Lecture 10

# Relational Algebra

Week 5

# Overview

- Relational algebra as a query language
- Selection, projection, cross product
- Union / intersection / set difference
- Attribute renaming
- Join
  - join
  - equijoin and natural join
- Division
- Simplifying relational algebra expressions

# Relational algebra as a query language

- *Objects*: relations (sets of tuples)
- *Operations* on relations result relations
- Relational algebra expressions can be evaluated, and the result is always a relation
- We can assign the result to a temporary relation

# Selection

$\sigma_{C}\ R$

select tuples from relation R
where each tuple satisfies condition C

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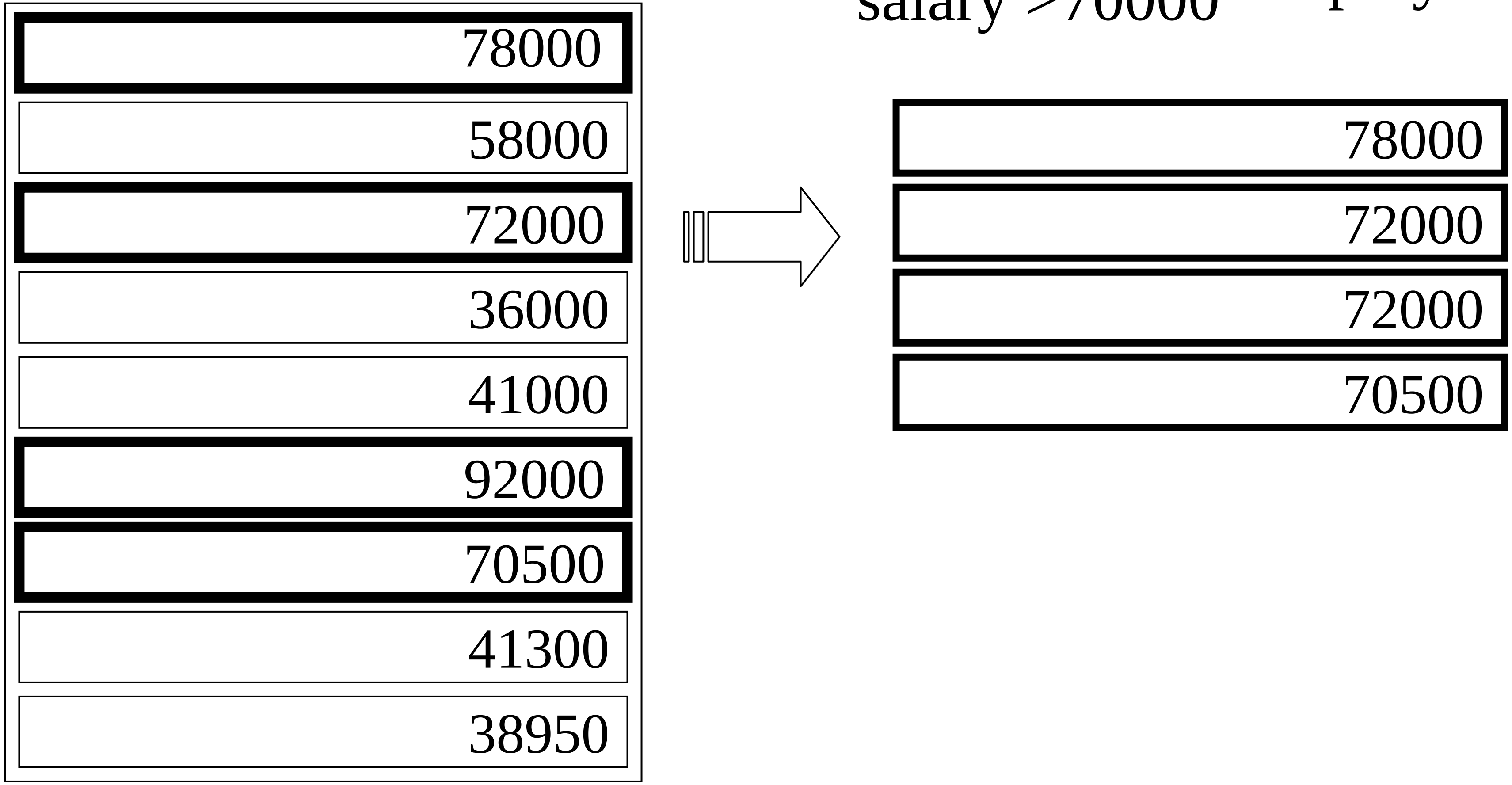
Example $\sigma_{salary > 70000}\ Employee$

Evaluates to a relation which is a subset of Employee, those employees who earn >70,000

Employee

$\sigma_{\text{salary} > 70000}$ Employee



78000

58000

72000

36000

41000

92000

70500

41300

38950

78000

72000

72000

70500

# Properties of selection

- number of tuples:

  $|\sigma_{C} R| \leq | R |$

- number of attributes:

  degree $(\sigma_{C} R)$ = degree ( R)

- commutative

  $\sigma_{C} \sigma_{D} R = \sigma_{D} \sigma_{C} R$ $(= \sigma_{C \text{ and } D} R)$

# Projection

$$\Pi_{L}\ R$$

take a subset of the attributes of a relation R, where $L \subseteq R$ ; and then eliminate duplicates

Example: $\Pi_{name,\ salary}$ Employee

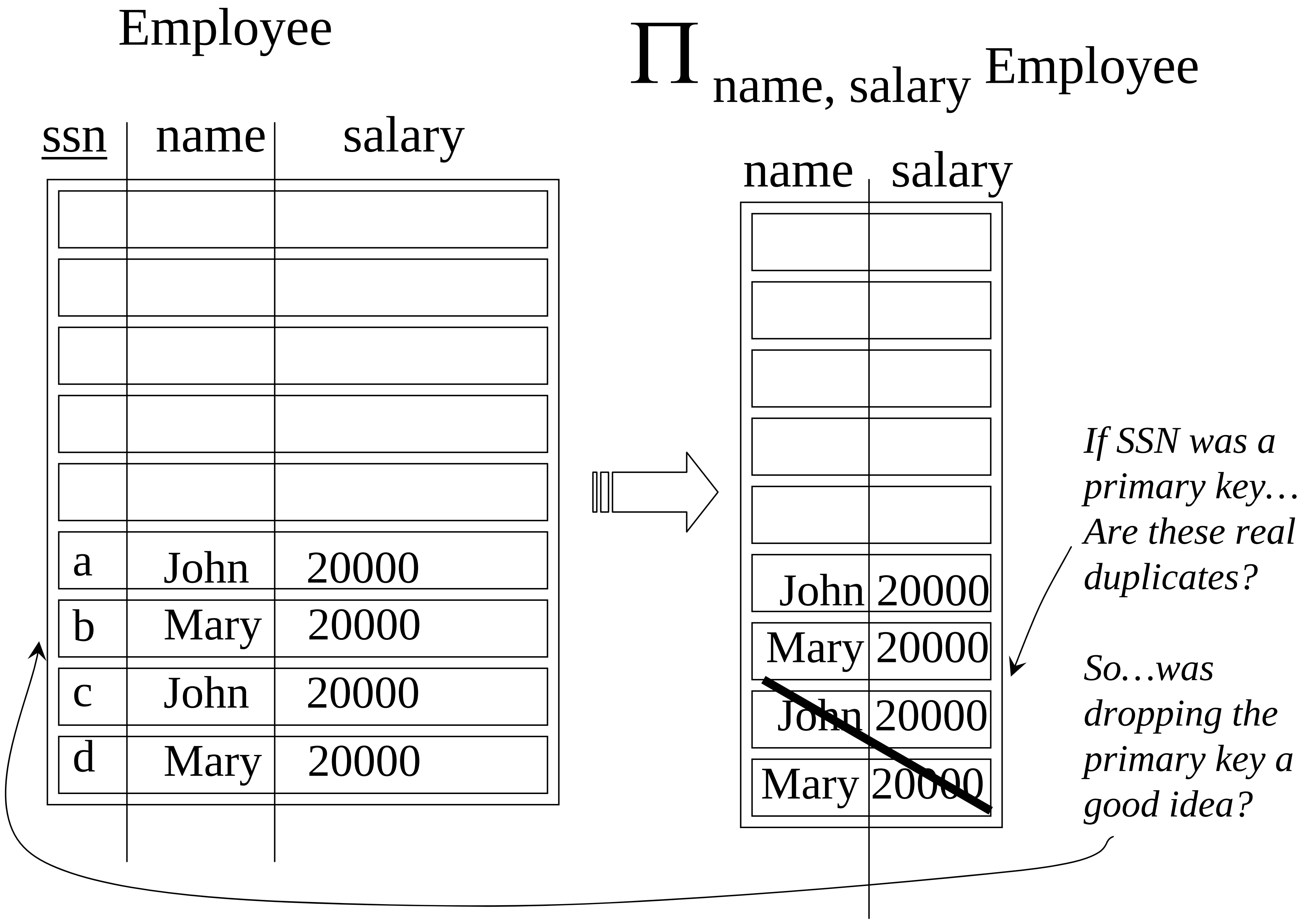
Evaluates to a relation with Employee names and salaries in it

Employee

| ssn | name | salary |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| a | John | 20000 |
| b | Mary | 20000 |
| c | John | 20000 |
| d | Mary | 20000 |

$\Pi_{\text{name, salary}}$ Employee

| name | salary |
|---|---|
| | |
| | |
| | |
| | |
| | |
| John | 20000 |
| Mary | 20000 |
| ~~John~~ | ~~20000~~ |
| ~~Mary~~ | ~~20000~~ |



Employee

| ssn | name | salary |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| a | John | 20000 |
| b | Mary | 20000 |
| c | John | 20000 |
| d | Mary | 20000 |

$\Pi_{\text{name, salary}}$ Employee

| name | salary |
|---|---|
| | |
| | |
| | |
| | |
| | |
| John | 20000 |
| Mary | 20000 |
| ~~John~~ | ~~20000~~ |
| ~~Mary~~ | ~~20000~~ |

*If SSN was a primary key… Are these real duplicates?*

*So…was dropping the primary key a good idea?*

# Properties of projection

- number of tuples:

  $|\Pi_L R| \leq |R|$

- number of attributes:

  degree $(\Pi_L R) \leq$ degree $(R)$

- *not* commutative

  $\Pi_L \Pi_S R \neq \Pi_S \Pi_L R$ (Note: $L \subseteq S$, but $S \not\subseteq L$)

- $\Pi_L \Pi_S R = \Pi_L R$ if $L \subseteq S$

# Note on constraint notation

- Remember the notation we used to describe foreign key constraints, e.g.

  **E(<u>k</u>,c)**

  **A(<u>k</u>,a)          fk: k is k in E**

  meaning that every k-value in A must be an actual k-value in E.

- We could have used relational algebra and logic to represent this constraint, i.e.

  **k is k in E**

  could have been written as

  $$\Pi_k A \subseteq \Pi_k E$$

# Set operations

$A \cup B$ Union

$A \cap B$ Intersection

$A \setminus B$ Difference

$A \times B$ Cartesian product

For union, intersection, and difference relations A and B must be *union compatible*:

- degree (A) = degree (B)
  operands have the same degree
- domain ($A_i$) = domain ($B_i$)
  (i.e. corresponding attributes must have the same value domain)

# Example

“Employees who work for department 5 *and* have salary >70000”

$$\sigma_{\text{DNO}=5}\ \text{Employee} \cap \sigma_{\text{SALARY}>70000}\ \text{Employee}$$

# Properties of union, intersection, difference

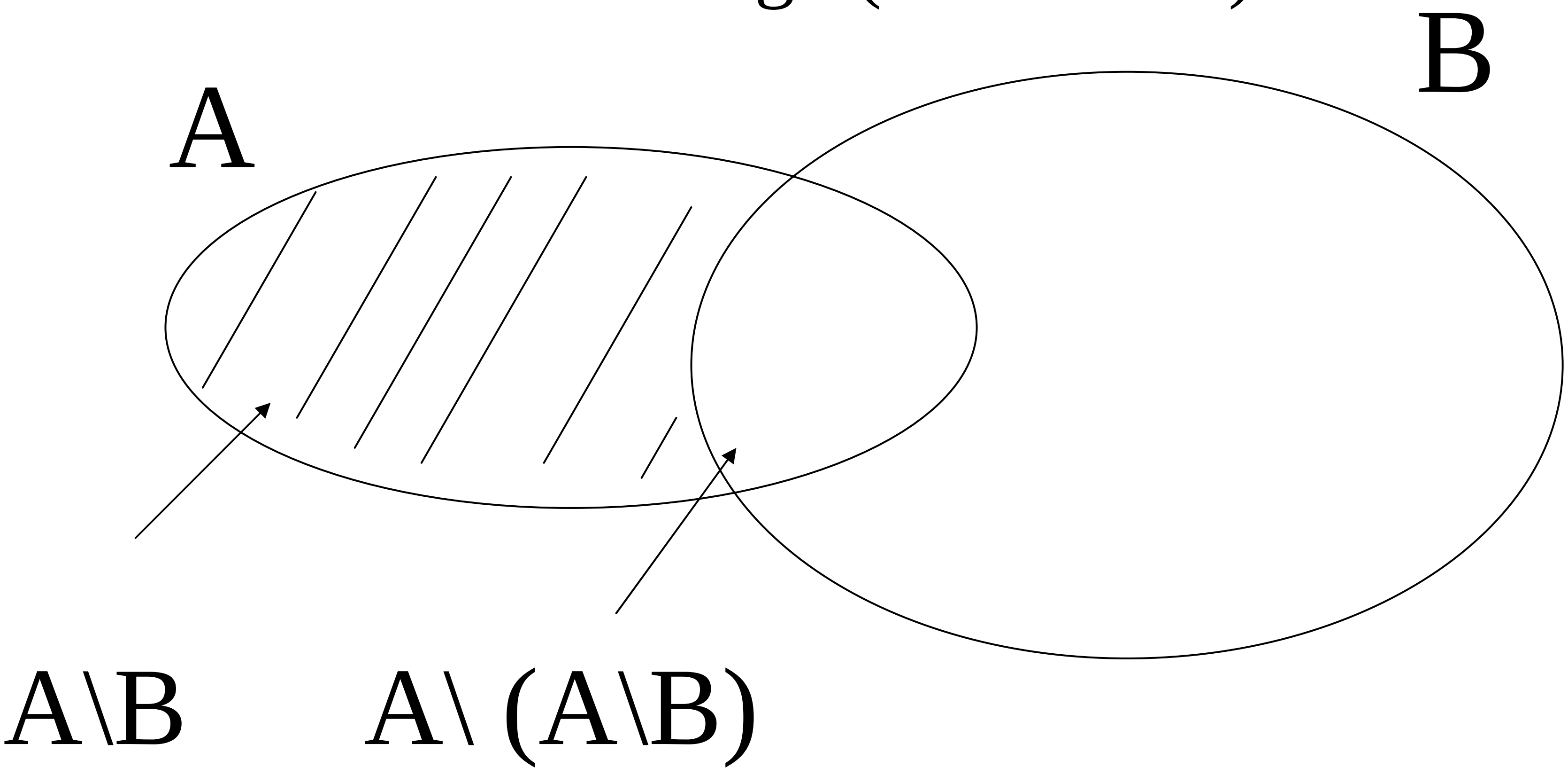
- number of tuples in intersection:

  $|A \cap B| \leq \mathrm{Min}\,(|A|, |B|)$
- number of tuples in union:

  $|A \cup B| \leq |A| + |B|$
- number of tuples in difference:

  $|A \setminus B| \leq |A|$
- degree ( A op B ) = degree (A) = degree (B)

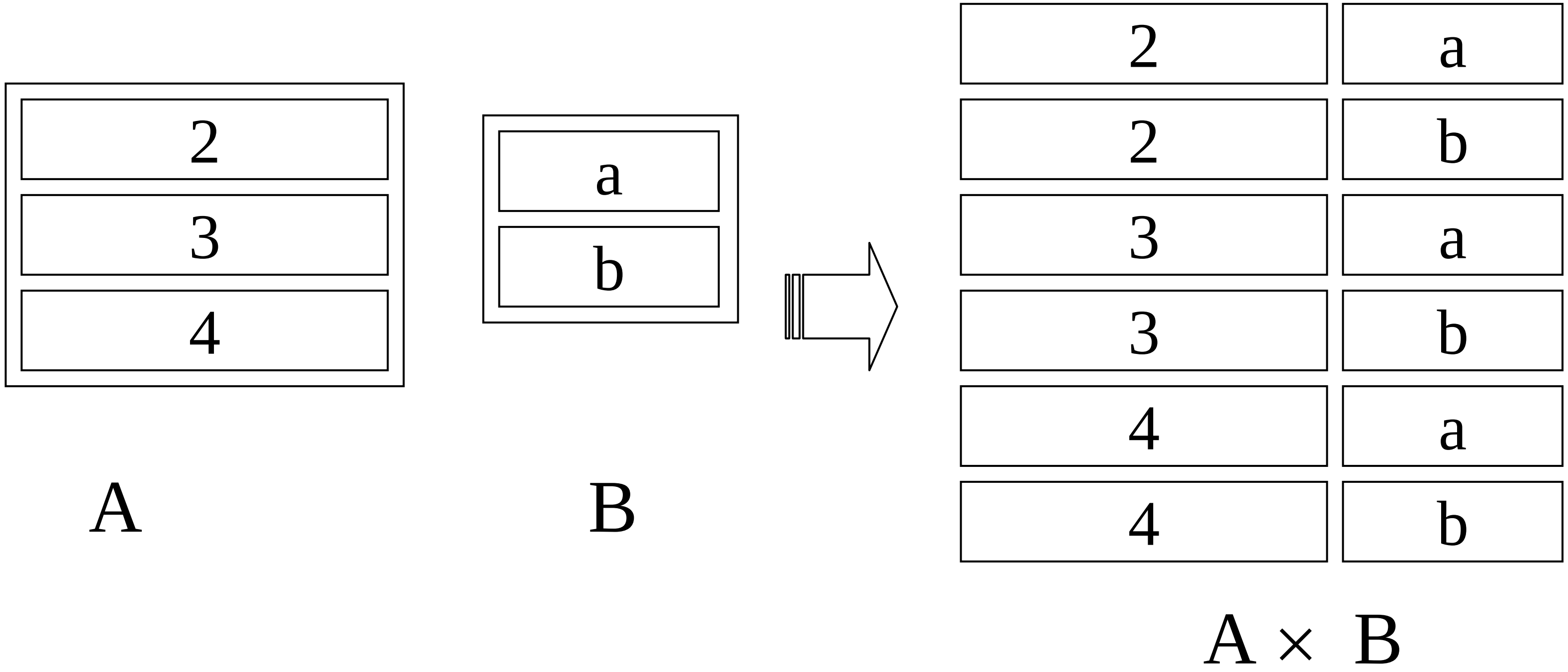
- Note that not all operations are necessary: $A \cap B = A \setminus (A \setminus B)$
- I.e. $\cup$ and \ is enough (or $\cap$ and \ )



# Cartesian product

A × B

concatenate the attributes of all tuples in A and B in *every possible way*



| |
|---|
| 2 |
| 3 |
| 4 |

A

| |
|---|
| a |
| b |

B

| | |
|---|---|
| 2 | a |
| 2 | b |
| 3 | a |
| 3 | b |
| 4 | a |
| 4 | b |

A × B

# Properties of cartesian product

- number of tuples in product:

  $| A \times B | = |A| * |B|$

  I.e. every possible combination is produced

- degree ( $A \times B$ ) = degree ($A$) + degree ($B$)

# Unary and binary operations

- Unary operations bind more strongly than binary ones, hence paratheses can be omitted

$$(\sigma_{dno=5}\ \text{Employee}\ ) \cap (\sigma_{salary>70000}\ R) =$$

$$\sigma_{dno=5}\ \text{Employee}\ \cap\ \sigma_{salary>70000}\ R$$

# Attribute renaming

- Sometimes we need to store the result of an expression and give the attributes a different name

$\text{Result(sssn, sname)} \leftarrow \Pi_{\text{ssn, name}} \text{Employee}$

# The end