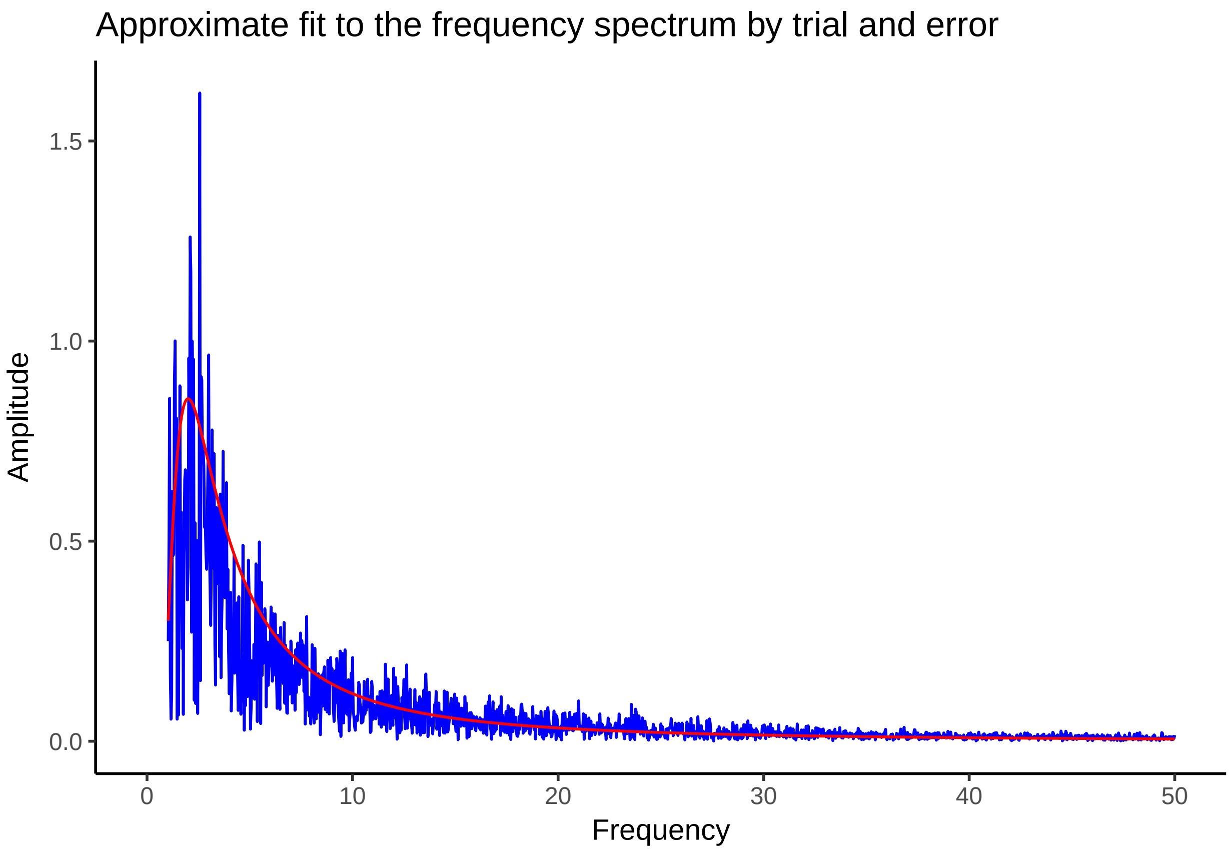
```
## Warning: Removed 148500 rows containing non-finite values (stat_smooth).

## Warning: Removed 148500 rows containing missing values (geom_path).
```



Since it didn't work very well, let's first use trial and error to find a Pearson Type IV density curve that approximately matches our data:

```r
# Add a column for the predicted amplitude values
fsp <-
    fsp %>%
    mutate(ampPredicted = area *
            dpearsonIV(freq, m = 1, nu = -6, location = 0.5, scale = 0.5))

# Plot the data with the Pearson Type IV curve
ggplot(aes(freq, amp), data = fsp) +
    geom_line(color = 'Blue') + xlim(0, 50) + theme_classic() +
    geom_line(aes(freq, ampPredicted), color = 'Red') +
    ggtitle("Approximate fit to the frequency spectrum by trial and error") +
    xlab("Frequency") + ylab("Amplitude")
```

```
## Warning: Removed 148500 rows containing missing values (geom_path).

## Warning: Removed 148500 rows containing missing values (geom_path).
```



Now we have initial guesses for the 4 parameters, let's use **nonlinear least squares** method to get a better fit to the data:

```
# Use nonlinear least squares to do curve fitting
model <-
    nls(formula = pearson4Curve, data = fsp,
        start = list(m = 1, nu = -6, location = 0.5, scale = 0.5))
model
```

```
## Nonlinear regression model
##   model: amp ~ area * dpearsonIV(freq, m, nu, location, scale)
##    data: fsp
##        m       nu location    scale
##   0.7343  -0.9472   1.3447   1.1341
##  residual sum-of-squares: 9.375
##
## Number of iterations to convergence: 24
## Achieved convergence tolerance: 9.677e-06
```

Finally, plot the data with the refined Pearson Type IV model

```
# Add a column for the predicted amplitude values
fsp <-
    fsp %>%
    mutate(ampPredicted = area *
            dpearsonIV(freq, m = 0.7343, nu = -0.9472,
                       location = 1.3447, scale = 1.1341))

# Plot the data with the Pearson Type IV curve
```

```r
ggplot(aes(freq, amp), data = fsp) +
    geom_line(color = 'Blue') + xlim(0, 50) + theme_classic() +
    geom_line(aes(freq, ampPredicted), color = 'Red') +
    ggtitle("Best fit to the frequency spectrum by nls") +
    xlab("Frequency") + ylab("Amplitude")
```

```
## Warning: Removed 148500 rows containing missing values (geom_path).

## Warning: Removed 148500 rows containing missing values (geom_path).
```