LaTeX under Review

Christopher D Miller

February 5 2019

1. Use the function $f_n(x) = \sum_{k=1}^{n-1} e^{x^k}$ to test that $n = 3$ is prime by computing $\frac{d^3 f}{dx^3}(0)$.

   To begin testing if 3 is a prime number we must first find the 3rd derivative of the following function.

$$
\begin{aligned}
f_3(x) &= \sum_{k=1}^{2} e^{x^k} \\
&= e^x + e^{x^2}
\end{aligned}
$$

   So we must differentiate $f_3(x)$ 3 times.

$$
\begin{aligned}
&\frac{df}{dx} = 2xe^{x^2} + e^x \\
&\frac{d^2 f}{d^2 x} = 2e^{x^2} + 4x^2 e^{x^2} + e^x \\
&\frac{d^3 f}{d^3 x} = 12xe^{x^2} + 8x^3 e^{x^2} + e^x \\
&\frac{d^3 f}{dx^3}(0) = 1
\end{aligned}
$$

2. Use this method to show that $n = 4$ is not prime.

   To begin testing if 4 is a prime number we must first find the 4th derivative of the following function.

$$
\begin{aligned}
f_4(x) &= \sum_{k=1}^{3} e^{x^k} \\
&= e^x + e^{x^2} + e^{x^3}
\end{aligned}
$$

So we must differentiate $f_4(x)$ 4 times.

$$\begin{aligned}
\frac{df}{dx} &= 2xe^{x^2} + e^x \\
\frac{d^2f}{d^2x} &= 2e^{x^2} + 4x^2e^{x^2} + e^x \\
\frac{d^3f}{d^3x} &= 12xe^{x^2} + 8x^3e^{x^2} + e^x \\
\frac{d^4f}{d^4x} &= 48x^2e^{x^2} + 12e^{x^2} + 16x^4e^{x^2} + e^x \\
\frac{d^3f}{dx^3}(0) &= 12 + 1 = 13
\end{aligned}$$

Thus 4 is not prime since the 4th derivative of this function was not 1.

3. Carefully state a theorem that the function $f_n(x)$ can be used to test whether n is prime. You do not need to include a proof.

Theorem 1. *For each positive integer $n > 1$, define the function g_n by $g_n(x) = \sum_{k=1}^{n-1} e^{x^k}$ A positive integer n is prime if and only if $\frac{d^3f}{dx^3}(0) = 1$.*