Pythagorean Triples

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Proposition 1.

$$a_{k+1} = a_k + 4$$
$$b_{k+1} = \frac{1}{2}a_k + b_k + 1$$
$$c_{k+1} = \frac{1}{2}a_k + c_k + 1$$

Generates Pythagorean Triples of Height 8 with $a_0 = 20, b_0 = 21, c_0 = 29$

Proof. We will prove by induction that Proposition 1 holds for all $k \geq 0$.
Base Case: Our base case is when k = 0.So when k =0 , by definition our formula gives us a Pythagorean Triple of height 8.So our proposition is true in this case.
Induction Step: Let k $\geq$ 0 be given and suppose our proposition is true for $n = k$. We need to show that proposition 1 holds for a_{k+1}. In other words we need to show that

$$a_{k+1}^2 + b_{k+1}^2 - c_{k+1}^2 = 0$$

.So using our induction hypothesis. We can write it in the following form:

$$\big(a_k + 4\big)^2 + \big(\frac{1}{2}a_k + b_k + 1\big)^2 - \big(\frac{1}{2}a_k + c_k + 1\big)^2$$

Applying some algebra we then simplify it further to obtain :

$$\big(\frac{1}{4} - \frac{1}{4}\big) = 0$$

Thus arriving at the thing we are trying to prove. □