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# 2023 HSC Mathematics Advanced Marking Guidelines

## Section I

### Multiple-choice Answer Key

Question	Answer
1	D
2	D
3	A
4	B
5	A
6	C
7	A
8	B
9	D
10	C

## Section II

### Question 11

Criteria	Marks
• Provides correct solution	2
• Finds the common difference, or equivalent merit	1

**Sample answer:**

$$d = 7 - 3 = 4$$

$$d = 4$$

$$a = 3$$

$$\begin{aligned}t_{15} &= a + (15 - 1)d = 3 + 14 \times 4 \\&= 59\end{aligned}$$

**Question 12 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	1

**Sample answer:**

$$\begin{aligned}
 E(X) &= \sum x P(x) \\
 &= 0 \times 0 + 1 \times 0.3 + 2 \times 0.5 + 3 \times 0.1 + 4 \times 0.1 \\
 &= 2
 \end{aligned}$$

**Question 12 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Attempts to find $\text{Var}(X)$ , or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 \text{Var}(X) &= E(X^2) - \mu^2 \\
 &= E(X^2) - [E(X)]^2 \\
 &= \sum x^2 P(x) - 4 \\
 &= 0 \times 0 + 1^2 \times 0.3 + 2^2 \times 0.5 + 3^2 \times 0.1 + 4^2 \times 0.1 - 4 \\
 &= 0.8
 \end{aligned}$$

$$\begin{aligned}
 \text{Standard deviation} &= \sqrt{0.8} \\
 &= 0.8944\dots \\
 &= 0.9 \quad (\text{to 1 decimal place})
 \end{aligned}$$

**Question 13**

Criteria	Marks
• Provides correct solution	2
• Attempts to find antiderivative, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}\frac{dP}{dt} &= 3000e^{2t} \\ \therefore P &= \frac{3000}{2}e^{2t} + C \\ &= 1500e^{2t} + C\end{aligned}$$

When  $t = 0$ ,  $P = 4000$

$$\begin{aligned}\therefore 4000 &= 1500e^{2 \times 0} + C \\ \therefore C &= 2500\end{aligned}$$

So  $P(t) = 1500e^{2t} + 2500$

**Question 14**

Criteria	Marks
• Provides correct solution	3
• Correctly finds slope of tangent, or equivalent merit	2
• Attempts to find the correct derivative, or equivalent merit	1

**Sample answer:**

$$y = (2x + 1)^3$$

$$y' = 3 \times (2x + 1)^2 \times 2$$

$$= 6(2x + 1)^2$$

$$\text{When } x = 0 \quad y' = 6$$

Equation of tangent given by:

$$y - 1 = 6(x - 0)$$

$$y - 1 = 6x$$

$$y = 6x + 1$$

**Question 15 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Identifies the correct factor from the table	1

**Sample answer:**

$$\begin{aligned} \text{Amount} &= \frac{\$450\,000}{13.181} \\ &= \$34\,140 \quad (\text{to the nearest dollar}) \end{aligned}$$

**Question 15 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Provides the correct interest rate and the correct number of periods, or equivalent merit	2
• Multiplies a factor from the table by \$8535, or equivalent merit	1

**Sample answer:**

$$\begin{aligned} r &= \frac{6}{4}\% \\ &= 1.5\% \end{aligned}$$

$$\begin{aligned} n &= 10 \times 4 \\ &= 40 \end{aligned}$$

$$\begin{aligned} \text{Amount} &= \$8535 \times 54.268 \\ &= \$463\,177.38 \end{aligned}$$

## Question 16

Criteria	Marks
• Provides correct solution	4
• Calculates the arc length AND the length of line segment $PQ$ , or equivalent merit	3
• Calculates the arc length OR the length of line segment $PQ$ , or equivalent merit	2
• Attempts to calculate the perimeter of the shape by adding some appropriate portions, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}\text{Arc length } PQ &= \frac{110}{360} \times 2 \times \pi \times 2.1 \\ &= 4.03171\dots\end{aligned}$$

$$\begin{aligned}\text{Length } PQ &= \sqrt{2.1^2 + 2.1^2 - 2 \times 2.1 \times 2.1 \times \cos 110^\circ} \\ &= 3.4404\dots\end{aligned}$$

$$\begin{aligned}\text{Total perimeter} &= (3.6 \times 2) + 8.0 + (8.0 - 3.4404) + 4.0317 \\ &= 23.7913 \\ &= 23.8 \text{ m}\end{aligned}$$

**Question 17**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Recognises the integral is of the form $k \int f'(x)[f(x)]^n dx$ , or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
& \int x(x^2 + 1)^{\frac{1}{2}} dx \\
&= \frac{1}{2} \int 2x(x^2 + 1)^{\frac{1}{2}} dx \\
&= \frac{1}{2} \left[ \frac{(x^2 + 1)^{\frac{3}{2}}}{\frac{3}{2}} \right] + C \\
&= \frac{1}{3}(x^2 + 1)^{\frac{3}{2}} + C
\end{aligned}$$

**Question 18 (a)**

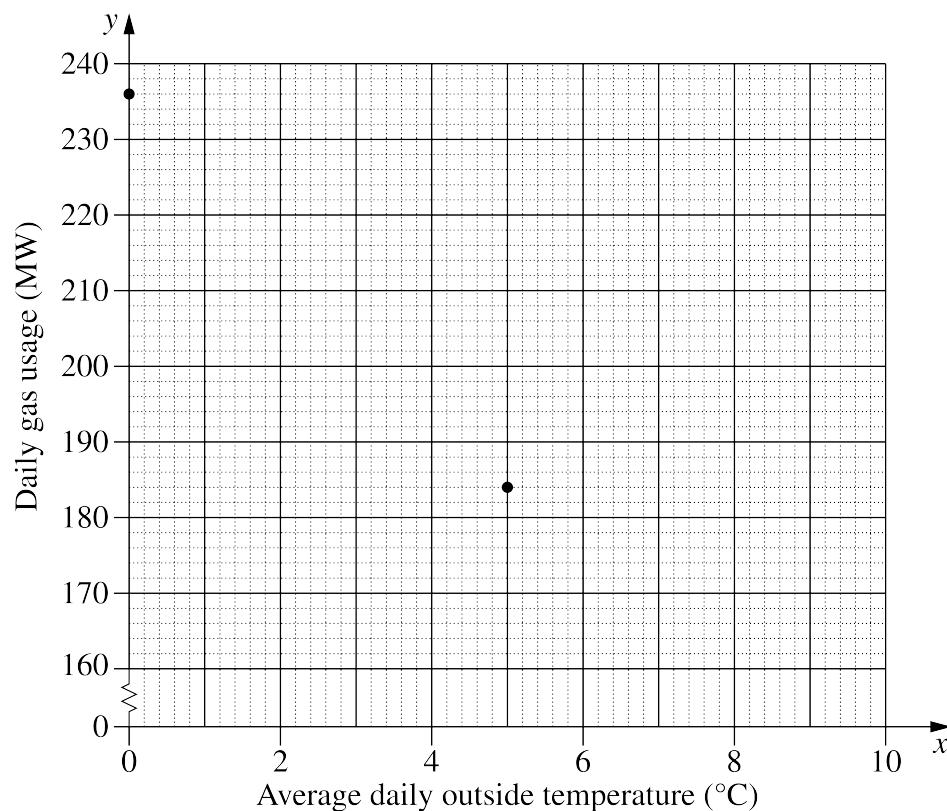
<b>Criteria</b>	<b>Marks</b>
• Correctly plots both points on the graph	3
• Calculates $\bar{x}$ and $\bar{y}$ , and plots this point on the grid, or equivalent merit	2
• Calculates $\bar{x}$ or $\bar{y}$ , or equivalent merit	1

**Sample answer:**

$$\begin{aligned}\bar{x} &= \frac{0+0+0+2+5+7+8+9+9+10}{10} \\ &= 5\end{aligned}$$

$$\begin{aligned}\bar{y} &= \frac{1840}{10} \\ &= 184\end{aligned}$$

$$\therefore (\bar{x}, \bar{y}) = (5, 184)$$



**Question 18 (b)**

Criteria	Marks
• Provides correct solution	2
• Finds the slope of the regression line, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}\text{Slope of regression line} &= \frac{184 - 236}{5} \\ &= -10.4\end{aligned}$$

Gas usage =  $236 - 10.4 \times \text{temperature}$

ie  $y = 236 - 10.4x$

**Question 18 (c)**

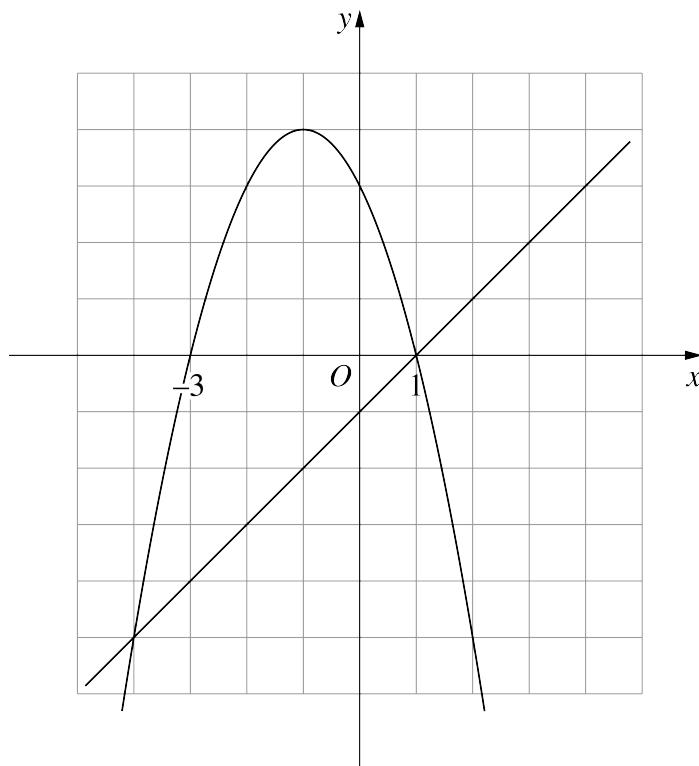
Criteria	Marks
• Identifies one problem with predicting using the regression line	1

**Sample answer:**

When temperature is  $23^\circ\text{C}$ , the regression equation provides a negative answer, which is not physically possible (negative gas usage).

**Question 19 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct graphs	2
• Provides a sketch of $f(x)$ , or equivalent merit	1

**Sample answer:****Question 19 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Finds that the graphs intersect at $x = -4$ , or equivalent merit	1

**Sample answer:**

The graphs meet when

$$x - 1 = (1 - x)(3 + x)$$

$$\begin{aligned} \therefore x - 1 &= 1 - x^2 - 2x \\ \text{ie } x + x^2 + 3x - 2 &= 0 \\ x^2 + 4x - 2 &= 0 \end{aligned}$$

From part (a),  $-4 < x < 1$ .

**Question 20**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Provides $\theta - 60^\circ = 240^\circ, 300^\circ$ , or equivalent merit	2
• Recognises that $\sin 60^\circ = \frac{\sqrt{3}}{2}$	1

**Sample answer:**

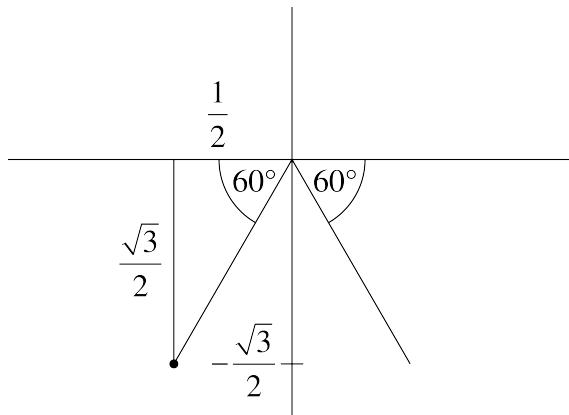
$$0^\circ \leq \theta \leq 360^\circ$$

$$-60^\circ \leq \theta - 60^\circ \leq 300^\circ$$

$$\sin(\theta - 60^\circ) = -\frac{\sqrt{3}}{2}$$

$$\theta - 60^\circ = 240^\circ \text{ or } 300^\circ \text{ or } -60^\circ$$

$$\theta = 300^\circ, 360^\circ, 0^\circ$$



**Question 21**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Finds $r^4$ , or equivalent merit	2
• Writes $ar^3 = 48$ , or equivalent merit	1

**Sample answer:**

Let  $a$  = first term and  $r$  = common ratio

$$\text{Then } ar^3 = 48 \quad \underline{\hspace{2cm}} \quad (1)$$

$$\text{and } ar^7 = \frac{3}{16} \quad \underline{\hspace{2cm}} \quad (2)$$

Dividing (2) by (1),

$$\frac{ar^7}{ar^3} = \frac{\frac{3}{16}}{48}$$

$$\therefore r^4 = \frac{1}{256}$$

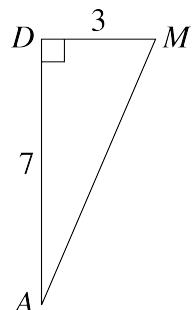
$$\therefore r = \pm \frac{1}{4}$$

$$\text{If } r = \frac{1}{4} \quad a\left(\frac{1}{4}\right)^3 = 48 \quad \therefore a = 3072$$

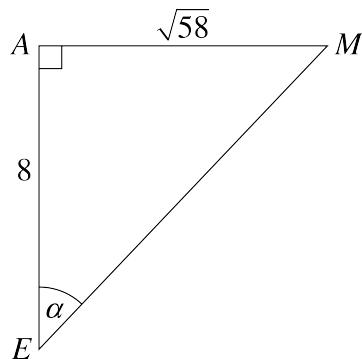
$$\text{If } r = -\frac{1}{4} \quad a\left(-\frac{1}{4}\right)^3 = 48 \quad \therefore a = -3072$$

**Question 22**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Finds the length of $AM$ , or equivalent merit	2
• Indicates that triangle $ADM$ is useful, or equivalent merit	1

**Sample answer:**Find  $AM$ 

$$\begin{aligned} AM^2 &= 3^2 + 7^2 \\ &= 9 + 49 \\ &= 58 \\ AM &= \sqrt{58} \end{aligned}$$

Triangle  $AME$ ,

$$\text{So } \tan \alpha = \frac{\sqrt{58}}{8}$$

$$\alpha = 43.59^\circ$$

so  $\alpha = 44^\circ$  (to the nearest degree)

**Question 23**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	4
• Finds the correct proportion of the group of koalas, or equivalent merit	3
• Finds the correct probability from the table, or equivalent merit	2
• Calculates the correct z value, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 z &= \frac{x - \mu}{\sigma} \\
 &= \frac{11.93 - 10.40}{1.15} \\
 &= 1.33 \quad (2 \text{ decimal places})
 \end{aligned}$$

∴ Probability from table = 0.9082

$$\begin{aligned}
 P(\text{more than } 11.93) &= 1 - 0.9082 \\
 &= 0.0918
 \end{aligned}$$

$$\begin{aligned}
 \text{Number of koalas} &= 0.0918 \times 400 \\
 &= 36.72 \\
 &= 36 \quad (\text{accept 37 as well})
 \end{aligned}$$

**Question 24 (a)**

Criteria	Marks
• Provides correct solution	1

**Sample answer:**

$$50 = (x - 2)(y - 1)$$

$$\frac{50}{x - 2} = y - 1$$

$$\text{So } y = \frac{50}{x - 2} + 1 \quad \text{as required.}$$

**Question 24 (b)**

Criteria	Marks
• Provides correct solution	4
• Finds $x = 12$ , or equivalent merit	3
• Finds $A'$ , or equivalent merit	2
• Finds an expression for the Area in terms of $x$ , or equivalent merit	1

**Sample answer:**Area of concrete path is  $2y + x - 2$ 

$$A = 2 \left( \frac{50}{x-2} + 1 \right) + x - 2$$

$$A = 2 \left( \frac{50}{x-2} \right) + 2 + x - 2$$

$$A = \frac{100}{x-2} + x$$

$$= 100(x-2)^{-1} + x$$

$$A' = 100(-1(x-2)^{-2}) + 1$$

$$= \frac{-100}{(x-2)^2} + 1$$

$$A' = 0 \quad \text{when} \quad \frac{-100}{(x-2)^2} = -1$$

$$100 = (x-2)^2$$

$$\pm 10 = x - 2$$

$$x = 12 \text{ or } -8$$

Since  $x$  is a distance, discard  $-8$ .Stationary point at  $x = 12$ .

$x$	11	12	13
$A'$	$\frac{-100}{81} + 1$	0	$\frac{-100}{121} + 1$
	\	-	/

So there is a minimum turning point at  $x = 12$ .The minimum area of the path is when  $x = 12$ .

**Question 25 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	1

**Sample answer:**

$$\begin{aligned}
 A_1 &= 10000(1.004) - M \\
 A_2 &= (10000(1.004) - M)(1.004) - M \\
 &= 10000(1.004)^2 - M(1.004) - M \quad \text{as required.}
 \end{aligned}$$

**Question 25 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Provides an expression for $A_n$ involving the sum of a geometric series, or equivalent merit	2
• Finds an expression for $A_n$ using part (a), or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 A_n &= 10000(1.004)^n - M(1 + 1.004 + \dots + 1.004^{n-1}) \\
 &= 10000(1.004)^n - \frac{M(1.004^n - 1)}{0.004} \\
 &= 10000(1.004)^n - \frac{M}{0.004} \times 1.004^n + \frac{M}{0.004} \\
 &= 10000(1.004)^n - 250M \times 1.004^n + 250M \\
 A_n &= (10000 - 250M)(1.004)^n + 250M
 \end{aligned}$$

**Question 25 (c)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Identifies $A_n > 0$ and $n = 100$ , or equivalent merit	1

**Sample answer:**

$$A_{100} > 0$$

$$(10\ 000 - 250M)(1.004)^{100} + 250M > 0$$

$$10\ 000 \times 1.004^{100} - 250M \times 1.004^{100} + 250M > 0$$

$$14\ 906.34886 - 250M(1.004^{100} - 1) > 0$$

$$14\ 906.34886 - 250M \times 0.4\ 9063 > 0$$

$$14\ 906.34886 > 122.6587\dots M$$

$$\frac{14\ 906.34886}{122.6587\dots} > M$$

$$121.527 > M$$

The largest amount Jia could withdraw is \$121.52.

**Question 26 (a)**

Criteria	Marks
• Provides correct solution	2
• Finds the antiderivative, or equivalent merit	1

**Sample answer:**

$$\frac{dx(t)}{dt} = -1.5\pi \sin\left(\frac{5\pi}{4}t\right)$$

$$\begin{aligned}x(t) &= \int -1.5\pi \sin\left(\frac{5\pi}{4}t\right) dt \\&= \frac{-1.5\pi}{\frac{5\pi}{4}} \times -\cos\left(\frac{5\pi}{4}t\right) + k\end{aligned}$$

$$\text{When } t = 0 \quad x = 11.2$$

$$\text{So } 11.2 = 1.2 \cos(0) + k$$

$$11.2 = 1.2 + k$$

$$k = 10$$

$$x(t) = 1.2 \cos\left(\frac{5\pi}{4}t\right) + 10$$

**Question 26 (b)**

Criteria	Marks
• Provides correct solution	2
• Finds the period, or equivalent merit	1

**Sample answer:**

$$\text{Period} = \frac{2\pi}{\frac{5\pi}{4}} = 1.6$$

$$10 \div 1.6 = 6.25$$

∴ Number of complete periods in 10 seconds is 6.

∴ Reaches closest point to camera 6 times.

**Question 27 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Finds the value of $b$ and $c$ , or equivalent merit	2
• Finds value of $c$ , or equivalent merit	1

**Sample answer:**

$c = 7$  Since the absolute value graph has been shifted by 7 vertically

$b = 6$  Shifted by 6 to the right

Let  $x = 3$ ,  $y = -5$

$$f(3) = a|3 - 6| + 7 = -5$$

$$3a + 7 = -5$$

$$3a = -12$$

$$a = -4$$

$$\therefore a = -4, b = 6, c = 7$$

**Question 27 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Finds that $m < \frac{7}{6}$ , or equivalent merit	1

**Sample answer:**

Line joining  $(6, 7)$  with  $(0, 0)$  has slope  $\frac{7}{6}$

$m$  must be less than  $\frac{7}{6}$  to cut the graph in two places.

Slope of right side of graph is  $-4$

$m$  must be greater than  $-4$  or it will only cut graph once

Hence  $-4 < m < \frac{7}{6}$ .

**Question 28**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	4
• Finds the $x$ -coordinate of $R$ AND the antiderivative for $y$	3
• Finds the $x$ -coordinate of $R$ OR the antiderivative for $y$	2
• Attempts to solve $\frac{dy}{dx} = 1$ , or equivalent merit	1

**Sample answer:**

$$\frac{dy}{dx} = 3x^2 - 6x - 8$$

Tangent at  $(-1, 6)$  is  $y = x + 7$ 

Slope of tangent is 1.

Solve  $\frac{dy}{dx} = 1$       ie       $3x^2 - 6x - 8 = 1$   
 $3x^2 - 6x - 9 = 0$   
 $3(x^2 - 2x - 3) = 0$   
 $3(x - 3)(x + 1) = 0$

So  $x$  coordinate of  $R$  is 3.

When  $\frac{dy}{dx} = 3x^2 - 6x - 8$   
 $y = x^3 - 3x^2 - 8x + k$       and when  $x = -1$   $y = 6$ .

So  $6 = (-1)^3 - 3(-1)^2 - 8(-1) + k$   
 $6 = -1 - 3 + 8 + k$   
 $6 - 4 = k$   
 $k = 2$

When  $x = 3$        $y = x^3 - 3x^2 - 8x + 2$   
 $= 27 - 27 - 24 + 2$   
 $= -22$

 $\therefore$  Coordinates of  $R$  are  $(3, -22)$ .

**Question 29 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Finds the derivative of $f(x)$ , or equivalent merit	1

**Sample answer:**

Mode of  $X$  will be when  $f(x)$  has a maximum.

$$f(x) = 12x^2 - 12x^3, \quad 0 \leq x \leq 1$$

$$\begin{aligned} f'(x) &= 24x - 36x^2 \\ &= 12x(2 - 3x) \end{aligned}$$

$$f'(x) = 0 \quad \text{when } x = 0 \quad \text{and when } 2 - 3x = 0$$

$$x = \frac{2}{3}$$

Discard  $x = 0$  since  $f(0) = 0$  so

the mode of  $X$  is  $\frac{2}{3}$ .

**Question 29 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Expresses $F(x)$ as an integral of $f(x)$ , or equivalent merit	1

**Sample answer:**

$$F(x) = \int_0^x 12t^2(1-t) dt$$

$$= \int_0^x 12t^2 - 12t^3 dt$$

$$= [4t^3 - 3t^4]_0^x$$

$$= 4x^3 - 3x^4$$

**Question 29 (c)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Substitutes the mode from part (a) into their cumulative distribution function, or equivalent merit	1

**Sample answer:**

$$\text{When } x = \frac{2}{3} \quad 4x^3 - 3x^4 = 4 \times \left(\frac{8}{27}\right) - 3 \times \left(\frac{16}{81}\right) \\ = 0.59$$

The probability of the variable being less than  $\frac{2}{3}$  is greater than 0.5, therefore the mode is greater than the median.

**Question 30 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Finds the $x$ values of the stationary points, or equivalent merit	2
• Finds correct derivative, or equivalent merit	1

**Sample answer:**

$$f(x) = e^{-x} \sin x$$

$$\begin{aligned} f'(x) &= e^{-x} \cos x + -e^{-x} \sin x \\ &= e^{-x}(\cos x - \sin x) \end{aligned}$$

$$f'(x) = 0 \quad \text{when} \quad \cos x = \sin x$$

$$x = \frac{\pi}{4} \text{ or } \frac{5\pi}{4}$$

$$\text{so } f(x) = e^{-\frac{\pi}{4}} \sin \frac{\pi}{4} \quad \text{or} \quad e^{-\frac{5\pi}{4}} \sin \frac{5\pi}{4}$$

The two stationary points are

$$\left( \frac{\pi}{4}, \frac{e^{-\frac{\pi}{4}}}{\sqrt{2}} \right) \quad \text{and} \quad \left( \frac{5\pi}{4}, \frac{-e^{-\frac{5\pi}{4}}}{\sqrt{2}} \right)$$

approx.

approx.

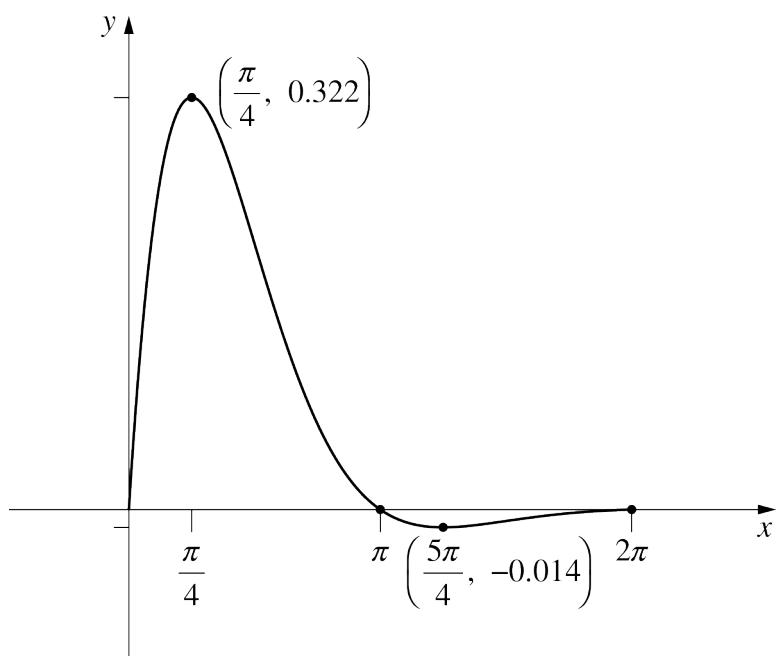
$$\left( \frac{\pi}{4}, 0.322 \right) \quad \text{and} \quad \left( \frac{5\pi}{4}, -0.014 \right)$$

**Question 30 (b)**

Criteria	Marks
• Provides correct graph	2
• Provides a graph with some correct features, or equivalent merit	1

**Sample answer:**

$$f(x) = 0 \quad \text{when} \quad \sin x = 0 \quad x = 0, \pi, 2\pi$$



**Question 31 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct reason	1

**Sample answer:**

No, since  $P(F|S) \neq P(F)$

**Question 31 (b)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	2
• Attempts to use conditional probability formula, or equivalent merit	1

**Sample answer:**

$$P(S|F) = \frac{P(S \cap F)}{P(F)}$$

$$\frac{1}{3} = \frac{P(S \cap F)}{\frac{3}{10}}$$

$$P(S \cap F) = \frac{1}{10}$$

$$P(F|S) = \frac{P(S \cap F)}{P(S)}$$

$$\frac{1}{8} = \frac{\frac{1}{10}}{P(S)} \quad (\text{since } P(S \cap F) = P(F \cap S))$$

$$P(S) = \frac{8}{10}$$

$$= \frac{4}{5}$$

**Question 31 (c)**

Criteria	Marks
• Provides correct answer	2
• Uses expression for complementary events, or equivalent merit	1

**Sample answer:**

$$1 - \left(\frac{4}{5}\right)^4 = 1 - \frac{256}{625} = \frac{369}{625} = 0.5904$$

**Question 32 (a)**

<b>Criteria</b>	<b>Marks</b>
• Provides correct solution	3
• Provides an antiderivative, or equivalent merit	2
• Provides an integral expression for the area, or equivalent merit	1

**Sample answer:**

$$\begin{aligned}
 \text{Shaded area} &= \int_0^{\ln 2} e^{-2x} - \left( e^{-x} - \frac{1}{4} \right) dx \\
 &= \int_0^{\ln 2} e^{-2x} - e^{-x} + \frac{1}{4} dx \\
 &= \left[ -\frac{1}{2}e^{-2x} + e^{-x} + \frac{1}{4}x \right]_0^{\ln 2} \\
 &= \left( -\frac{1}{2}e^{-2\ln 2} + e^{-\ln 2} + \frac{1}{4}\ln 2 \right) - \left( -\frac{1}{2} + 1 + 0 \right) \\
 &= -\frac{1}{2}e^{\ln(2^{-2})} + e^{\ln(2^{-1})} + \frac{1}{4}\ln 2 - \frac{1}{2} \\
 &= -\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} + \frac{1}{4}\ln 2 - \frac{1}{2} \\
 &= \frac{1}{4}\ln 2 - \frac{1}{8}
 \end{aligned}$$

**Question 32 (b)**

Criteria	Marks
• Provides correct solution	3
• Uses the discriminant to find $k > -\frac{1}{4}$ , or equivalent merit	2
• Attempts to form a quadratic equation, or equivalent merit	1

**Sample answer:**

We want the equation  $e^{-2x} = e^{-x} + k$  to have 2 solutions.

$$\text{ie } e^{-2x} - e^{-x} - k = 0 \quad \text{has 2 solutions}$$

$$\text{ie } (e^{-x})^2 - (e^{-x}) - k = 0 \quad \text{has 2 solutions}$$

Using the quadratic formula,

$$e^{-x} = \frac{1 \pm \sqrt{1+4k}}{2}$$

For two real solutions we want  $1+4k > 0$ , ie  $k > -\frac{1}{4}$

But for two solutions for  $e^{-x}$ , both the real solutions to the quadratic must be positive.

$$\therefore \sqrt{1+4k} < 1$$

$$\therefore 1+4k < 1$$

$$\therefore k < 0$$

$$\text{Hence } -\frac{1}{4} < k < 0.$$

# 2023 HSC Mathematics Advanced

## Mapping Grid

### Section I

Question	Marks	Content	Syllabus outcomes
1	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
2	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
3	1	MA-F1 Working with Functions	MA11-1
4	1	MA-F1 Working with Functions	MA11-1
5	1	MA-C4 Integral Calculus	MA12-7
6	1	MA-C3 Applications of Differentiation	MA12-6
7	1	MA-C2 Differential Calculus	MA12-6
8	1	MA-E1 Logarithms and Exponentials	MA11-6
9	1	MA-F1 Working with Functions	MA11-2
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### Section II

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12 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-7
12 (b)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
13	2	MA-C4 Integral Calculus	MA12-7
14	3	MA-C2 Differential Calculus	MA12-6
15 (a)	2	MA-M1 Modelling Financial Situations	MA12-2
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16	4	MA-T1 Trigonometry and Measure of Angles	MA11-3
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18 (a)	3	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-8
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18 (c)	1	MA-S2 Descriptive Statistics and Bivariate Data Analysis	MA12-10
19 (a)	2	MA-F2 Graphing Techniques	MA12-1
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20	3	MA-T2 Trigonometric Functions and Identities	MA11-1
21	3	MA-M1 Modelling Financial Situations	MA12-4
22	3	MA- T1 Trigonometry and Measure of Angles	MA11-3
23	4	MA-S3 Random Variables	MA12-8
24 (a)	1	MA-F1 Working with Functions	MA11-2
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25 (a)	1	MA-M1 Modelling Financial Situations	MA12-10
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<b>Question</b>	<b>Marks</b>	<b>Content</b>	<b>Syllabus outcomes</b>
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30 (a)	3	MA-C3 Applications of Differentiation	MA12-6
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31 (a)	1	MA-S1 Probability and Discrete Probability Distributions	MA11-9
31 (b)	2	MA-S1 Probability and Discrete Probability Distributions	MA11-7
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32 (a)	3	MA-C4 Integral Calculus	MA12-7
32 (b)	3	MA-F1 Working with Functions	MA11-1