

1.8

Let $\mathbf{u} \in \mathbb{R}^n$. Prove that the normalization of \mathbf{u} has norm 1.

Proof. Let $\mathbf{u} \in \mathbb{R}^n$. The normalization of a vector \mathbf{u} is the vector \mathbf{v} given by

$$\mathbf{v} = \frac{\mathbf{u}}{\|\mathbf{u}\|}.$$

Also notice to calculate the norm of a vector \mathbf{v} , we need to use the following formula

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{\sum_{k=1}^n v_k^2} \\ &= \sqrt{\sum_{k=1}^n \frac{u_k^2}{\|\mathbf{u}\|^2}} \\ &= \sqrt{\sum_{k=1}^n \frac{u_k^2}{\|\mathbf{u}\|^2}} \\ &= \sqrt{\frac{1}{\|\mathbf{u}\|^2} \sum_{k=1}^n \frac{u_k^2}{1}} \\ &= \frac{1}{\|\mathbf{u}\|} \sqrt{\sum_{k=1}^n \frac{u_k^2}{1}} \\ &= \frac{1}{\|\mathbf{u}\|} \|\mathbf{u}\| \\ &= 1\end{aligned}$$

□