

DEVELOPMENT AND FEASIBILITY OF OPEN-SOURCE HARDWARE  
AND SOFTWARE IN CONTROL THEORY APPLICATION

by

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# Abstract

Control theory is a methodology investigated by many mechanical and electrical engineering students throughout most universities in the world. Because of control theory's broad and interdisciplinary nature, it necessitates further study by application through laboratory practice. Typically the hardware used to connect the theoretical aspects of controls to the practical can be expensive, big, and time consuming to the students and instructors teaching on the equipment. This is due to the fact that connecting various hardware components such as sensors, encoders, amplifiers, and motors can lead to data that does not fit perfectly the theoretical mold developed in the controls classroom, further dissuading students of the idea that there exists a connection between developed theoretical models and what is seen in practice.

There is a recent trend in universities wishing to develop open-source, inexpensive hardware for various applications. This thesis will investigate and conduct a multitude of experiments on an apparatus known as the Motorlab to determine the feasibility of such equipment in the field of control theory application. The results will be compared against time-tested hardware to demonstrate the practicality of open-source, inexpensive hardware.

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# Acronyms

<b>ARM</b>	Advanced RISC Machine
<b>DAEC</b>	Dynamic Angle Error Compensation
<b>MPU</b>	Microprocessor Unit
<b>BLDC</b>	Brushless DC
<b>GUI</b>	Graphical User Interface
<b>NERMLAB</b>	New Earth Robotics Motor Lab
<b>back-emf</b>	back electromotive force
<b>RPM</b>	Rotations Per Minute



# Acknowledgments

Enter the text for your Acknowledgements page in the `acknowledge.tex` file. The Acknowledgements page is optional. If you wish to remove it, see the comments in the `etdrtemplate.tex` file.

# Chapter 1

## Introduction

Current research indicates a growing need for laboratory components for introductory control theory classes. However, many hurdles like budget, class size, and space limitations arise when laboratories are added to lecture components in universities [1]. This thesis will address the issues, like budget of laboratory hardware, class sizes, and space limitations, and try to assess the feasibility of utilizing low budget, smaller, and portable laboratory hardware for introductory control classes. The introduction of this thesis will be broken up into sections to address these issues individually. It's important to note, that while this thesis does recognize the importance of having laboratory components to control theory lectures, it will not be the main focus, rather importance will be given to addressing the feasibility of low budget portable hardware, more specifically the Motorlab, for control applications. This chapter serves more as a background to why lower budget, portable hardware is important to control classes, and in turn demonstrates a need for these low budget devices.

### 1.1 Hardware and Software Budget

### 1.2 Space Limitations

# Chapter 2

## Apparatus

Two pieces of apparatus were used to conduct the experiments in this thesis. This chapter will detail the purpose, design and recreation of the equipment. Section 2.1 will cover the new Motorlab, including the hardware implementation, design of components, and basic functionality. Section 2.1.3 will detail how a new type of position sensor works that is used for the position measurements of the Motorlab. Then, the older Motorlab will be discussed and compared to the new Motorlab in section 2.2.

### 2.1 New Motorlab

The new Motorlab is a reimplementation of older laboratory hardware created by Dr. Schin-stock and Dr. White for Control of Mechanical Systems I at Kansas State University. The Motorlab allows users to connect the theoretical ideas of control theory with those in practice. (Maybe include applications of the motorlab and its use in the laboratory).

#### 2.1.1 Hardware

The new Motorlab consists of several key pieces of hardware, namely a Microprocessor Unit (MPU), motor driver, and a Brushless DC (BLDC) motor. The main MPU of the Motorlab is the STM32 Nucleo, which allows Arduino attachment shields and other STM boards to

be attached for added functionality. For the purposes of the Motorlab, a motor driver was required to drive a brushless DC motor, namely a RCTIMER GBM2804. An X-Nucleo-IHM07M1 (a three-phase brushless DC motor driver) was selected to be the primary driver for the Motorlab.

### 2.1.2 Position Sensor

The main purpose of the Motorlab is to conduct control laboratory experiments, as a result, feedback via sensor readings is necessary to do such control. The typical way to do position and speed control of mechanical systems and motors is to use position feedback via an encoder. An encoder is a device that converts angular position of a motor shaft to an analog or digital signal that can be processed by an MPU. In the case of the Motorlab, an on-axis magnetic encoder is used to do position feedback. Special equipment had to be designed in order to use this type of encoder, and will be detailed in section 2.1.3.

The encoder that is being used on the Motorlab consists of 14-bit on-axis magnetic rotary position sensor chip, specifically the AS5047D by AMS <sup>1</sup>. The position sensor chip provides high resolution absolute angle measurements through a full 360 degree range <sup>2</sup>. In addition to the fast absolute angle measurement system that the position sensor provides, it also has Dynamic Angle Error Compensation (DAEC) that provides position control systems with near 0 latency [2].

The AS5047D chip is a magnetic sensor that utilizes the Hall-effect. The chip works by taking the Hall sensors and converting the perpendicular magnetic field on the surface of the chip to a voltage. The voltage signals are filtered and amplified in order to calculate the angle of the magnetic vector. In order for position measurements to be taken, a small diametrically opposed magnet must be placed on the shaft of the equipment being measured. The magnet and AS5047D are contactless, meaning there is a small air gap between the chip and magnet. As the magnet rotates above the chip (Figure 2.1), angle measurements are calculate and

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<sup>1</sup>AMS is an Austrian analog sensor and semi-conductor manufacturer

<sup>2</sup>These chips typically provide a maximum resolution of 2000 steps/revolution in decimal mode and 2048 steps/revolution in binary mode

transmitted through the chip [2]. The Motorlab uses the AS5047D chip primarily as a position and speed control system.

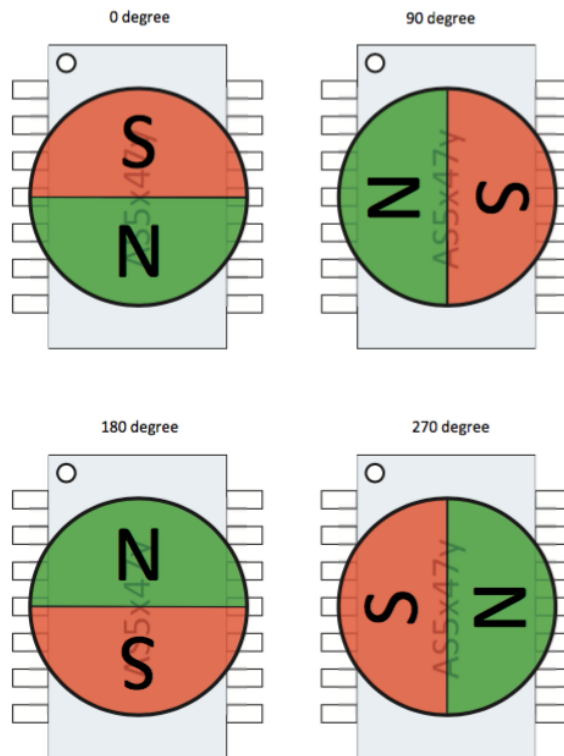


Figure 2.1: Magnet and AS5047D [2]

### 2.1.3 Motorlab Parts

Along with the hardware mentioned in section 2.1.1, three components were needed to be developed in order to bring the Motorlab to fruition: a printed circuit board that houses the on-axis magnetic rotary position sensor, a spacer to put distance between the circuit board and the motor, and a magnet holder, which holds one diametrically opposed magnet<sup>3</sup>. Both the spacer and magnet holder which can be seen in figure 2.2 had to be 3D printed in order to achieve the required specifications of the apparatus setup. Detailed drawings of these two parts can be found in Appendix B, figures B.1 and B.2, if reproduction is desired. Along

<sup>3</sup>Diametrically opposed meaning the north and south poles of the magnet are in-plane as opposed to top/bottom poles. Reference figure 2.1 for further clarification

with the two 3D printed parts, a printed circuit board had to be designed by Eric Patterson of Kansas State University to allow the position sensor to communicate with the rest of the hardware.

Because of variability in resolution of current 3D printers, care was given to the design of the magnet holder <sup>4</sup>. A spline was used for both the shaft of the magnet holder and the section that holds the magnet itself. The spline allowed for greater tolerances in the parts, meaning the magnet holder could be easier to press fit into the motor, and likewise allowed easier removal of the diametrically opposed magnet.

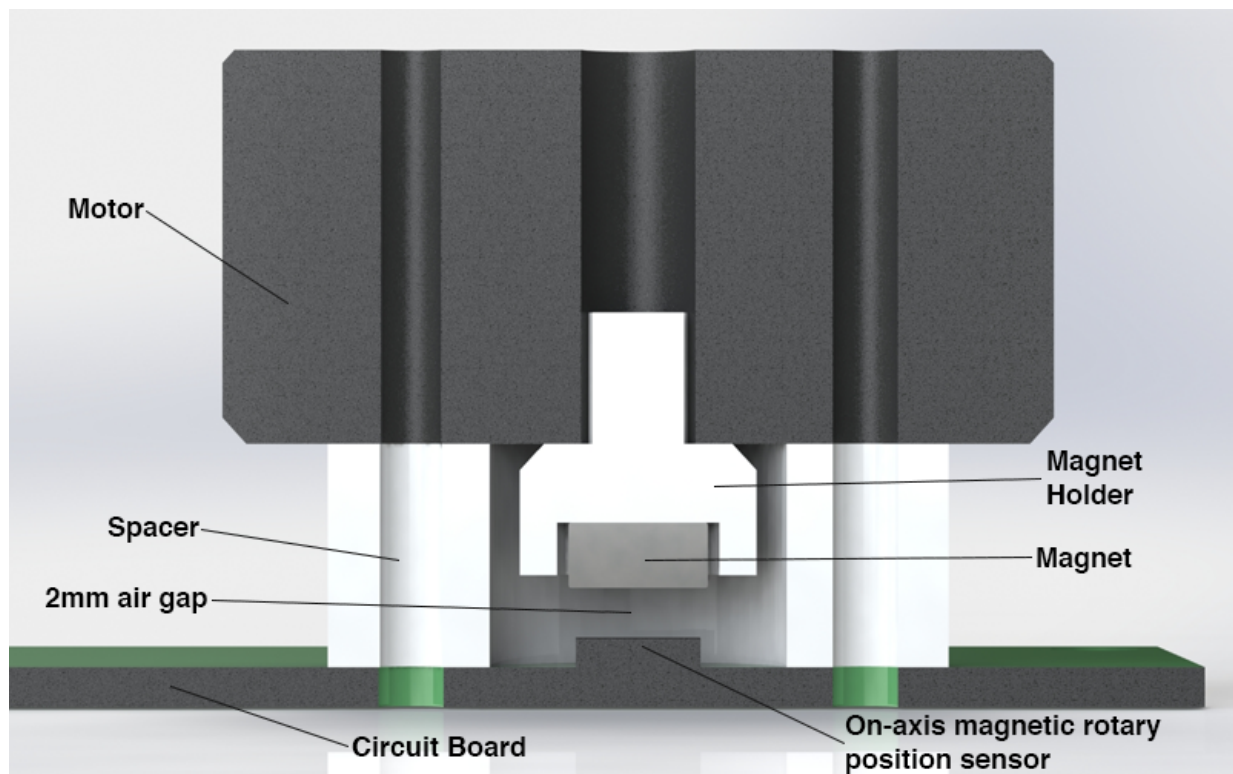


Figure 2.2: Section View of Motorlab Assembly

## 2.1.4 Motorlab Cost

Text here.

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<sup>4</sup>Because of this variability in resolution, the magnet holder was printed in iterations, varying the diameter of the spline to insure a tight fit in the motor shaft

Table 2.1: Motorlab expenditure report

Component	Brand/Manufacture	Cost
BLDC Motor	RCTIMER GBM2804	11.94 USD
Position Sensor	AS5047D AMS	4.21 USD
ST32 Nucleo	STMicroelectronics	10.12 USD
X-Nucleo-IHM07M1	STMicroelectronics	9.80 USD
Magnet	-	3.00 USD
Printed Circuit Board	-	30.00 USD
	<b>TOTAL COST</b>	<b>69.07 USD</b>

### 2.1.5 Motorlab GUI

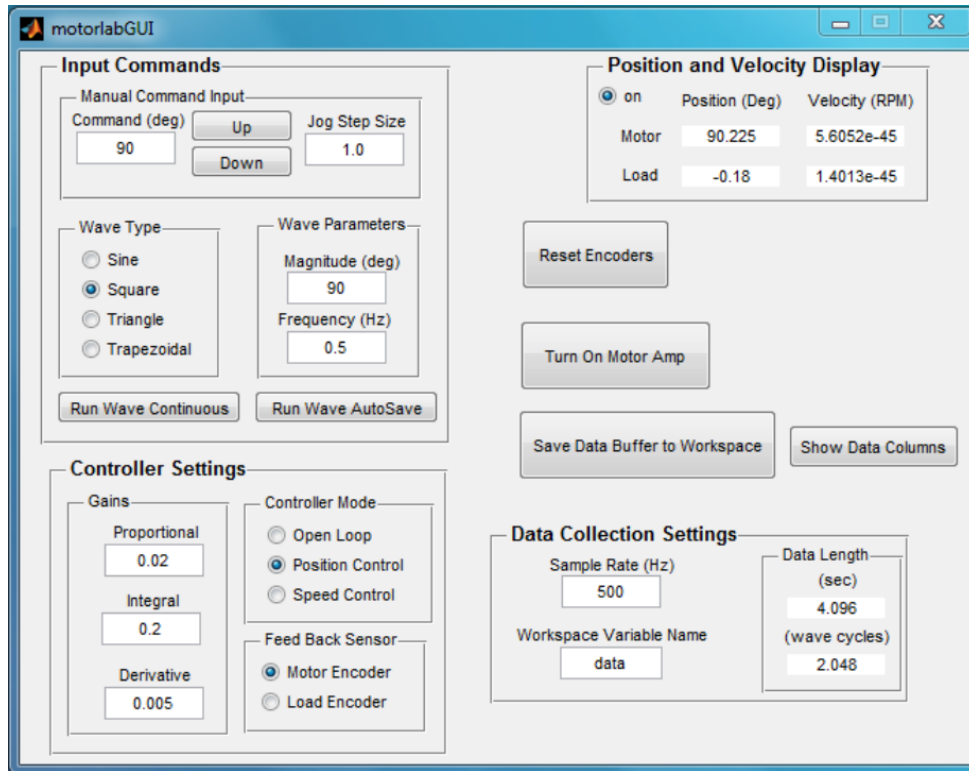


Figure 2.3: Motorlab GUI in MATLAB

The Motorlab interfaces with a Graphical User Interface (GUI) coded in MATLAB to allow users to run various laboratory experiments on the hardware. It allows the selection of various wave types, frequency, controller gains, and sample rate that get sent to the Motorlab. After the parameters of the experiment are setup, the GUI can run the Motorlab,

which in turn sends the experimental data to the workspace of MATLAB in the form of a matrix. The MATLAB GUI can be seen in figure [2.3](#).

## 2.2 Motorlab

The Motorlab is a piece of laboratory equipment developed by Dr. Dale Schinstock and Dr. Warren N. White at Kansas State University. It has been in service at the university for over 15 years, and as a result is a time-tested piece of laboratory hardware that has proven to be reliable in terms of providing quality practical control theory application to students enrolled in the class Control of Mechanical Systems I. Because the Motorlab's hardware components were designed and created to insure a very clear translation of laboratory results, in addition to the Motorlab having a much larger operating limit and bandwidth than students use in laboratory practice, this represents to a good base model to compare the results of the apparatus in this thesis too.

Various hardware make up the Motorlab, namely, a high quality BLDC motor, BLDC servo amplifier by Copley Controls Corp., and a ST Discovery board <sup>5</sup>. Typically the Motorlabs run a cost of about 700 USD per lab station [\[3\]](#).

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<sup>5</sup>STMicroelectronics is a Switzerland based micro-controller manufacturer



# Chapter 3

## Model Development

Chapter 3 will be dedicated to developing the various parameters that make up the NERMLAB such as the motor torque constant, back electromotive force ([back-emf](#)), inductance, and max voltage. Each section in Chapter 3 will detail the process of how the various parameters were measured, calculated, and experimentally determined. Nomenclature for various constants and parameters are detailed in the table [3.1](#).

Table 3.1: Motor parameters

Parameter	Description
V	Motor Voltage
$k_t$	Motor Torque Constant per Phase
$k_T$	Overall Motor Torque Constant
$K_e$	Back Electromotive Force Constant per Phase
$K_{e,LL}$	Line-Line Back Electromotive Force Constant
J	Mass Moment of Inertia
L	Motor Inductance
R	Motor Phase Resistance
$R_{LL}$	Motor Line-Line Resistance
$\tau$	Time Constant
T	Motor Torque
$\omega_m$	Motor Speed

## 3.1 Motor Resistance

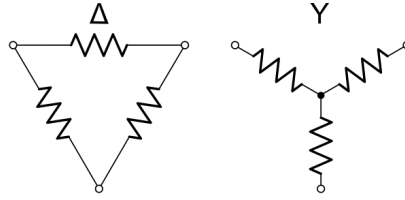


Figure 3.1: Motor Connection Configuration

BLDC motors are typically connected in two wiring configurations, WYE (Y) or delta ( $\Delta$ ) which can be seen in figure 3.1. The RCTIMER GBM2804 utilizes the WYE (Y) configuration and will be analyzed as such. Due to the wiring of WYE systems, the neutral connection is typically unavailable for measurement on most motors. As a result it is common to measure resistance by a line-line reading. However in terms of motor control it is the phase resistance and not the line-line resistance that is of importance. Converting between the phase and line-line resistance is quite simple and can be done by dividing the line-line resistance by two (equation 3.1).

$$R = \frac{R_{LL}}{2} \quad (3.1)$$

### 3.1.1 Procedure and Results

In order to gather a good estimate for the phase resistance of the RCTIMER GBM2804 motor, the resistance was measured line-line across all 3 phases. Each set of motor leads were hooked up to a digital multimeter and the values were tabulated for each phase component in table 3.3. An average was then calculated between the different line-line resistances to get the overall resistance of the motor.

Using equation 3.1 it is then possible to find the overall phase resistance of the motor.

$$R = 4.955 \approx 5\Omega$$

Table 3.2: Measured motor resistance

A-B	A-C	B-C
9.870 $\Omega$	9.900 $\Omega$	9.950 $\Omega$
	Average:	9.91 $\Omega$

## 3.2 Motor Torque Constant and Back EMF

The motor torque constant ( $k_t$ ) is a common parameter used in BLDC motors. It relates the armature current to the torque produced by a motor:  $T = k_t i$ . Many methods exist to determine the torque constant, including relating the motor velocity constant  $k_v$  which is inversely related to the torque constant by  $k_T = \frac{1}{k_v}$ , or by measuring the line-line back-emf voltage per phase ( $K_e$ ).  $K_e$  is the peak value of the back-emf per angular velocity measured from line-neutral. However since line-neutral is typically unavailable on most BLDC motors, the back-emf constant is often represented as a line measurement,  $K_{e,LL}$ . The overall torque constant can then be related to the line measurement back-emf voltage for sinusoidal type outputs by equation 3.2 or for trapezoidal outputs by equation 3.3. [4].

$$k_T = \frac{\sqrt{3}}{2} K_{e,LL} \quad (3.2)$$

$$k_T = K_{e,LL} \quad (3.3)$$

Because  $K_{e,LL}$  can be experimentally determined, it is possible to find the overall motor torque constant for a BLDC motor. One simply needs to measure the line-line sinusoidal or trapezoidal back-emf voltage at various speeds to get a good estimate of  $K_{e,LL}$ . With equation 3.2 or 3.3,  $k_T$  can then be determined.

### 3.2.1 Procedure and Results

In order to calculate the back-emf of the RCTIMER GBM2804 BLDC motor an experiment had to be set up to measure the voltage generated by the motor. Three pieces of equipment

were needed: an oscilloscope, the Motorlab, and a torque transmission shaft. The torque transmission shaft was a 3D printed part<sup>1</sup> that allowed the Motorlab to spin the RCTIMER GBM2804 at a constant speed to generate a back voltage. A line-line voltage (peak-peak) was then read from the leads of the RCTIMER GBM2804 by an oscilloscope<sup>2</sup>. The data collected is tabulated in table 3.3.

Table 3.3: Measured back voltage

Speed (RPM)	Speed $\omega_m$ (rad/s)	Peak-Peak Voltage (V)	Peak Voltage (V)
300	31.41	4.64	2.32
500	52.36	7.60	3.8
1000	104.72	15.1	7.55
1500	157.10	22.0	11.0
2000	209.44	30.0	15.0

There is a fairly linear relationship between the peak voltage and speed. Due to this fact  $K_{e,LL}$  can be approximated from the slope of  $\frac{V}{\omega_m}$ . The normal equation from the least-squares method was employed to find the best fit for the data in table 3.3. Two matrices were constructed from the data, namely  $\mathbf{V}$  and  $\omega_m$ .

$$K_{e,LL} = (\omega_m \omega_m^T)^{-1} \omega_m \mathbf{V}^T \quad (3.4)$$

From equation 3.4, the back-emf constant was found to be:

$$K_{e,LL} = 0.0713 \quad \frac{V \cdot s}{rad}$$

To verify that  $K_{e,LL}$  was the best fit to that data,  $K_{e,LL}$  was plotted against the collected data in figure 3.2.

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<sup>1</sup>Appendix B figure B.3

<sup>2</sup>Appendix A figure A.1

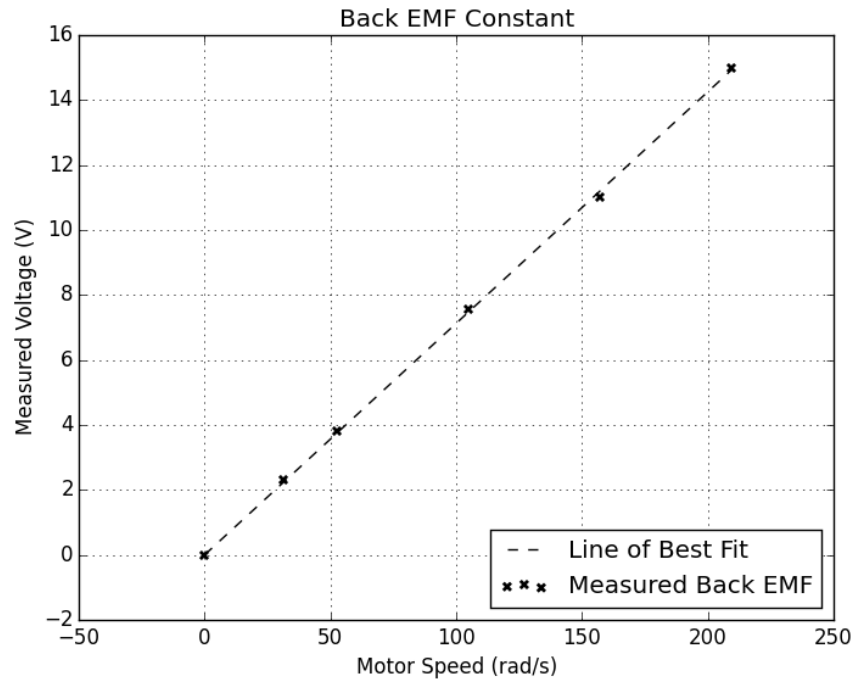


Figure 3.2: Measured Back EMF vs Speed

Since the relationship between  $k_T$  and  $K_{e,LL}$  is known by equation 3.2 and 3.3,  $k_T$  can now be calculated.

$$k_T = 0.0617 \approx 0.06 \quad \frac{N \cdot m}{s} \quad [Sinusoidal]$$

$$k_T = 0.0713 \approx 0.07 \quad \frac{N \cdot m}{s} \quad [Trapezoidal]$$

### 3.3 Mass Moment of Inertia Estimation

Mass moment of inertia  $J$  is the equivalent to mass in a rotational system (commonly referred to as angular mass). More formally it is defined as  $J = \int r^2 dm$ , where  $r$  is the distance to some mass from an axis of rotation.

The angular mass of the NERMLAB will be determined in two ways: experimentally determining  $J$  through software modeling, and approximating  $J$  through mathematical formulation. For both setups the mass of the rotating inertia had to be measured.

#### 3.3.1 Software Modeling of Mass Moment of Inertia

#### 3.3.2 Mathematical Approximation of Mass Moment of Inertia

To simplify the mathematical analysis of the mass moment of inertia calculation of the angular mass of the NERMLAB, an engineering assumption will be made that the angular mass is a rotating ring mass. This assumption is valid for the particular motor used in this thesis due to the fact that most of the mass is concentrated around the outside parameter of the motor. The outside ring mass of the motor contributes the most to the inertial load, so the mathematical formulation would result in the following equation:

$$J_z = \frac{m}{2}(r_1^2 + r_2^2) = mr_2^2(1 - t + \frac{t^2}{2})$$

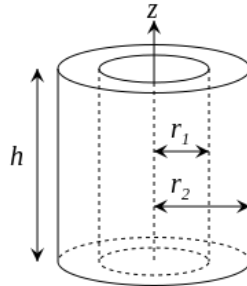


Figure 3.3: Thick Walled Cylinder ( $J$ )

## 3.4 Motor Inductance

$$L = R\tau$$

# Chapter 4

## Experiment One

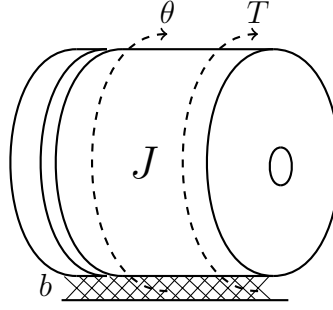
Chapter 4 will experiment with a position control system on the NERMLAB. In this experiment a model of the closed-loop position control system will be developed and will be used to predict the response of the NERMLAB. The main purpose of this experiment is not to develop a better position control system but rather demonstrate the concept of changing pole locations and the characteristic of different response to a changing proportional gain. This will be done by looking frequency of oscillation and decay rate of the oscillations of the various responses. Results produced by the NERMLAB will then be compared against the Motorlab, which is the basis of comparison of all experiments in this thesis.

### 4.1 Mathematical Model of a Closed-Loop Position Control System

To simplify the mathematical formulation the closed-loop current control system is assumed to be much faster than the mechanical dynamics. As a result of this assumption only the mechanical dynamics and controller will be in the model development. The best way to start the formulation is to begin with a time domain differential equation of the NERMLAB system. The NERMLAB is composed of only an angular mass and viscous friction in this experiment which can be seen in figure 4.1.



Figure 4.1: NERMLAB Model



$$T = k_T i(t) = b\dot{\theta}(t) + J\ddot{\theta}(t) \quad (4.1)$$

Taking the Laplace transform of equation 4.1:

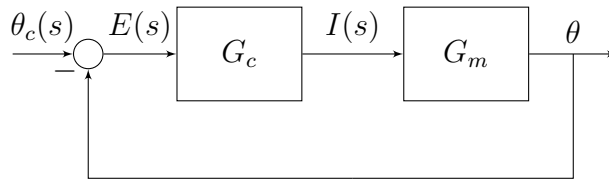
$$k_T I(s) = (bs + Js^2)\theta(s) \quad (4.2)$$

The transfer function can then be developed for  $G_m$  from equation 4.2.

$$\frac{\theta(s)}{I(s)} = \frac{k_T}{Js^2 + bs} \quad (4.3)$$

As for the controller transfer function, a proportional gain  $K_p$  is being used. To further develop the closed loop transfer function, the block diagram in figure 4.2 can be used. It is as simple as doing a block diagram reduction by merging the blocks in series and then performing a feedback calculation.

Figure 4.2: Closed Loop Control System



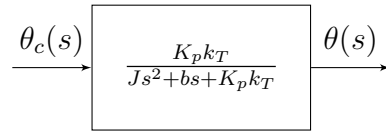
Reducing the blocks in series gives the result:

$$G = G_c G_m = \frac{K_p k_T}{Js^2 + bs} \quad (4.4)$$

The final reduction is performing a feedback calculation:

$$\frac{\theta(s)}{\theta_c(s)} = \frac{G}{1 + G} = \frac{K_p k_T}{Js^2 + bs + K_p k_T} \quad (4.5)$$

Figure 4.3: Block Diagram Reduction



# Chapter 5

## Experiment Two

Chapter 5 will

# Chapter 6

## Experiment Three

# Chapter 7

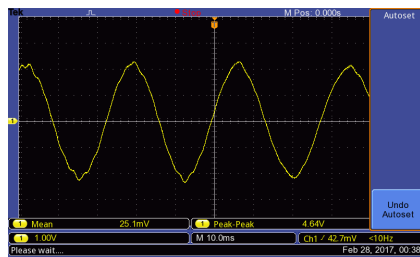
## Experiment Four

# Bibliography

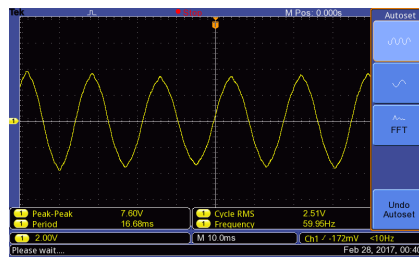
- [1] R. M. Reck and R. S. Screenivas, “Developing a new affordable dc motor laboratory kit for an existing undergraduate controls course,” in *American Control Conference (ACC)*, (Chicago, IL), pp. 2801–2806, 2015.
- [2] AMS, *AS5047D 14-Bit On-Axis Magnetic Rotary Position Sensor with 11-Bit Decimal and Binary Incremental Pulse Count*. AMS, April 2016.
- [3] S. R. Smith, “Demonstrating introductory control systems concepts on inexpensive hardware,” Master’s thesis, Kansas State University, 2017.
- [4] J. R. Mevey, “Sensorless field oriented control of brushless permanent magnet synchronous motors,” Master’s thesis, Kansas State University, 2009.

# Appendix A

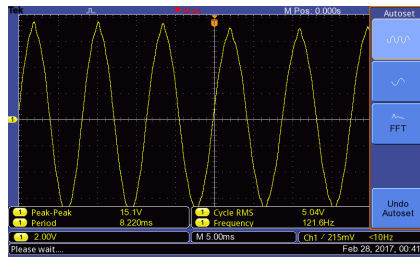
## Experiment Documentation



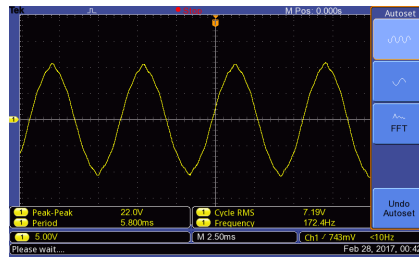
(a) 300 RPM



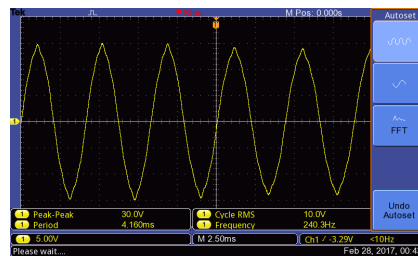
(b) 500 RPM



(c) 1000 RPM



(d) 1500 RPM



(e) 2000 RPM

Figure A.1: Back emf at various motor speeds

# Appendix B

## Part Drawings



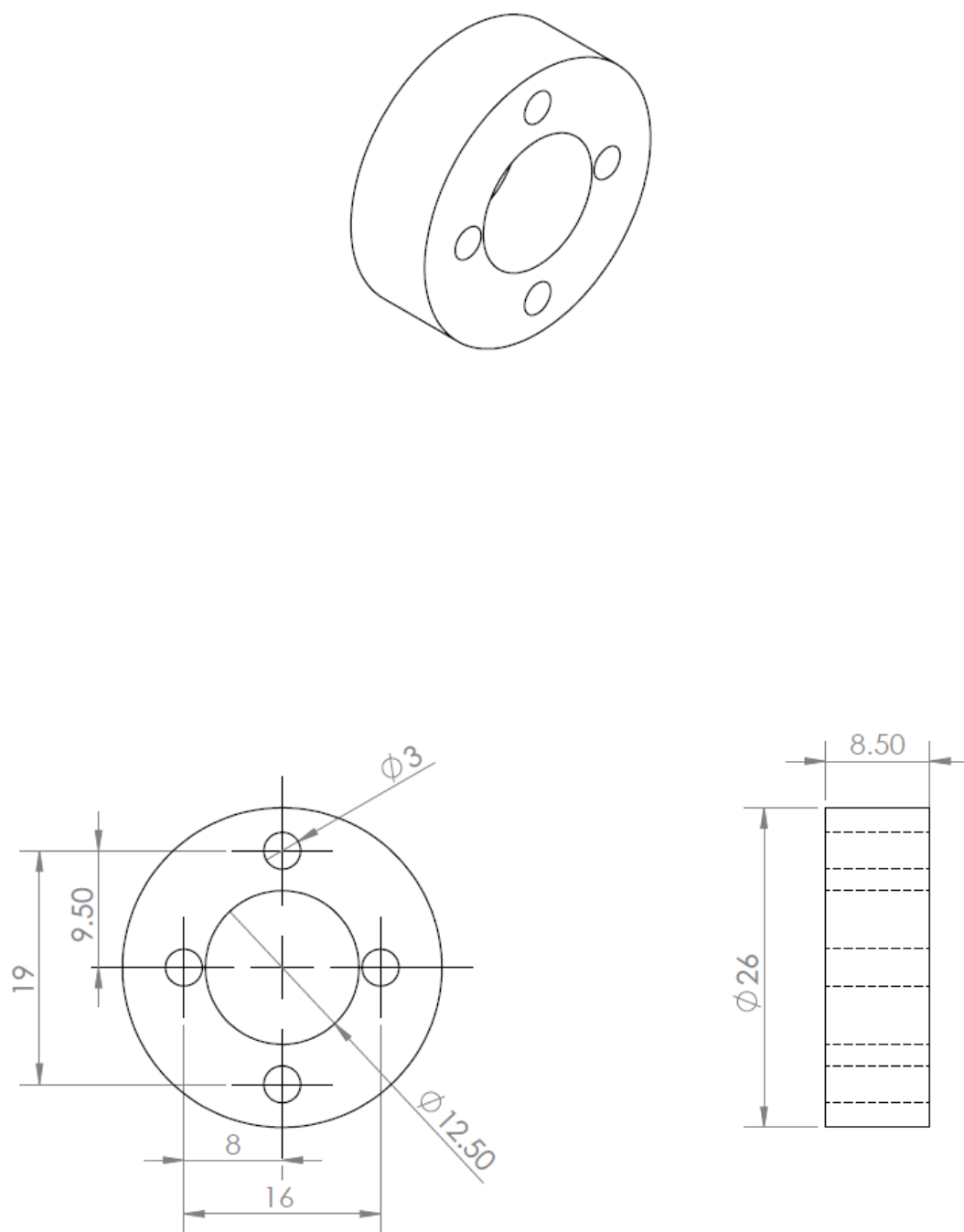


Figure B.1: Standoff for NERMLAB [mm]

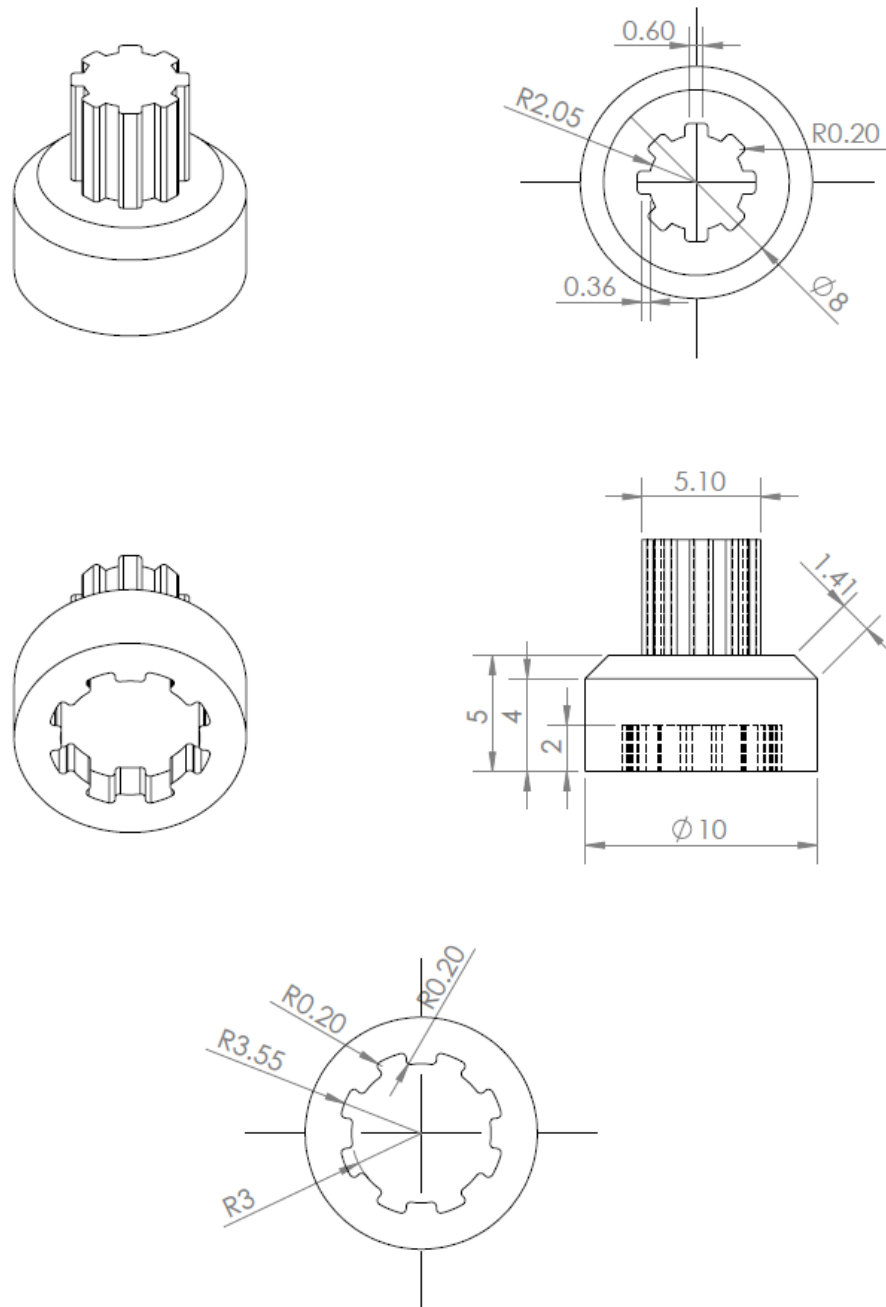


Figure B.2: Magnet holder for NERMLAB [mm]

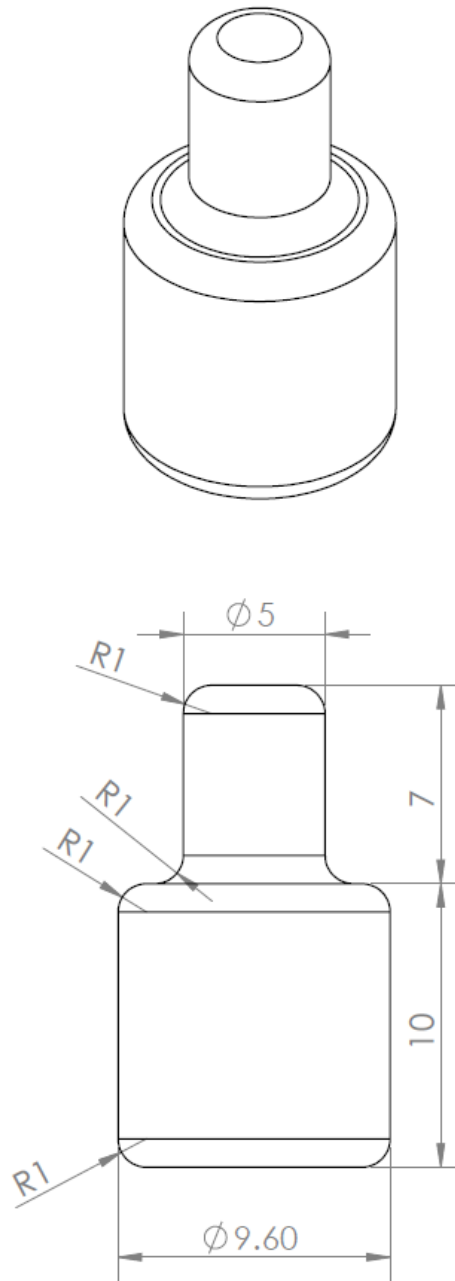


Figure B.3: Torque Transmission Shaft for NERMLAB [mm]