Tessellations

Christopher D Miller

February 5 2019

Theorem 1. *There are exactly three regular tessellations of the plane.*

Proof. A regular tessallation is a being able to cover a plane with regular polygons without any gaps or overlaps. A regular polygon is a polygon with the same length and every angle is equal. To cover a plane without gaps then the shapes interior angle has to divide 360.The formula for the interior angle of a regular polygon is

$$\frac{180(n-2)}{n}\text{where } n \text{ is the number of sides.}$$

So we need the interior angle of the regular polygon to divide 360. In other words there exist a k such that

$$\frac{180(n-2)}{n}k = 360.$$

.

$$\begin{aligned}
\frac{180(n-2)}{n}k &= 360 \\
\frac{(n-2)}{n}k &= 2 \\
(n-2)k &= 2n \\
kn - 2k &= 2n \\
kn - 2k + 4 &= 2n + 4 \\
(n-2)(k-2) &= 4
\end{aligned}$$

In order to solve this equation notice that $(n-2) \leq 4$ and $(k-2) \leq 4$. This gives us $3 \leq n \leq 6$ and $3 \leq n \leq 6$. The only solutions are $n = 3, k = 6$, $n = 6, k = 3$, $n = 4, k = 4$. So there must be only 3 regular polygons that can tesselate a plane. $\square$