Exercises 5

1. Consider the two MA(1) processes

$$\begin{aligned} X(t) &= Z(t) + \beta_1 Z(t-1), \\ Y(t) &= Z(t) + \beta_2 Z(t-1) \end{aligned}$$

where $Z(t)$ is white noise with variance σ^2 and $\left|\beta_j\right| < 1$ for $j = 1, 2$. Find the cross–covariance function $\gamma_{XY}(k)$ here, and hence the corresponding cross–correlation function. Determine the cross–correlation when $\beta_1 = 0.6$, $\beta_2 = -0.6$.

2. Consider the AR(1) process

$$(1 - \alpha B) X(t) = Z(t),$$

where $Z(t)$ is white noise with variance σ^2, and $|\alpha| < 1$. Find the cross-correlation function $\rho_{ZX}(k)$. Plot this function when $\alpha = 0.6$.

3. The multivariate time series data set `EuStockMarkets` contains 1860 consecutive (business) day closing prices of four major European stock indices: DAX (Germany), CAC (France), SMI (Switzerland) and FTSE (UK). Read these data into R using the following commands:

```
> data(EuStockMarkets)

> DAX <- EuStockMarkets[,''DAX'']

> FSTE <- EuStockMarkets[,''FTSE'']
```

Plot these two series, and examine their sample cross–correlation, as well as their individual autocorrelations. Pre-whiten by applying a five–point moving average filter to each series and study the residual autocorrelations and cross–correlations once the smoothed version has been subtracted from the raw data.