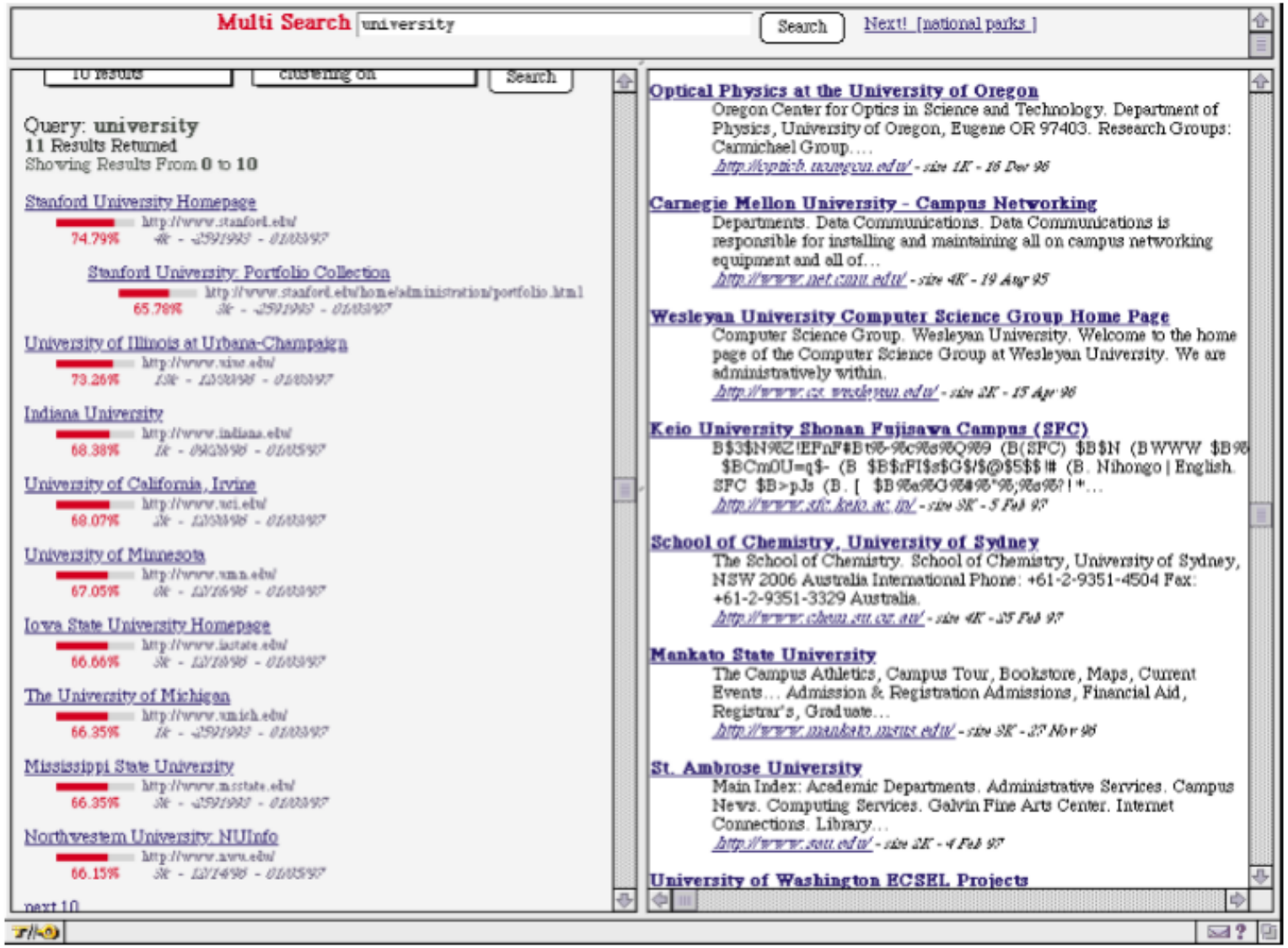
# CPSC 340:
# Machine Learning and Data Mining

PageRank

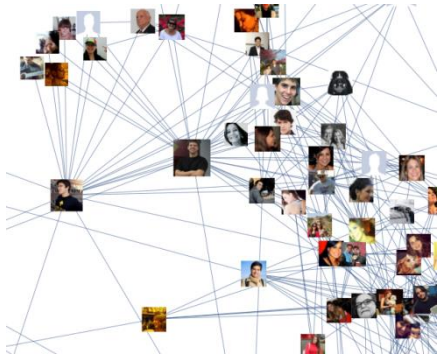
Fall 2019

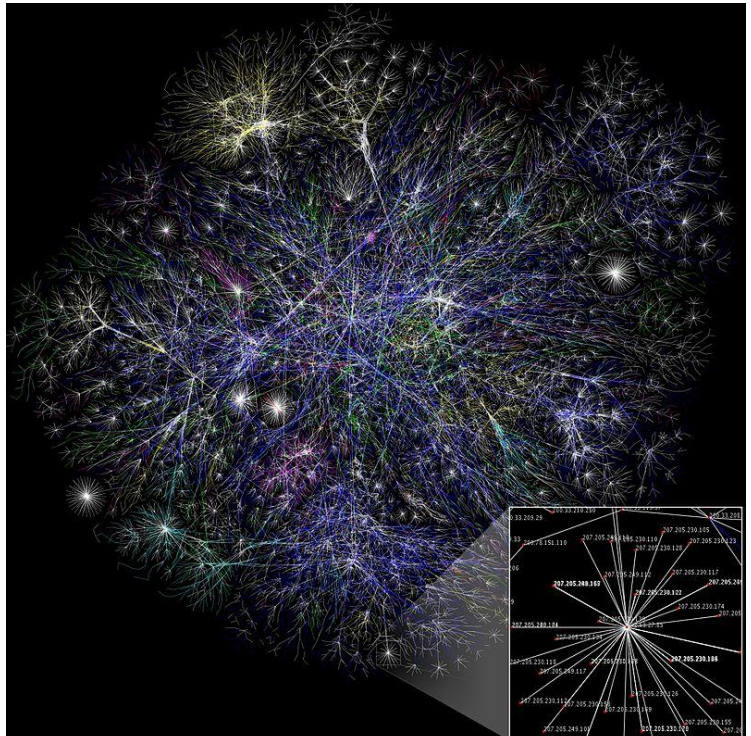
# Web Search before Google



# Unsupervised Graph-Based Ranking

- We want to rank "importance" based on graph between examples.
  - Every webpage is a node, and every web-link is an edge.
  - Every paper is a node, and every citation is an edge.
  - Every Facebook user is a node, and every "friendship" is an edge.



https://en.wikipedia.org/wiki/Scale-free_network
http://blog.revolutionanalytics.com/2010/12/facebooks-social-network-graph.html
http://mathematica.stackexchange.com/questions/11673/how-to-play-with-facebook-data-inside-mathematica

# Unsupervised Graph-Based Ranking

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  - Every paper is a node, and every citation is an edge.
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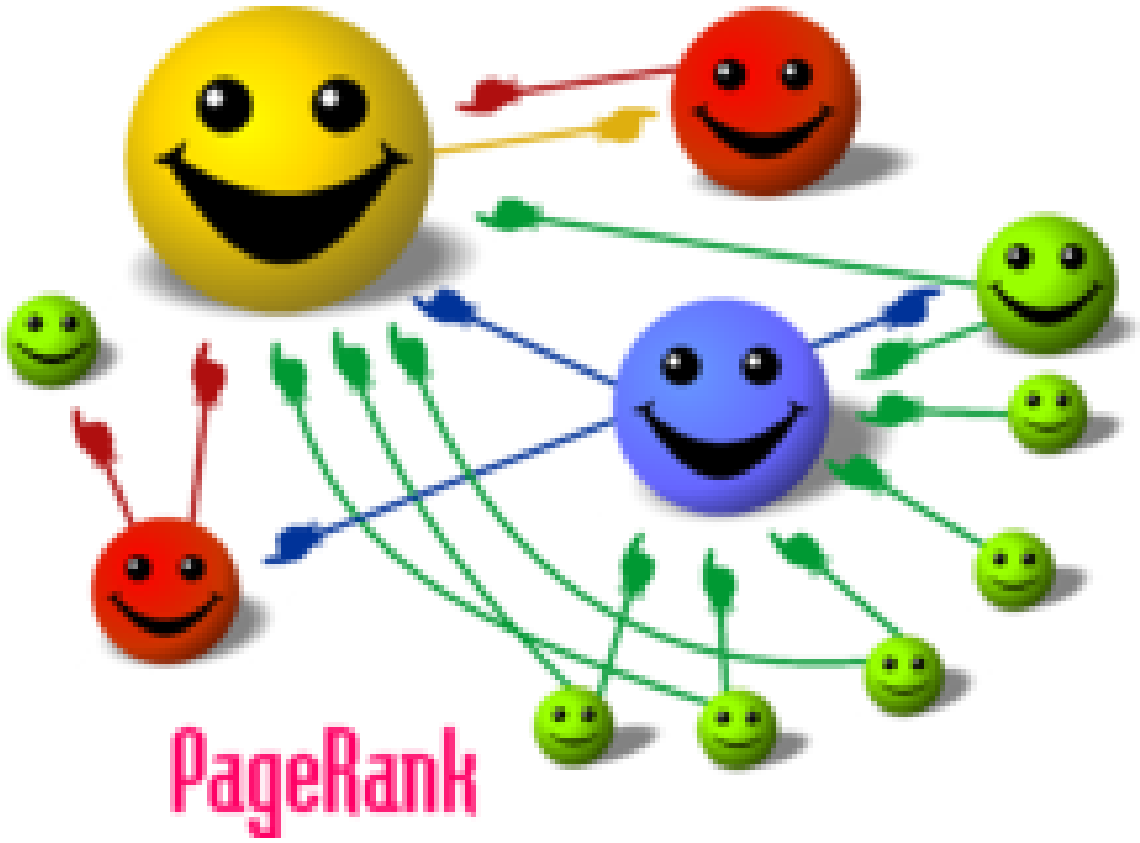
- Key idea: use links (edges) to predict importance of nodes.
- Many link analysis methods, usually with recursive definitions:
  - A journal is "influential" if it is cited by "influential" journals.
- We will discuss PageRank, Google's original ranking algorithm.

# PageRank

- Wikipedia's cartoon illustration of PageRank:
  - Large face => higher rank.



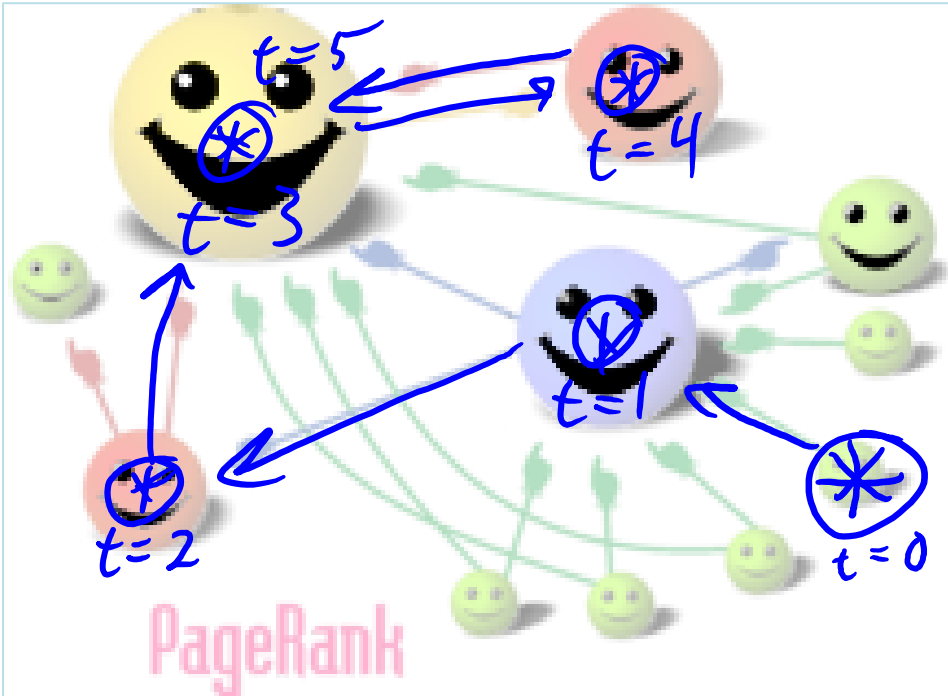

- Key ideas:
  - Important webpages are linked from other important webpages.
  - Link is more meaningful if a webpage has few links.

https://en.wikipedia.org/wiki/PageRank

# Random Walk View of PageRank

- PageRank algorithm can be interpreted as a random walk:
  - At time t=0, start at a random webpage.
  - At time t=1, follow a random link on the current page.
  - At time t=2, follow a random link on the current page.

    ....
- PageRank:
  - Probability of landing on page as t->∞.
- Obvious problem:
  - Pages with no in-links have a rank of 0.
  - Algorithm can get "stuck" in part of the graph.




https://en.wikipedia.org/wiki/PageRank

# Random Walk View of PageRank

- Fix: add small probability of going to a random webpage at time ‘t’.
- Damped PageRank algorithm:
  - At time t=0, start at a random webpage.
  - At time t=1:
    - With probability α (like 10%): go to a random webpage.
    - With probability (1- α): follow a random link on the current page.
  - At time t=2, follow a random link on the current page.
    - With probability α: go to a random webpage.
    - With probability (1- α): follow a random link on the current page.
- PageRank:
  - Probability of landing on page as t->∞.

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# PageRank Computation

- "Monte Carlo" method for computing PageRank:
  - Just run the random walk algorithm a really long time.
  - Count the number of times you visit each webpage.
    - Maybe include a "burn in" time at the start where you don't count pages.
    - Can parallelize by using 'm' independent surfers.
  - Intuitive but slow.

- It can also be solved analytically with SVD:
  - But $O(n^3)$ for 'n' webpages.

- Google's approach is the power method:
  - Repeated multiplication by transition matrix: O(nLinks) per iteration.

# Application: Game of Thrones

- PageRank can be used for other applications.
- “Who is the main character in the Game of Thrones books?”

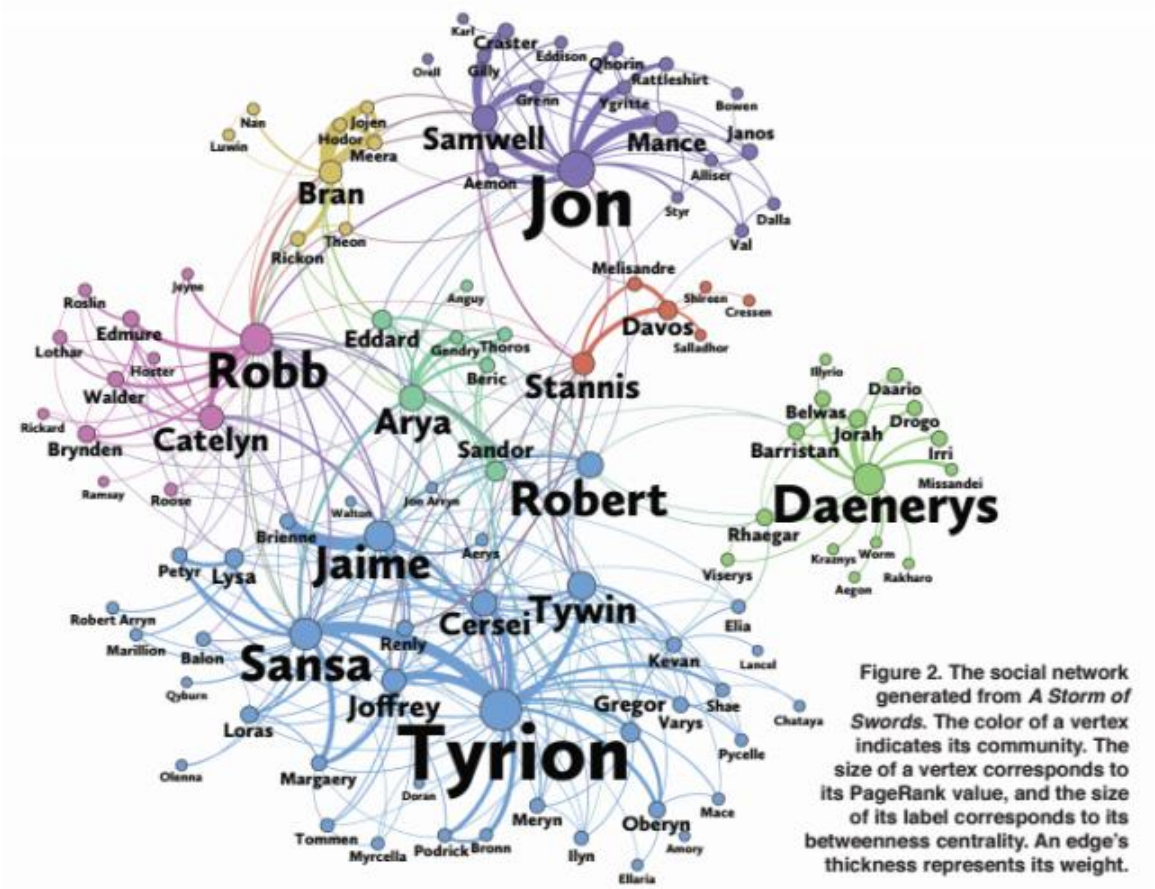



Figure 2. The social network generated from *A Storm of Swords*. The color of a vertex indicates its community. The size of a vertex corresponds to its PageRank value, and the size of its label corresponds to its betweenness centrality. An edge's thickness represents its weight.

http://qz.com/650796/mathematicians-mapped-out-every-game-of-thrones-relationship-to-find-the-main-character

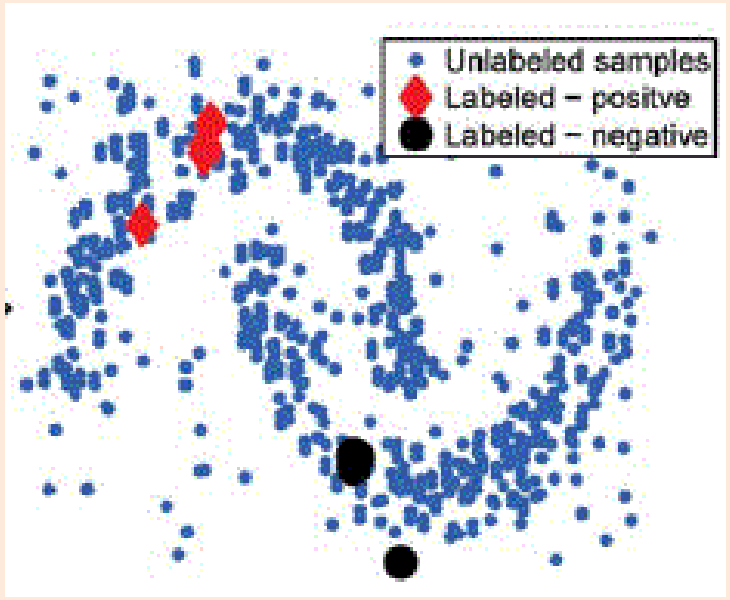
# Ranking Discussion

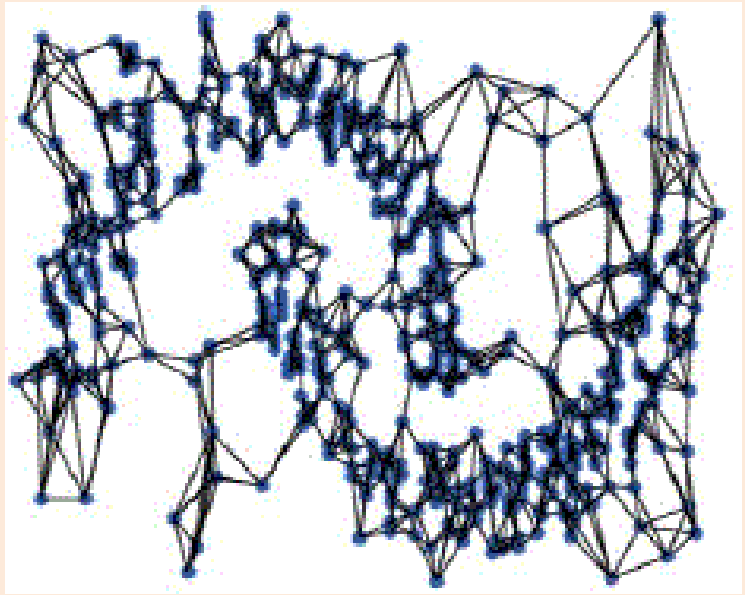
- Modern ranking methods are more advanced:
  - Guarding against methods that exploit algorithm.
  - Removing offensive/illegal content.
  - Supervised and personalized ranking methods.
  - Take into account that you often only care about top rankings.
  - Also work on diversity of rankings:
    - E.g., divide objects into sub-topics and do weighted “covering” of topics.
  - Persistence/freshness as in recommender systems (news articles).

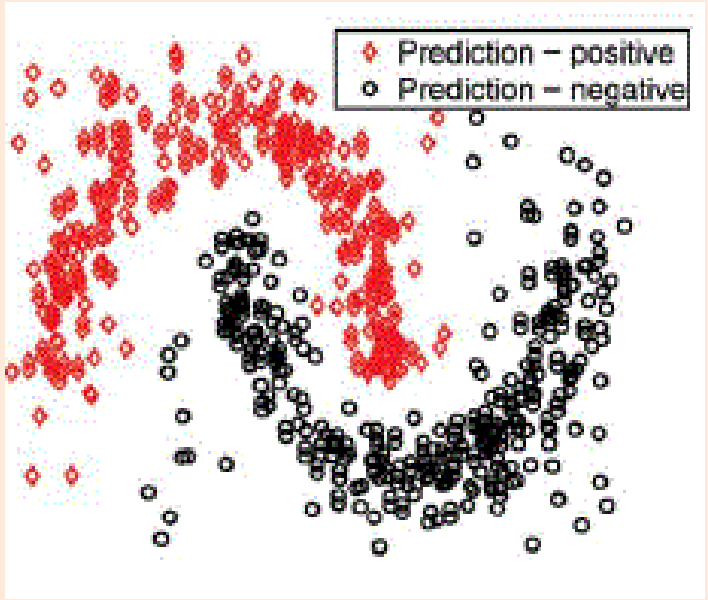
(pause)

# Previously: Graph-Based Semi-Supervised Learning

- Graph-based semi-supervised learning:
  - Define weighted graph on training examples:
    - For example, use KNN graph or points within radius ‘ε’.
    - Weight is how ‘important’ it is for nodes to share label.



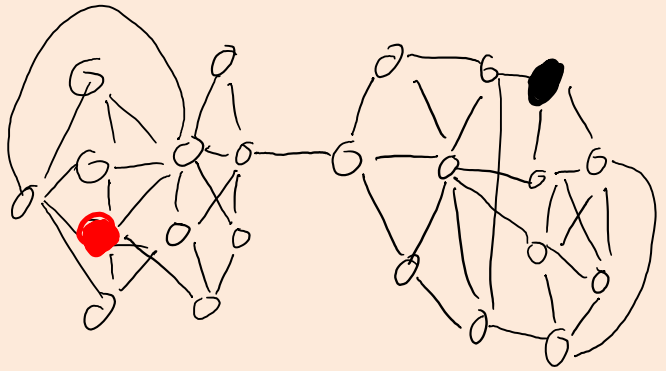






http://www.ee.columbia.edu/ln/dvmm/pubs/publications.html

# PageRank, Label Propagation, and Random Walks

- Standard graph-based SSL also has a random walk interpretation:
  - At time t = 0, set your state to the node you want to label.
  - At time t > 0, move to a random neighbor.
    - With probability proportional to $w_{ij}$ (how much we want them to be similar).
  - If you land on a labeled node, choose that label for this "round".
- Final predictions are probabilities of outputting each label.



https://en.wikipedia.org/wiki/PageRank

# What else can we do with random walks?

- We've discussed random walks for ranking and SSL.
  - Useful for problems defined on graphs.
  - We can convert from features to graphs using things like KNN graphs.
- Random walks for other tasks:
  - Outlier detection with outrank:
    - Examples with low PageRank are considered outliers (can detect outlier clusters).

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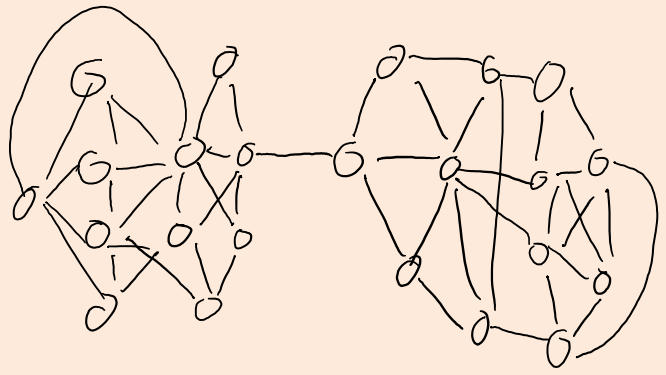
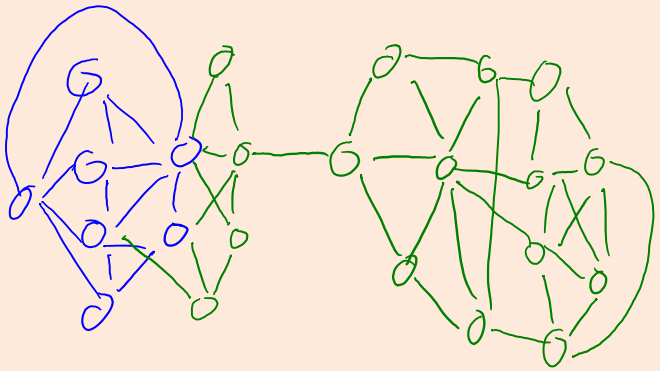
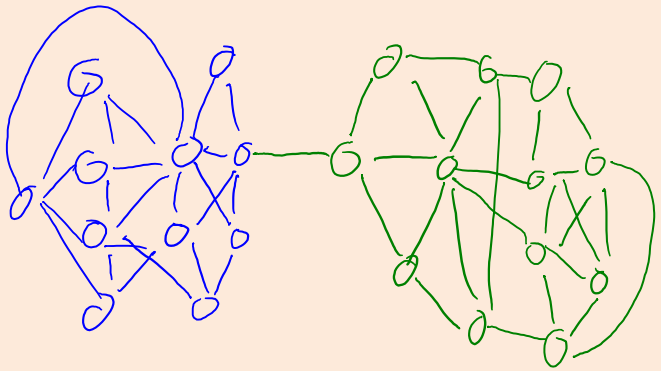
- We've discussed random walks for ranking and SSL.
  - Useful for problems defined on graphs.
  - We can convert from features to graphs using things like KNN graphs.
- Random walks for other tasks:
  - Clustering with spectal clustering (and "spectral graph theory):
    - "If we start in cluster 'c', random walk should tend to stay in cluster 'c'".



Graph representation of data

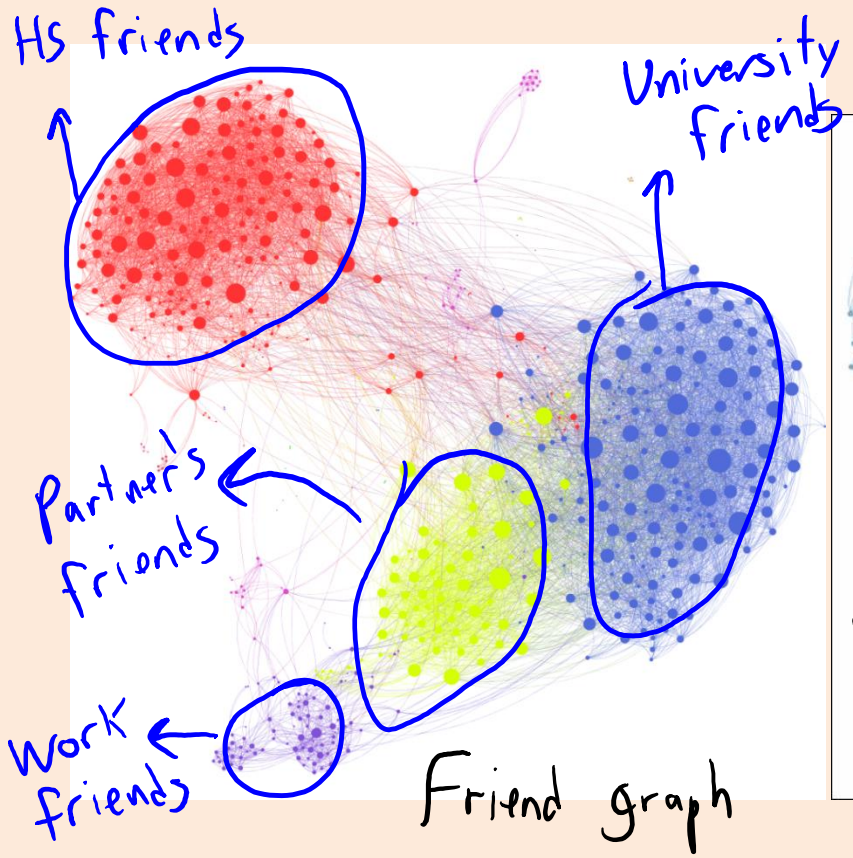


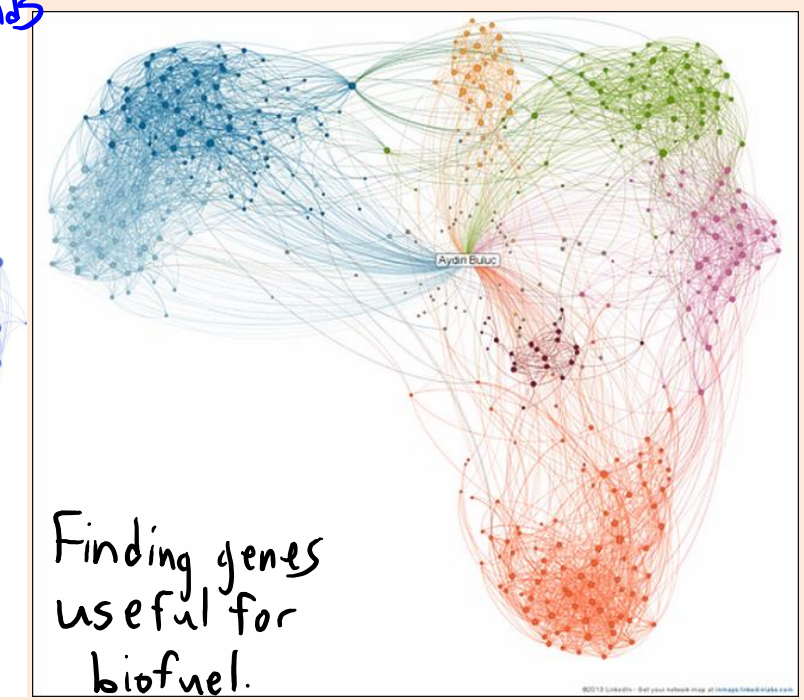
Bad clustering

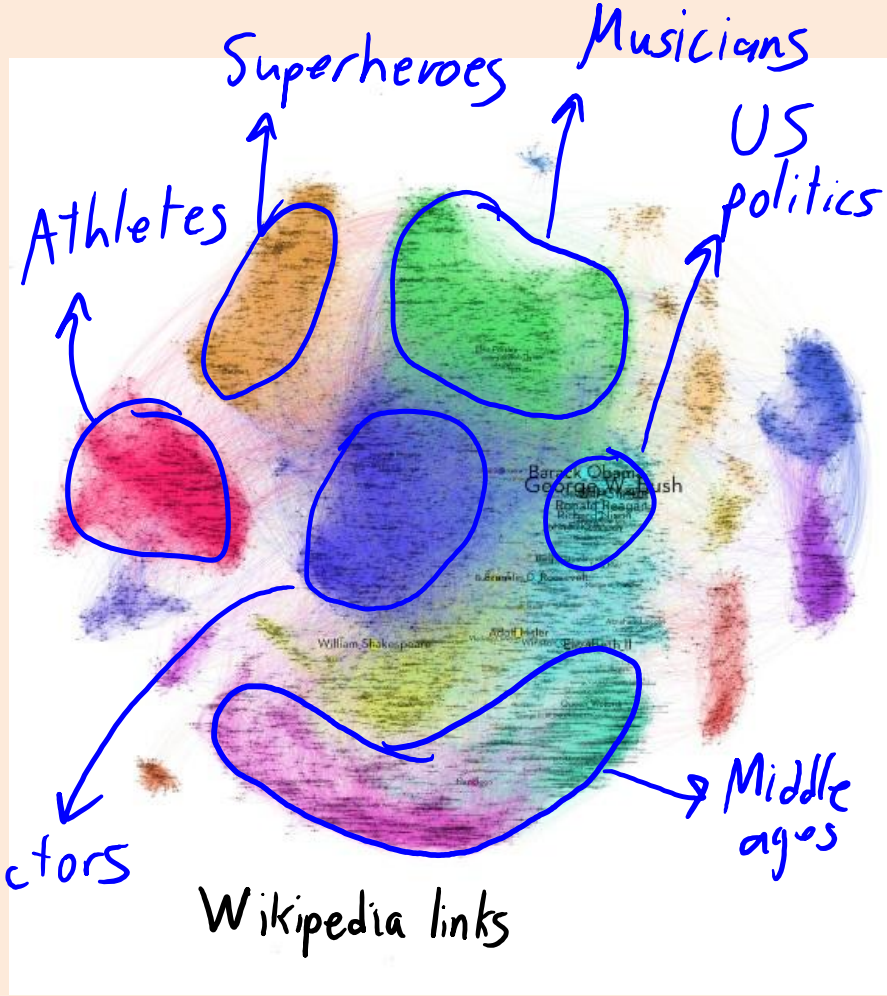


Good clustering.

# Graph-Based Clustering Methods



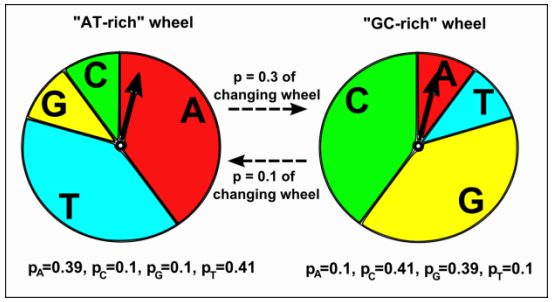



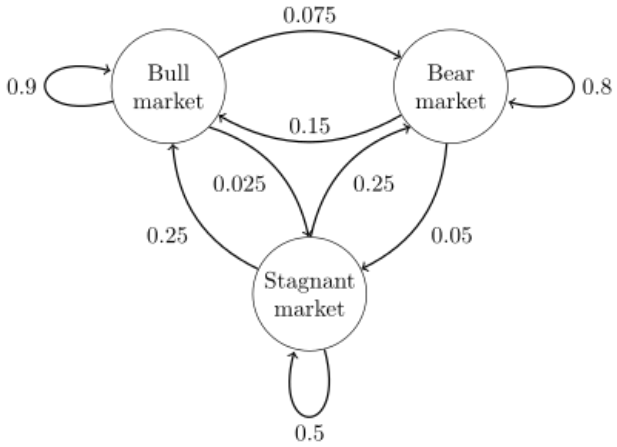



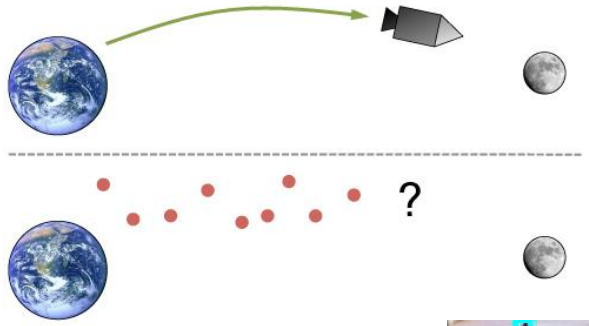

https://griffsgraphs.wordpress.com/tag/clustering/
http://ascr-discovery.science.doe.gov/2013/09/sifting-genomes/
https://www.hackdiary.com/2012/04/05/extracting-a-social-graph-from-wikipedia-people-pages/

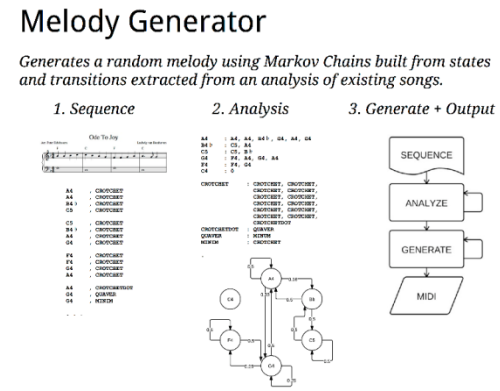
# Markov Chains

- These random walk algorithms are special cases of Markov chains:
  - Most common framework for modeling sequences.
    - Bioinformatics, physics/chemistry, speech recognition, predator-prey models, language tagging/generation, computing integrals, economic models, flying airplanes, tracking missiles/players, modeling music.




https://en.wikipedia.org/wiki/Markov_chain
http://www.cs.uml.edu/ecg/index.php/AIfall11/MarkovMelodyGenerator
http://a-little-book-of-r-for-bioinformatics.readthedocs.org/en/latest/src/chapter10.html
https://plus.maths.org/content/understanding-unseen
http://www.cs.ubc.ca/~okumak/research.html

# Summary

- Graph-based ranking uses links to solve ranking queries.
  - PageRank is based on a model of a random web user.