Christopher David Miller
FSMAT 201
Problem 2.1
Version 1

2.1
Show that if $B = W_M A W_N^T$, then $W_M^T B W_N = A$.

Solution

Let A be a $M \times N$ matrix.

Suppose $B = W_M A W_N^T$.

We intend to show that the above equation implies $W_M^T B W_N = A$.

$$B = W_M A W_N^T$$

Our first step is to multiply on the left sides by W_M^T,

$$W_M^T \cdot B = W_M^T \cdot W_M A W_N^T.$$

Now note that since the transformation is invertible then

$$W_M^T \cdot W_M = I.$$

After simplifying we now have

$$W_M^T B = A W_N^T.$$

Our next step is to multiply by W_N on the right sides,

$$W_M^T B \cdot W_N = A W_N^T \cdot W_N.$$

Finnaly after simplifying we are left with our desired result:

$$W_M^T B W_N = A$$