DEFINITION

Norm on a Linear Space
Normed Space

DEFINITION

Inner Product

DEFINITION

Linear Transformation/Operator

A real-valued function $||x||$ defined on a linear space X, where $x \in X$, is said to be a *norm on* X if

Positivity $||x|| \geq 0$,

Triangle Inequality $||x + y|| \leq ||x|| + ||y||$,

Homogeneity $||\alpha x|| = |\alpha| \, ||x||$, α an arbitrary scalar,

Positive Definiteness $||x|| = 0$ if and only if $x = 0$,

where x and y are arbitrary points in X.

A linear/vector space with a norm is called a *normed space*.

Let X be a complex linear space. An *inner product* on X is a mapping that associates to each pair of vectors x, y a scalar, denoted (x, y), that satisfies the following properties:

Additivity $(x + y, z) = (x, z) + (y, z)$,

Homogeneity $(\alpha \, x, y) = \alpha(x, y)$,

Symmetry $(x, y) = \overline{(y, x)}$,

Positive Definiteness $(x, x) > 0$, when $x \neq 0$.

A transformation L of (operator on) a linear space X into a linear space Y, where X and Y have the same scalar field, is said to be a *linear transformation (operator)* if

1. $L(\alpha x) = \alpha L(x), \forall x \in X$ and $\forall$ scalars α, and
2. $L(x_1 + x_2) = L(x_1) + L(x_2)$ for all $x_1, x_2 \in X$.