Cantor Set

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Say we have the closed interval $[0, 1]$. Lets define a set C_1 defined by taking the middle third out of the closed interval $[0, 1]$. So we have the C_1 consists of the closed intervals $\left[0, \frac{1}{3}\right]$ and $\left[\frac{2}{3}, 1\right]$. Now lets repeat this process on C_1 to obtain the set C_2. Taking the middle third out of the closed intervals $\left[0, \frac{1}{3}\right]$ and $\left[\frac{2}{3}, 1\right]$ leaving us with the closed intervals $\left[0, \frac{1}{9}\right]$, $\left[\frac{2}{9}, \frac{1}{3}\right]$,$\left[\frac{2}{3}, \frac{7}{9}\right]$, $\left[\frac{8}{9}, 1\right]$. Now for each of these closed intervals you want to repeat this process. Each iteration taking the middle third out of the closed intervals you obtain. The cantor set is the intersection of all these sets. Now an interesting fact is that this set is non-empty because the numbers 0 and 1 are included in every set. Since the set is closed.