Lecture 16

Principles of Relational Database Design (cont.)

Week 9

To learn how to design good quality database schemata we study the theory of ***functional dependencies***

Overview

- Functional dependencies
- Inference rules for FDs
- The closure of a functional dependency set
- Functional dependencies and keys
- Closure of set of attributes under an FD set
- Minimal FD set
- Equivalence of FD sets

Functional dependencies

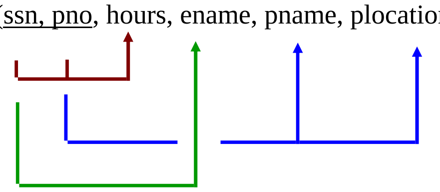
Employee_p (<u>ssn, pno</u>, hours, ename, pname, plocation)

In *any* instance of this relation it will be true that
if two tuples agree on the value of **ssn**,
then these two tuples also must agree on
the value of **ename**!

We say that ssn <u>functionally determines</u> ename
We write **ssn → ename**

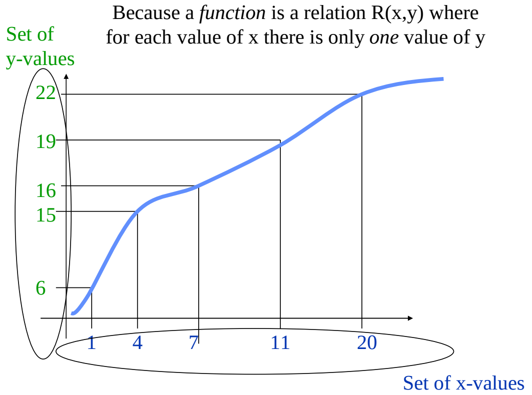
Dependencies in Employee_p

Employee_p
(<u>ssn, pno</u>, hours, ename, pname, plocation)



ssn → ename
pno → pname, plocation
ssn, pno → hours

Why is it called ‘functional’ dependency?
Because a *function* is a relation R(x,y) where
for each value of x there is only *one* value of y



Inference Rules for FDs

- Consider the well known company database, in which we know:

ssn → dno
dno → dname
ssn → dname

1. transitivity

Inference Rules for FDs

- ssn,pno → ssn — **2.** reflexivity
- ssn → dno
  ssn,pno → dno,pno — **3**. augmentation
- pno → pname, plocation
  pno → pname
  (and also pno → plocation) — **4**. decomposition

Inference Rules for FDs

- pno → pname
  pno → plocation
  pno → pname,plocation — **5.** additive (union)

Several inference rules exist, but rules 1,2, and 3 are enough[*] **-** the rest follows.

* 1,2,3 known as Armstrong's Axioms

The Closure of a Functional Dependency Set

Given a set of functional dependencies **F** , the **closure F+** of this set is obtained by applying these inference rules to derive all possible consequences of **F.**

(The result is usually a ***very large*** set, with many trivial dependencies. We never actually calculate the closure of an FD set, but use the **concept** in the sequel)

Functional Dependencies and Keys...

- Superkey

If a set of attributes **X** functionally determines **all** other attributes in a relation schema **R**, then that set is a **superkey of R.** Eg. in Employee_p

{ssn, pno, ename} is (one) superkey

ssn, pno, ename → all attributes of Employee

Functional Dependencies and Keys...

- Of course we could have left out **ename** from {ssn, pno, ename} and **still** the result would be a superkey.
- If we keep leaving out attributes (but *still have a superkey*) such that we get a **minimal superkey**, then we obtain a **candidate key**:
- **Definition: Candidate key** is a minimal superkey

  Eg. **{ssn, pno} is a candidate key** of Employee_p since neither {ssn} nor {pno} alone determine all attributes of this relation

Algorithms to determine all candidate keys of a relation:

- top-down: eliminate attributes from trivial superkey sets while **preserving** the superkey feature;
- bottom up: start from a necessary set of attributes which **must** be part of a candidate key (ck), test if they are **ck** themselves, if not try **extending** the set until superkeys are found

Closure of a Set of Attributes under a Set of FDs

- Given a set of attributes **X** we often would like to determine what other attributes are functionally determined by these?

  We call this determined set the **closure of X under a set of FDs F**, noted X_F+ .

Closure of Attributes under FDs (cont.)

- Provided the set of FDs is given, this is an easy task: all we have to do is try to apply the known functional dependencies and according to them, extend the determined set
- Let R = ABCD and F = A → B, B → C

  Question

  if X= {A,D} what is the closure X_F+ ?

Closure of Attributes under FDs (cont.)

R = ABCD and F = {A → B; B → C}

- In other words {AD} → ?

  Apply inference rules:

  {AD} → {AD}
  {AD} → {ADB} by FD1
  {AD} → {ADB} → {ADBC} by FD2

  therefore

  $\boxed{\{AD\}_F+ = \{ABCD\}}$

Minimal Set of Functional Dependencies

- Consider the FD set

  This FD is redundant in itself - why ?

  F = {AB → B, B → C, C → BA}

  What about **B → A** ? Would **it** be redundant ?
- Clearly, this is redundant, because we could simplify F without losing any information.

  Eg. H = {B → C, C → B, C → A} is equivalent with F, ie. F+ = H+

Minimal Set of Functional Dependencies (minimal Cover)

Definition: A set of functional dependencies is *minimal* if:

- All FDs have *one* attribute on the right hand side
- No FD can be omitted without losing information
- All left hand sides are minimal (no attribute can be omitted without losing information)

Minimal Set of Functional Dependencies (minimal Cover)

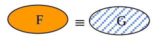
- The outline of the algorithm to determine a **minimal cover** (a minimal set of dependencies) of an FD set
  1. Split all FDs into atomic ones, instead of
     $A \rightarrow BC$ write $A \rightarrow B$, $A \rightarrow C$
  2. Try to omit FDs which are redundant
  3. Try to omit attributes from left hand sides of the remaining FDs (this is usually possible when other FDs may be identified **within** the left side, e.g.: $AB \rightarrow C$ is equivalent to $A \rightarrow C$ if A also $\rightarrow B$)

Equivalence of FD sets

- Suppose that two analysts go out and try to establish the FDs of a database schema. If they come back with two different results **F** and **G**, which one is right?

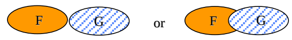
Possibilities:

- Both are right, because F and G are equivalent.
  (ie F+ = G+)



- One covers the other .
  (eg F+ $\supseteq$ G+ : F says at least as much as G, and **more**)

- None covers the other or partially cover each other
  (need their union)



Equivalence of FD sets

- There is a simple algorithm to test if two FD sets are equivalent.
- We test if **F covers G**, i.e. if F+ $\supseteq$ G+ and if **G covers F**, i.e. if G+ $\supseteq$ F+
- If both are true, then F+ = G+ i.e. F and G **are equivalent**.

Outline of algorithm

- For each FD $X \rightarrow Y$ in F determine the closure of X under F and under G *:

  If for each X $X_F + \subseteq X_G +$ then F+ $\subseteq$ G+

- For each FD $Z \rightarrow Q$ in G determine the closure of Z under G and under F :

  If for each Z $Z_G + \subseteq Z_F +$ then G+ $\subseteq$ F+

* That is, what attributes are functionally determined by the set X under the FD sets F and G. $X_F +$ and $X_G +$ are in the form of {..attributes..}

Conclusion

- Implemented relational schemata should be non-redundant and free from update anomalies;
- functional dependency, superkeys and candidate keys have been defined;
- useful concepts related to functional dependencies which assist relational database design, have also been defined:

  Closure of a **set** of FDs F (F+);
  Closure of a **set of attributes** under a set of FDs F (X_F +)
  Minimal set of FDs (or **minimal cover** of a set of FDs);
  Equivalence of FD sets (F+ = G+ implies F equiv. to G)

The end