Lab 3 - Continue Modeling MotorLab System

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1 Introduction

2 Dynamic Model with Spring Added

$$T = J\ddot{\theta}(t) \ + b\dot{\theta}(t) + k_s\theta(t)$$

3 MotorLab Coefficients

3.1 Coefficients Needed to be Found

- ζ ('zeta' or 'damping ratio')
- w_n ('natural frequency')
- k_s ('spring constant')

3.2 How to Find k_s

- Like Lab 2, we will be looking at steady state
- Unlike Lab 2, we have a spring attached. This means we have a constant position opposed to constant velocity like Lab 2 when we apply amperage to the motor

$$T(t) = k_t i(t) = J\ddot{\theta}(t) \ + b\dot{\theta}(t) + k_s\theta(t)$$

$$T(t) = k_t i(t) = J\cancelto{0}{\ddot{\theta}(t)} + b\cancelto{0}{\dot{\theta}(t)} + k_s\theta(t)$$

$$k_t i(t) = k_s\theta(t)$$

$$k_s = k_t\frac{i(t)}{\theta(t)} = \frac{T(t)}{\theta(t)}$$

- This means we can find k_s from the slope of $T(t)$ and $\theta(t)$
- We will find theta by commanding current to the motor and then measuring the position of the motor shaft (Demonstrated in Lab)
- To estimate k_s, the data should come out to be linear, so all you have to do is estimate the slope to find k_s

  You can play around with k_s manually to estimate slope

  You can export data to excel and use linear regression

  You can use the 'Normal Equation'

$$^{1}k_s = (\boldsymbol{\theta}\boldsymbol{\theta}^{\boldsymbol{T}})^{-1}\boldsymbol{\theta}\boldsymbol{T}^{\boldsymbol{T}}$$

[1] $\boldsymbol{\theta}$ and $\boldsymbol{T}$ defined already in code after you have ran the experiment and put in values for position

3.3 How to Find ζ - Logarithmic Decrement

- Log. Decrement is used for finding the damping ratio of an underdamped system
- Defined as the natural log of the ratio of amplitude of two peaks, n periods apart, namely $ln(P_i/P_{i+n})$
- Will us this ratio to estimate the damping ratio
- You will need to find three points using data cursors in MATLAB (How to do this will be covered)

3.4 Finding w_n, T_{osc}, w_d, b

- We can find rest of the coefficients of the MotorLab by equating a standard 2^{nd} order system with our model

$$G(s) = \underbrace{\frac{k_t}{Js^2 + bs + k_s}}_{\text{Motorlab Model}} = \underbrace{\frac{k_{dc}w_n^2}{s^2 + 2\zeta w_n s + w_n^2}}_{\text{Standard } 2^{nd} \text{ Order System}}$$

- Equate the coefficients of the two equations to solve for w_n, b, etc.
- REMEMBER: THE COEFFICIENTS ARE NOT EQUIVILENT UNTIL YOU TAKE CARE OF J IN FRONT OF s^2 IN THE MOTORLAB MODEL!

4 Final Thoughts

- Make sure you've read the entire lab. It has information that will help you finish this lab and fill out the code
- We are looking for numeric answers for the coefficients table, not equations
- Have the instructors sign off the questions before you leave