

Writing Skills 2

Typesetting Matrices and Vectors

An $m \times n$ matrix is an array of m rows and n columns. The notation a_{ij} is used to denote the entry in the i th row and the j th column of a matrix A , so a general $m \times n$ matrix A has the form

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

A vector $\mathbf{u} \in \mathbb{R}^n$ is an $n \times 1$ matrix. Thus, \mathbf{u} has the form

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix},$$

where u_1, u_2, \dots, u_n are real numbers. To save space, it is often convenient to express a vector in terms of its transpose, so $\mathbf{u} = [u_1, u_2, \dots, u_n]^T$ or $\mathbf{u}^T = [u_1, u_2, \dots, u_n]$.

Given $\mathbf{u} = [u_1, u_2, \dots, u_n]^T$ and $\mathbf{v} = [v_1, v_2, \dots, v_n]^T$, the inner product of \mathbf{u} and \mathbf{v} , $\mathbf{u} \cdot \mathbf{v}$, is defined by

$$\mathbf{u} \cdot \mathbf{v} = \sum_{k=1}^n u_k v_k = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n.$$

The *norm* of a vector \mathbf{u} is denoted by

$$\|\mathbf{u}\| = \sqrt{\sum_{k=1}^n u_k^2} = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}.$$

Note that $\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$.

The matrix

$$\begin{bmatrix} 3 & 0 & -1 & 5 & 9 & -2 \\ -5 & 2 & 4 & 0 & -3 & 1 \\ -8 & -6 & 3 & 1 & 7 & -4 \end{bmatrix}.$$

can also be written, for example, as the 2×3 partitioned matrix or block matrix

$$\left[\begin{array}{c|c} M & N \\ \hline R & S \end{array} \right]$$