ME565 HW5

1. $\ddot{x} + 3\dot{x} + 2x = u(t)$

(a)

using Laplace Transform $\mathcal{L}\{x(t)\} = X(s)$

$$\Rightarrow s^2 X(s) + 3s X(s) + 2X(s) = U(s)$$

$$\Rightarrow (s^2 + 3s + 2) X(s) = U(s)$$

(i) $\mathcal{L}\{1(t)\} = \frac{1}{s} = U(s)$

$$X(s) = \frac{1}{s(s^2+3s+2)} = \frac{1}{s(s+2)(s+1)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s+2} + \frac{k_3}{s+1}$$

$$k_1 = \lim_{s\to 0} sX(s) = \frac{1}{2}$$

$$k_2 = \lim_{s\to -2} (s+2)X(s) = \frac{1}{2}$$

$$k_3 = \lim_{s\to -1} (s+1)X(s) = -1$$

$$X(s) = \frac{1}{2}\frac{1}{s} + \frac{1}{2}\frac{1}{s+2} - \frac{1}{s+1}$$

$$\Rightarrow \boxed{x(t) = \mathcal{L}^{-1}\{X(s)\} = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}} \quad \#$$

(iii) $\mathcal{L}\{\delta(t)\} = 1 = U(s)$

$$X(s) = \frac{1}{s^2+3s+2} = \frac{1}{(s+1)(s+2)}$$

$$= \frac{k_1}{s+1} + \frac{k_2}{s+2}$$

$$k_1 = \lim_{s\to -1} (s+1)X(s) = 1$$

$$k_2 = \lim_{s\to -2} (s+2)X(s) = -1$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$\Rightarrow \boxed{x(t) = \mathcal{L}^{-1}\{X(s)\} = e^{-t} - e^{-2t}} \#$$

(b) $\ddot{x} + 2\dot{x} + 2x = u(t)$

using Laplace Transform $\mathcal{L}\{x(t)\} = X(s)$

$$\Rightarrow (s^2 + 2s + 2)X(s) = U(s)$$

(i) $\mathcal{L}\{1(t)\} = \frac{1}{s} = U(s)$

$$X(s) = \frac{1}{s(s^2+2s+2)} = \frac{1}{s[(s+1)^2+1]}$$

$$= \frac{k_1}{s} + \frac{k_2(s+1)+k_3}{(s+1)^2+1}$$

$$k_1(s^2+2s+2) + k_2 s^2 + (k_2+k_3)s = 1$$

$$\Rightarrow (k_1+k_2)s^2+(2k_1+k_2+k_3)s+2k_1=1$$

$$\Rightarrow k_1=\frac{1}{2} \quad k_2=-\frac{1}{2} \quad k_3=-\frac{1}{2}$$

$$X(s)=\frac{1}{2}\frac{1}{s}-\frac{1}{2}\frac{s+1}{(s+1)^2+1}-\frac{1}{2}\frac{1}{(s+1)^2+1}$$

$$\Rightarrow \boxed{x(t)=\mathcal{L}\{X(s)\}=\frac{1}{2}-\frac{1}{2}e^{-t}\cos t-\frac{1}{2}e^{-t}\sin t} \quad \text{※}$$

(iii) $\mathcal{L}\{\delta(t)\}=1=U(s)$

$$X(s)=\frac{1}{s^2+2s+2}=\frac{1}{(s+1)^2+1}$$

$$\Rightarrow \boxed{x(t)=\mathcal{L}\{X(s)\}=e^{-t}\sin t} \quad \#$$