Christopher David Miller
FSMAT 201
Problem 3.1
Version 1

3.1

Let $z = a + bi$ and $w = c + di$. Prove that $\overline{zw} = \overline{z} \cdot \overline{w}$.

To prove this statement we need to introduce the definition of the conjugate of a complex number. Let q be a complex number, in other words $q = c + di$ where c and d are real numbers. The conjugate of q, denoted by $\overline{q}$, is defined by

$$\overline{q} = c - di.$$

Now we have enough definitions to finish our proof.

Proof. Let $z = a+bi$ and $w = c+di$. In order to show that $\overline{zw} = \overline{z} \cdot \overline{w}$. We need to calculate $\overline{zw}$ and $\overline{z} \cdot \overline{w}$.

$$\begin{aligned} \overline{zw} &= \overline{(a+bi)(c+di)} \\ &= \overline{ac + adi + bci - bd} \\ &= \overline{(ac - bd) + i(ad + bc)} \\ &= (ac - bd) - i(ad + bc) \end{aligned} \qquad \begin{aligned} \overline{z} \cdot \overline{w} &= \overline{(a+bi)} \cdot \overline{(c+di)} \\ &= (a - bi) \cdot (c - di) \\ &= ac - adi - bci - bd \\ &= (ac - bd) - i(ad + bc) \end{aligned}$$

We have shown that $\overline{zw} = \overline{z} \cdot \overline{w}$

$\square$

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Problem 3.2
Version 1

3.2
Let $z = a + bi$ and $w = c + di$. Prove that $|zw| = |z| \cdot |w|$.

To prove this statement we need to introduce the definition of the modulus of a complex number. Let q be a complex number, in other words $q = c + di$ where c and d are real numbers. The modulus of q, denoted by $|q|$, is defined by

$$|q| = \sqrt{c^2 + d^2}$$

Now we have enough definitions to finish our proof.

Proof. Let $z = a + bi$ and $w = c + di$. In order to show that $|zw| = |z| \cdot |w|$. We need to calculate $|zw|$ and $|z| \cdot |w|$.

$$\begin{aligned} |z| \cdot |w| &= \sqrt{a^2 + b^2} \cdot \sqrt{c^2 + d^2} \\ &= \sqrt{(a^2 + b^2)(c^2 + d^2)} \\ &= \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2} \end{aligned} \qquad \begin{aligned} |zw| &= \overline{(a + bi)} \cdot \overline{(c + di)} \\ &= (a - bi) \cdot (c - di) \\ &= ac - adi - bci - bd \\ &= (ac - bd) - i(ad + bc) \end{aligned}$$

We have shown that $\overline{zw} = \overline{z} \cdot \overline{w}$

$\square$