

## Writing Skills 2

### Typesetting Matrices and Vectors

An  $m \times n$  matrix is an array of  $m$  rows and  $n$  columns. The notation  $a_{ij}$  is used to denote the entry in the  $i$ th row and the  $j$ th column of a matrix  $A$ , so a general  $m \times n$  matrix  $A$  has the form

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}.$$

A vector  $\mathbf{u} \in \mathbb{R}^n$  is an  $n \times 1$  matrix. Thus,  $\mathbf{u}$  has the form

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix},$$

where  $u_1, u_2, \dots, u_n$  are real numbers. To save space, it is often convenient to express a vector in terms of its transpose, so  $\mathbf{u} = [u_1, u_2, \dots, u_n]^T$  or  $\mathbf{u}^T = [u_1, u_2, \dots, u_n]$ .

Given  $\mathbf{u} = [u_1, u_2, \dots, u_n]^T$  and  $\mathbf{v} = [v_1, v_2, \dots, v_n]^T$ , the inner product of  $\mathbf{u}$  and  $\mathbf{v}$ ,  $\mathbf{u} \cdot \mathbf{v}$ , is defined by

$$\mathbf{u} \cdot \mathbf{v} = \sum_{k=1}^n u_k v_k = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n.$$

The *norm* of a vector  $\mathbf{u}$  is denoted by  $\|\mathbf{u}\|$  and defined by

$$\|\mathbf{u}\| = \sqrt{\sum_{k=1}^n u_k^2} = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}.$$

Note that  $\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{u}}$ .

The matrix

$$\begin{bmatrix} 3 & 0 & -1 & 5 & 9 & -2 \\ -5 & 2 & 4 & 0 & -3 & 1 \\ -8 & -6 & 3 & 1 & 7 & -4 \end{bmatrix}$$

can also be written, for example, as the  $2 \times 3$  partitioned matrix or block matrix

$$\left[ \begin{array}{ccc|cc|c} 3 & 0 & -1 & 5 & 9 & -2 \\ -5 & 2 & 4 & 0 & -3 & 1 \\ \hline -8 & -6 & 3 & 1 & 7 & -4 \end{array} \right]$$