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Oil market volatility and stock market volatility

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ABSTRACT

This paper studies the comovement between volatility of the equity market and the oil market, both for implied and realized volatilities. The wavelet methodology enables us to study this relationship on various time scales. We find that there is a strong comovement between the volatilities of the two markets. However, this comovement is time-varying and depends on the time scale. It is strong at yearly horizon, but much weaker at horizons of a few days. Moreover, implied volatility of the stock market leads the implied volatility of the oil market, whereas no such relationship is observed for realized volatilities.

1. Introduction

Participants in financial markets are subject to the volatility of their investments. Therefore, volatility plays a crucial role in portfolio optimization, risk management, hedging, derivative pricing and particularly option pricing. In 1993, the CBOE introduced a volatility index for the US stock market, the VIX index. The VIX index, often called a fear index, has become one of the most followed indicators in the financial markets.

Due to the huge success and importance of the VIX index, similar indices have been introduced not only for other equity markets (Bugge et al., 2016), but also for commodities (Birkelund et al., 2015). The unique feature of implied volatility is that it is forward-looking, whereas volatility models based on historical data are backward-looking. The implied volatility is a measure of a market risk, and it is forward-looking since it contains investors expectation of future market changes. Hence, studies based on implied volatility can help us understand how the expectations about risk are transferred from one market to another. Such studies have become increasingly popular, for example, Sari et al. (2011) found that VIX had a significantly suppressing effect on oil prices in the long run and Qadan and Yagil (2012) concluded that VIX index has significant impact on gold prices.

In recent years, commodities have become a more and more important part of many portfolios. Crude oil is probably the most important commodity in the world. Oil has a weight above 50% in the general commodity index. Moreover, oil prices have a strong impact on many other commodities. Understanding oil price volatility is important for several reasons. Not only has oil price volatility an impact on company investments (Henriques and Sadorsky, 2011) and other macroeconomic variables (Rafiq et al., 2009), but it also has a direct impact on the economies of oil importing and oil exporting countries.

Since the stock market can be considered as a proxy for the general economy, and oil is the most important commodity, we therefore study the relationship between implied volatility for the equity market (the VIX index) and implied volatility for crude oil (the OVX index). There are several papers related to our study. Ji and Fan (2012) found that the crude oil market has significant volatility spillover effects on non-energy commodity markets and that the overall level of correlation strengthened after the crisis. Guo and Ji (2013) found a significant impact of Google search query volumes on oil volatility. Haugom et al. (2014) found that

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volatility model for oil is improved when implied volatility is included in the model. Dutta et al. (2017) found that the OVX index predicts volatility of Middle East and African stock markets. Luo and Qin (2017) studied the impact of oil volatility on the Chinese stock market and confirmed the importance of forward-looking implied volatility by finding that implied volatility has significant and negative effects on the Chinese stock market while the impact of realized volatility shocks is negligible.

Papers most closely related to our work are those by Liu et al. (2013) and Maghyereh et al. (2016). Liu et al. (2013) studied transmission between the oil implied volatility (OVX) and stock market implied volatility (VIX), euro/dollar exchange rate implied volatility (EVZ) and gold price implied volatility (GVZ). Maghyereh et al. (2016) analysed the relationship between implied volatility of oil and implied volatility of various stock markets. Both these papers are based on variance decomposition and analyse the relationships on daily frequency.

We utilize the wavelet methodology, which has the advantage of enabling us to investigate the relationship between variables on various time scales. In other words, we investigate the comovement and the lead-lag relationship between VIX and OVX not just on one arbitrary time scale (e.g. daily), but on various time scales from daily to yearly. We find that the implied volatility of the equity market (VIX) and the implied volatility of the oil market (OVX) are highly correlated.

Our main contribution is the finding that the relationship between implied volatility of oil and implied volatility of stock market depends on time scale. There is only contemporary correlation (no lead/lag relationship) between VIX and OVX on short time scales (high frequencies), but there is a significant lead/lag relationship on longer time scales (lower frequencies). Our results have implications both for general understanding of financial markets as well as for traders and other market participants exposed to the volatility of the oil market.

The rest of the paper is organized as follows. Section 2 introduces the continuous wavelet transform, wavelet power, squared wavelet coherence, relative phase and cross-wavelet gain. Section 3 presents the data used in the analysis. Section 4 reports and discusses the results of the analysis. Section 5 concludes.

2. Wavelet characteristics

The continuous wavelet transform (CWT) and measures derived from the CWT, such as the cross-wavelet transform, cross-wavelet power, squared wavelet coherence and relative phase (see e.g., Torrence and Compo, 1998 or Grinsted et al., 2004) provide a favorite set of tools used to explore time-varying relationships between two time series.

While introducing the continuous wavelet transform and related measures, we make use of the introduction available in Bašta et al. (2017). Specifically, we assume the Morlet wavelet defined as

$$\psi_0(\eta) = \pi^{-1/4}\exp(i\omega_0\eta)\exp\left(-\frac{1}{2}\eta^2\right), \tag{1}$$

where ω_0 is equal to 6 and where η is a dimensionless time (Grinsted et al., 2004).

The continuous wavelet transform of a time series $\{X_t: t = 0, \ldots, N-1\}$ of length N at time t and at scale $s > 0$ is defined as (Grinsted et al., 2004)

$$W_{t,s}^X = \frac{1}{\sqrt{s}}\sum_{k=0}^{N-1} X_k\psi_0^*\left(\frac{k-t}{s}\right), \tag{2}$$

where * denotes complex conjugation. The wavelet coefficient $W_{t,s}^X$ captures how scale s contributes to the dynamics of $\{X_t\}$ at time t. Since $W_{t,s}^X$ is complex, it is advantageous to introduce the wavelet power spectrum at time t and scale s defined as $|W_{t,s}^X|^2$, i.e. as the square of the modulus of $W_{t,s}^X$. $|W_{t,s}^X|^2$ is related to the variability of $\{X_t\}$ at time t and scale s, large/small values of $|W_{t,s}^X|^2$ meaning large/small variability of $\{X_t\}$ at time t and scale s. Since $|W_{t,s}^X|^2$ is biased in favour of large-scale features, Liu et al. (2007) suggested to use the *corrected* wavelet power spectrum given as $|W_{t,s}^X|^2/s$.

To study the comovement of two time series $\{X_t: t = 0, \ldots, N-1\}$ and $\{Y_t: t = 0, \ldots, N-1\}$, the cross-wavelet transform between $\{X_t\}$ and $\{Y_t\}$ can be used, being defined as (Grinsted et al., 2004)

$$W_{t,s}^{XY} = W_{t,s}^X W_{t,s}^{Y*}. \tag{3}$$

Generally, $W_{t,s}^{XY}$ is a complex number. The modulus of $W_{t,s}^{XY}$ is called the cross-wavelet power and can be considered as the absolute covariance between $\{X_t\}$ and $\{Y_t\}$ at time t and scale s. The argument of $W_{t,s}^{XY}$ is called the relative phase and can take any value from $-\pi$ to π. It captures the lead/lag relationship between $\{X_t\}$ and $\{Y_t\}$ at time t and scale s. More specifically, if relative phase is zero, no lead or lag is present. On the other hand, positive values of the relative phase imply that $\{X_t\}$ leads $\{Y_t\}$, while negative values imply that $\{X_t\}$ lags behind $\{Y_t\}$. The relative phase can be converted to time ΔT by which $\{X_t\}$ leads $\{Y_t\}$ (if $\Delta T > 0$), or lags behind it ($\Delta T < 0$). Specifically,

$$\Delta T = \frac{\arg(W_{t,s}^{XY})}{2\pi f}, \tag{4}$$

where f denotes the Fourier frequency associated with scale s which is, for the Morlet wavelet assumed in Eq. (1), given as (Torrence and Compo, 1998) $f = 1/(1.03\ s)$.

The squared wavelet coherence between two time series $\{X_t: t = 0, \ldots, N-1\}$ and $\{Y_t: t = 0, \ldots, N-1\}$ at time t and scale s is defined as (Grinsted et al., 2004)

$$R_{t,s}^2 = \frac{\left|S\left(\frac{1}{s}W_{t,s}^{XY}\right)\right|^2}{S\left(\frac{1}{s}|W_{t,s}^X|^2\right)S\left(\frac{1}{s}|W_{t,s}^Y|^2\right)}, \tag{5}$$

where the S operator in Eq. (5) defines smoothing in time and scale. More specifically, the operator is defined as (Grinsted et al., 2004)

$$S(W_{t,s}) = S_{scale}(S_{time}(W_{t,s})), \tag{6}$$

where

$$S_{time}(W_{t,s})|_s = \left(W_{t,s} * c_1^{-\frac{t^2}{2s^2}}\right)\Bigg|_s, \tag{7}$$

$$S_{scale}(W_{t,s})|_t = (W_{t,s} * c_2\Pi(0.6s))|_t, \tag{8}$$

where c_1 and c_2 are normalization constants and Π is the boxcar function. In Eqs. (7) and (8), the * operator denotes convolution. Squared wavelet coherence can attain any value in the range from 0 to 1 and can be considered as a local (at time t and scale s) squared correlation between the time series $\{X_t\}$ and $\{Y_t\}$. If the squared wavelet coherence is close to one, a strong linear relationship is suggested at the given time t and scale s. On the other hand, squared wavelet coherence close to zero denotes a very weak linear relationship between the time series at time t and scale s.

Ge (2008) suggested that relative phase, i.e. the argument of the cross-wavelet transform, should be explored (only) in those regions (in the time-scale plane) for which the values of the squared wavelet coherence are rather high.

Mandler and Scharnagl (2014) propose to use the cross-wavelet gain defined as

$$G_{t,s} = \frac{\left|S\left(\frac{1}{s}W_{t,s}^{XY}\right)\right|}{S\left(\frac{1}{s}|W_{t,s}^X|^2\right)}, \tag{9}$$

which can be interpreted as a local absolute value of the regression coefficient of $\{Y_t\}$ on $\{X_t\}$.

Since wavelet coefficients are calculated by linearly filtering the time series using a non-causal linear filter (see Eq. (2)), the coefficients, the (corrected) wavelet power spectrum, wavelet coherence, relative phase and cross-wavelet gain cannot be directly obtained for regions close to the beginning and the end of the time series. As a result, artificial boundary conditions have to be introduced so that all the characteristics associated with times corresponding to the beginning and the end of the time series can be calculated. The region where the values of these characteristics are affected to a non-negligible extent by these artificial boundary conditions is called the *cone of influence* and the results in the cone must be interpreted with caution since they may not reflect the true underlying dynamics.

3. Data

The data were downloaded from finance.yahoo.com. As previously mentioned, the VIX index records 30-day volatility implied by options written on the S&P 500 index. The OVX index records 30-day volatility implied by options written on the United States Oil Fund.

The United States Oil Fund (USO) is an exchange traded fund that seeks to provide investors with easy exposure to the oil market. Since investing in physical oil would be too costly, it invests in oil-related financial instruments, mostly oil futures. The USO's investment objective is to track percentage changes in the price of West Texas Intermediate (WTI) light, sweet crude oil delivered to Cushing, Oklahoma, as measured by the changes in the price of the futures contract for light, sweet crude oil traded on the New York Mercantile Exchange (the "NYMEX"), less the USO's expenses. The USO is the most actively traded commodity exchange traded fund, and is the 7th most traded of all the exchange traded funds.[1] We consider the daily time series of VIX and OVX for the period from May 10, 2007 till July 28, 2016.

Besides the implied volatility indices VIX and OVX, we also consider the time series of realized volatility of S&P 500 and USO in the period from May 10, 2007 till July 28, 2016. Even though our main interest is the comovement between the implied volatility of the oil and stock market, we conduct the same analysis for realized volatilities. Implied volatility reflects primarily the expectations about the future, whereas realized volatility reflects what really happened. Therefore, investigating both types of volatility provides us with a more complete picture of comovement between volatility of oil and stock market.

The Garman and Klass (1980) estimate of realized volatility (variance) for trading day t is adjusted for opening jump (Molnár, 2012) and calculated as:

$$GK_t = 0.5(h_t - l_t)^2 - (2\log 2 - 1)c_t^2 + j_t^2, \tag{10}$$

[1] According to http://etfdb.com/compare/volume/, accessed on July 30, 2017.

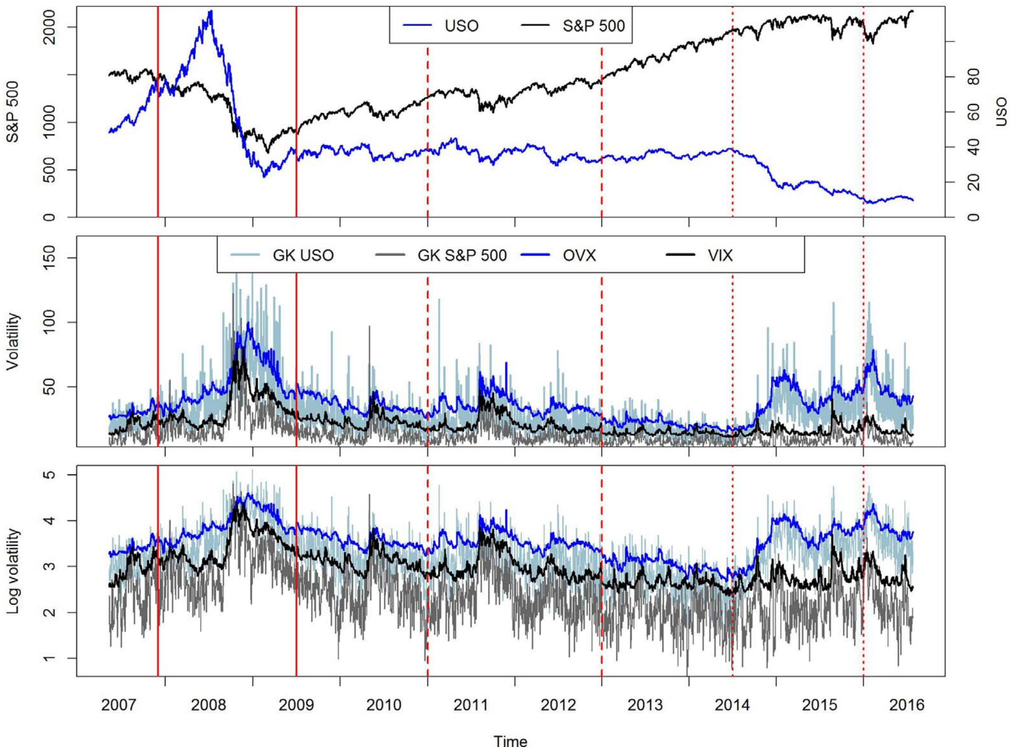

Fig. 1. *Top plot:* S&P 500 (black) and USO (blue). *Middle plot*: VIX (black), GK S&P 500 (gray), OVX (blue) and GK USO (lightblue). *Bottom plot*: LVIX (black), LGK S&P 500 (gray), LOVX (blue) and LGK USO (lightblue). The Great Recession, the Arab spring and the Oil price fall are depicted by the solid, dashed and dotted red vertical lines. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

where $c_t = \log(C_t) - \log(O_t)$, $h_t = \log(H_t) - \log(O_t)$, $l_t = \log(L_t) - \log(O_t)$ and $j_t = \log(O_t) - \log(C_{t-1})$, where log denotes the natural logarithm and C_t, O_t, H_t and L_t are the close, open, high and low price of the asset (S&P 500 or USO) for day t. Implied volatility measures volatility over a 30-day period over the whole 24 hours of each day, not just over periods from open to close. Therefore, opening jump needs to be included in the estimate of realized volatility to make it comparable with implied volatility.[2]

Implied volatility indices VIX and OVX are annualized standard deviations of the underlying assets (S&P 500 and USO) quoted as percentages, whereas the Garman–Klass estimates of realized volatilities of S&P 500 and USO calculated according to Eq. (10) are variances on a daily scale. Consequently, we take the square root of the time series of the Garman–Klass estimates of realized volatilities of S&P 500 and USO, multiply the results by the square root of 252 and further by 100, and refer to these newly obtained variables (time series) as GK S&P 500 and GK USO. GK S&P 500 and GK USO are comparable to VIX and OVX since they are measured on the same scale.

In the top plot of Fig. 1 S&P 500 and USO are plotted, in the middle plot VIX, OVX, GK S&P 500 and GK USO are displayed, and in the bottom plot of the figure the natural logarithms of VIX, OVX, GK S&P 500 and GK USO are presented, hereafter denoted as LVIX, LOVX, LGK S&P 500 and LGK USO. The summary statistics for the four time series in the middle plot of Fig. 1 are given in Table 1, whereas the summary statistics for the four time series in the bottom plot of Fig. 1 are given in Table 2.

We can observe that GK S&P 500 and GK USO (or LGK S&P 500 and LGK USO) are rather noisy compared to VIX and OVX (or LVIX and LOVX). This is not surprising, since GK are estimates of volatility for a particular day, whereas implied volatility is calculated for a 30-day period, therefore effectively averaging out differences between individual days.

From Tables 1 and 2 we can also see that the realized volatility time series have lower mean values compared to the implied volatility time series. The reason for this is that even though implied volatility is often considered as an expectation of future volatility, it is not expected volatility. Implied volatility usually reflects both expected volatility and volatility risk premium. In other words, an option trader is willing to sell options to buyers only as long as he, on average, earns some profit by doing so. If he would price options in such a way that implied volatility would be equal to expected volatility, he would, on average, not earn any profit from it. Therefore, options are usually priced in such a way that implied volatility is higher than expected volatility. It is also obvious that OVX (LOVX) is generally higher than VIX (LVIX), which implies that oil market is generally more volatile than stock market.

The period of Great Recession from the end of 2007 till the middle of 2009 is depicted by the two red vertical solid lines in Fig. 1.

[2] The necessity to include the opening jump is the reason why we do not utilize realized volatility calculated from high-frequency data. High-frequency data allow for more precise estimation of volatility, but only during the trading part of the day. Since we need to include the opening jump, high-frequency data would not be of much help to us.

Table 1
Summary statistics (mean, median, standard deviation, interquartile range, minimum, maximum, skewness and excess kurtosis) for VIX, OVX, GK S&P 500 and GK USO.

	mean	median	st. dev.	IQR	min	max	skew	ex. k.
VIX	21.4	18.5	9.8	9.5	10.3	80.9	2.3	6.8
OVX	37.6	34.6	14.5	15.8	14.5	100.4	1.3	2.2
GK S&P	13.0	10.3	10.1	8.2	1.9	124.4	3.6	21.9
GK USO	30.7	25.8	18.9	19.7	3.1	161.5	1.9	5.1

Table 2
Summary statistics (mean, median, standard deviation, interquartile range, minimum, maximum, skewness and excess kurtosis) for LVIX, LOVX, LGK S&P 500 and LGK USO.

	mean	median	st. dev.	IQR	min	max	skew	ex. k.
LVIX	2.98	2.92	0.37	0.49	2.33	4.39	0.99	0.88
LOVX	3.56	3.54	0.37	0.44	2.67	4.61	0.08	-0.01
LGK S&P	2.37	2.33	0.59	0.77	0.65	4.82	0.42	0.48
LGK USO	3.26	3.25	0.56	0.75	1.13	5.08	0.13	-0.07

This period is closely related to the period of financial crisis of 2007–2008 and the period of the subprime mortgage crisis of 2007–2009. During these periods, the stock market was highly volatile. The decreased economic growth in many sectors (manufacturing, transportation etc.) also decreased the demand for energy products, which led to falling and volatile oil prices.

Recovery from the Great Recession started in 2009 but the Arab Spring of 2011–2012 was a major geo-political event which played its role in the oil market due to the uncertainty in the level of oil production in the affected countries. This period is depicted by the two red vertical dashed lines in Fig. 1.

The period of oil price fall from the middle of 2014 till 2015 due to (or accompanied by) a long-term slowdown of several major economies such as China, Russia etc., alternative ways of reaching oil resources (hydraulic fracking) and OPEC's members decision as of November 2014 to maintain oil production at the usual levels is depicted by the two red vertical dotted lines in Fig. 1. During this period, volatility increased slightly in the stock market and more profoundly in the oil market. This period also overlaps with the Russian financial crisis (2014–2017) and with the Chinese stock market crisis (2015–2016).

4. Results

From Fig. 1 and Tables 1 and 2, it is obvious that LVIX, LOVX, LGK S&P 500 and LGK USO are less skewed and more Gaussian compared to VIX, OVX, GK S&P 500 and GK USO. Consequently, the logarithmic time series will be used in further analysis. Using the logarithmic time series also offers an appealing interpretation since changes in these time series can be directly interpreted as percentage changes in the original time series.

The corrected wavelet power spectra[3] for LVIX and LOVX are presented in Fig. 2. Time is given on the horizontal axis, while Fourier period[4] P in (trading) days is depicted on the vertical axis. The colour in the plots captures the values of the corrected wavelet power spectrum (see the colour bars on the right side of the plots) and the cone of influence is separated by the U-shaped white curve.

Fig. 2 reveals that major variability in LVIX occurs at Fourier periods larger than 32 days. The variability at these Fourier periods is pronounced especially during the Great Recession, during the start of the recovery from the Great Recession, and during the Arab Spring. Noticeable variability in LVIX at Fourier periods shorter than 32 days is present during the Oil price fall. In LVOX, a pronounced variability at Fourier periods larger than 64 days occurs during the Oil price fall, but is also present during the Great Recession and the Arab Spring. Not much variability is present at Fourier periods shorter than 64 days.

Because of the noise present in LGK S&P 500 and LGK USO, corrected wavelet power spectra for LGK S&P 500 and LGK USO (not presented in any figures) exhibit major variability not only at large Fourier periods (64 days and larger), but also at short ones (2–16 days).

Squared wavelet coherence between LVIX and LOVX is depicted in Fig. 3. The red colour in the figure denotes values of wavelet coherence close to one, whereas the blue colour denotes values of wavelet coherence close to zero (see the colour bar on the right side of the figure). Relative phase is depicted by arrows only in those regions - in agreement with the suggestion of Ge (2008), see Section 2 - where the corresponding squared wavelet coherence is larger or equal to the 70th percentile of the distribution of the squared wavelet coherence. Generally, the value of relative phase is equal to the angle formed by the positive horizontal axis (the initial side) and the line of the arrow (the terminal side). The angle can take any value from $-\pi$ to and π. For example, if the arrow points to the right, relative phase is equal to zero and no delay is present between the time series. If the arrow points upwards, relative

[3] The R software (R Core Team, 2017) and the biwavelet R package (Gouhier et al., 2016) have been used to obtain various wavelet-analysis figures presented in the text.

[4] The Fourier period P is given as $P = 1/f$.

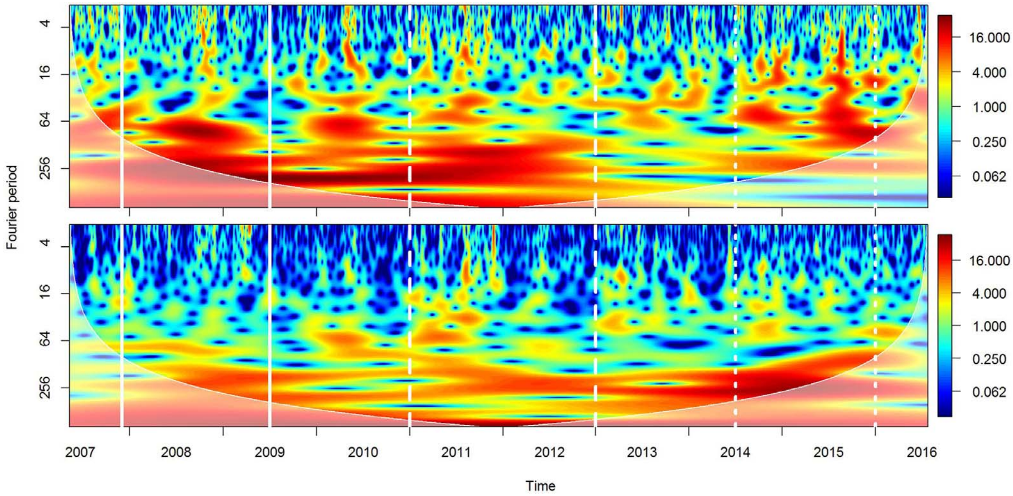

Fig. 2. Corrected wavelet power spectra for LVIX (top plot) and LOVX (bottom plot). The value of the corrected wavelet power spectrum is depicted in colour. The semi transparent regions at the left and right boundary of the plots separated by the white U-shaped curves are the cones of influence. The Great Recession, the Arab spring and the Oil price fall are depicted by the solid, dashed and dotted white vertical lines. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

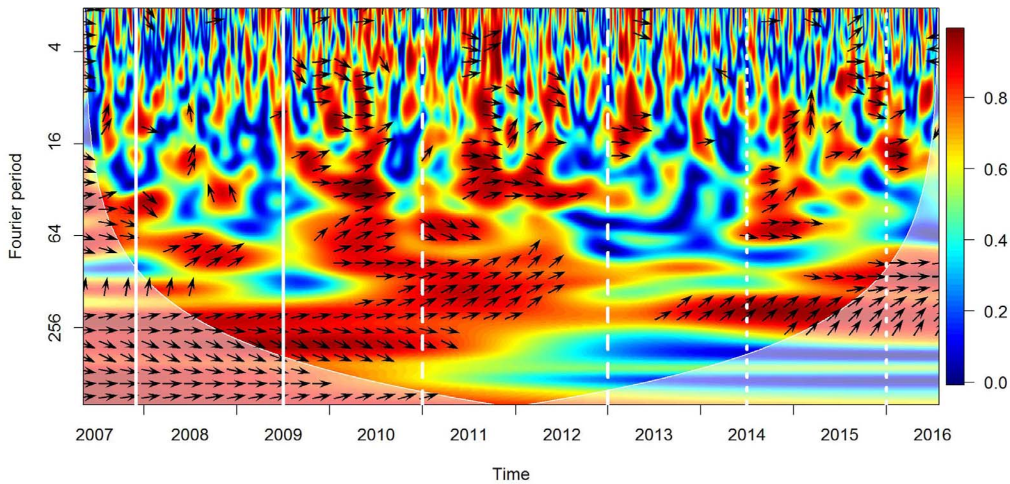

Fig. 3. Squared wavelet coherence and relative phase between LVIX and LOVX. The value of squared wavelet coherence is depicted in colour and the value of relative phase by arrows. The semi transparent region at the left and right boundary of the figure separated by the white U-shaped curve is the cone of influence. The Great Recession, the Arab spring and the Oil price fall are depicted by the solid, dashed and dotted white vertical lines. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

phase is $\pi/2$ and LVIX leads LOVX by $\pi/2$ in phase. If the arrow points downwards, relative phase is $-\pi/2$ and LVIX lags behind LOVX by $\pi/2$ in phase. Results of squared wavelet coherence and relative phase for LGK S&P 500 and LGK USO are presented in Fig. 4.

Let us discuss the results for LVIX and LOVX at first. The relationship is time-varying. LVIX and LOVX are strongly correlated at most Fourier periods from the middle of the year 2009 till the middle of the year 2012 (which covers the start of the recovery from the Great Recession and the start of the Arab Spring), whereas outside these dates the correlation is mostly strong at Fourier periods larger than 32 days and mostly not so strong at shorter ones. This conclusion holds in general - specifically, the squared wavelet coherence mostly increases with Fourier period, which can be observed in the top plot of Fig. 5 where the squared wavelet coherence averaged *over time* is depicted by the black line. The averaged squared wavelet coherence peaks at Fourier periods close to 1 year and decreases as we move towards shorter Fourier periods, such as 1 month or 1 week, or towards Fourier periods longer than 1 year. This suggests that in the long run (i.e. in the dynamics associated with approx. 1-year Fourier periods) LVIX and LOVX are much more tightly connected than in the medium or short run (i.e. in the dynamics associated with approx. 1-month or 1-week Fourier periods).

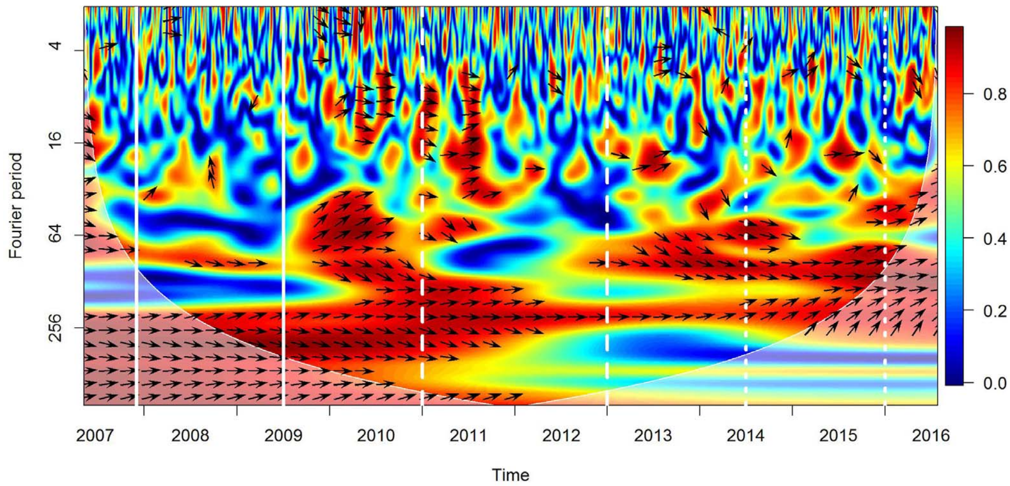

Fig. 4. Squared wavelet coherence and relative phase between LGK S&P 500 and LGK USO. The meaning of the colours, arrows, the U-shaped region and the white vertical lines is analogous to that of Fig. 3.

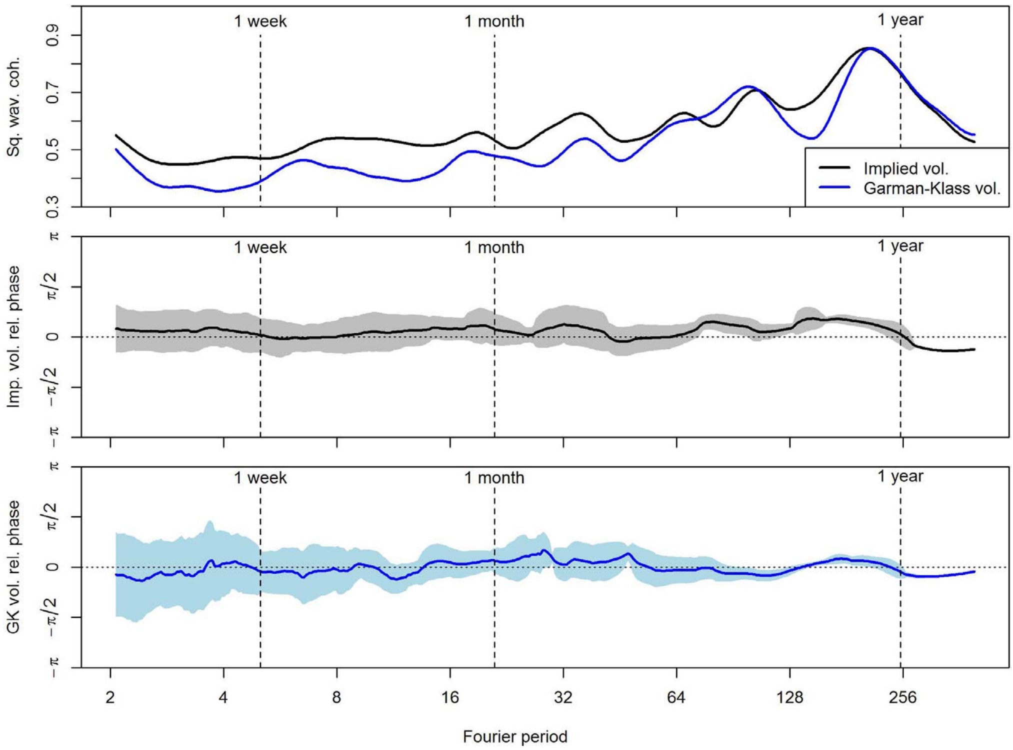

Fig. 5. *Top plot*: Squared wavelet coherence between LVIX and LOVX (black curve) and between LGK S&P 500 and LGK USO averaged over time (blue curve). Note that the range of the y-axis is from 0.3 to 1. *Middle plot*: Relative phase between LVIX and LOVX averaged over time (black curve) ± the corresponding standard deviation (the gray region). *Bottom plot*: Relative phase between LGK S&P 500 and LGK USO averaged over time (the blue curve) ± the corresponding standard deviation (the light blue region). *Note*: Values of the squared wavelet coherence in the cone of influence are excluded from the calculation of the average in the top plot. Values of relative phase in the cone of influence as well as those which correspond to regions with squared wavelet coherence below the 70th percentile of the distribution of squared wavelet coherence are excluded from the calculation of average phase and from the calculation of the corresponding standard deviation in the middle and bottom plot. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

Relative phase (in regions with sufficiently large squared wavelet coherence) seems to be very close to zero at all Fourier periods with some events in the time-frequency plane where LVIX seems to slightly lead LOVX.[5] This is confirmed by the middle plot of Fig. 5 where relative phase is averaged over time[6] and the average is plotted against Fourier period; see the black curve in the middle plot of Fig. 5. The boundaries of the gray regions in the plot correspond to the average relative phase ± the standard deviation[7] of the relative phase. The width of the gray region at a particular Fourier period tells us how the distribution of relative phase is variable. Taking the standard deviation of relative phase into account, we can see that relative phase is on average effectively zero at all Fourier periods. The only exceptions are Fourier periods slightly shorter than 100 days or 200 days where the average values of relative phase are above zero and the standard deviation is relatively small. These exceptions are associated with the following regions in Fig. 3: a.) the Great Recession and Fourier periods slightly shorter than 100 days, b.) the Arab Spring and Fourier periods a little shorter than 200 days, and c.) the epoch starting approx. one year before the Oil price fall and Fourier periods round 200 days. In all these regions LVIX seems to slightly lead LOVX.

A likely reason why the VIX index slightly leads the OVX index is the fact that the VIX index is in general much more important. VIX index is calculated from options which are more liquid than options used to calculate the OVX index. Moreover, neither VIX nor OVX are tradable (nobody can buy either of these indices directly). However, various derivatives (futures, options, exchange traded funds and exchange traded notes) with the VIX index as an underlying asset are actively traded (Bordonado et al., 2017), whereas no such derivatives exist for the OVX index as an underlying asset. All these factors may cause that the VIX index reacts to new information faster than the OVX index, and therefore leads the OVX index.

By comparing Fig. 4 with Fig. 3, it can easily be discerned that concerning the dynamics at Fourier periods *larger than 64 days*, the squared wavelet coherence and relative phase between LKG S&P 500 and LGK USO are very similar to those between LVIX and LOVX, the difference being that LKG S&P 500 and LGK USO seem to be a little less correlated during the Arab Spring and slightly more correlated during the Oil price fall and the epoch prior to this fall when compared to the correlation between LVIX and LOVX. Moreover, there is a smaller total area in the time-frequency plane where LKG S&P 500 leads LGK USO compared to the area where LVIX leads LOVX. This could indicate that in some cases, expectations about future volatility, which were transmitted from equity to oil market, did not materialize in the actual (realized) volatility.

We can also note that LGK USO is correlated with and leads LGK S&P 500 at the scale of approximately 64 days from the beginning of 2013 through the beginning of 2014 (see Fig. 4), while such a relationship is not observed between implied volatilities (Fig. 3). In order to interpret this, we have to again remember that implied volatilities reflect the expectations about the future. This means that the behavior which happened in realized volatilities was not anticipated beforehand by implied volatilities. Further, a possible reason why realized volatility of the oil market was correlated with and leading the realized volatility of the stock market could be that the oil price played a more important role during the calm year 2013 than in other years.

At Fourier periods *shorter than 64 days* LGK S&P 500 and LGK USO are seen to be less correlated and generally not so steady in relative phase when compared to LVIX and LOVX. This is also confirmed by Fig. 5 where the time-averaged squared wavelet coherence and the time-averaged relative phase are plotted as a function of Fourier period for both pairs of time series. The reason for the lower squared wavelet coherence and for the "unsteady" relative phase between LKG S&P 500 and LGK USO (when compared to the case of LVIX and LOVX) can presumably be explained by noticing that both the time series of Garman–Klass estimates of realized volatility are rather noisy and the noise dominates the dynamics at Fourier periods of 32 days and less. The presence of the noise generally leads to a decrease in the correlation between LKG S&P 500 and LGK USO and to an unsteady relative phase at Fourier periods lower than 1 week.

As documented in Fig. 5, the strongest comovement between the implied as well as between the realized volatility time series occurs round the Fourier period of 210 days. Consequently, in Fig. 6 we plot the wavelet squared coherence and relative phase for the Fourier period of one year (252 trading days) as a function of time. It is obvious that the relationship between the implied as well as between the realized volatility time series is time-varying at this Fourier period. Further, LVIX seems to slightly lead LOVX at this Fourier period, while the lead of LGK S&P 500 before LGK USO is less pronounced. This is in accordance with what we observed previously.

It should also be noted that missing arrows in Fig. 3 do *not* imply that *no* lead/lag relationship is present since arrows are plotted only in those regions where the corresponding squared wavelet coherence is sufficiently large as explained in the previous text. This can be demonstrated, for example, for the Fourier period of one year at the end of 2012 and the beginning of 2013 where no arrows are plotted in Fig. 3 since the squared wavelet coherence has decreased a lot, but LVIX still leads LOVX (see Fig. 6). The likely reason for the decrease in the squared wavelet coherence is the aftermath of the Arab Spring, during which oil price was subject to more idiosyncratic shocks which translated into weaker comovement with the stock market.

Next we look at the comovement between the VIX and OVX from a viewpoint of VIX being the independent, and OVX being the dependent variable. Cross-wavelet gain between LOVX (response variable) and LVIX (explanatory variable) is depicted in Fig. 7. Regions associated with the green colour correspond to events where a change in LVIX is accompanied by a change in LOVX of a similar size. Light/dark blue colours correspond to regions where a change in LVIX is accompanied by a slightly smaller/much smaller change in LOVX. On the other hand, light/dark red colours correspond to regions where a change in LVIX is accompanied by a

[5] There are also some events in the time-frequency plane where LOVX seems to lead LVIX. However, these events seem to be less frequent than the events where LVIX leads LOVX.

[6] Circular average is used to calculate the average relative phase.

[7] Circular standard deviation is used to calculate the standard deviation of relative phase.

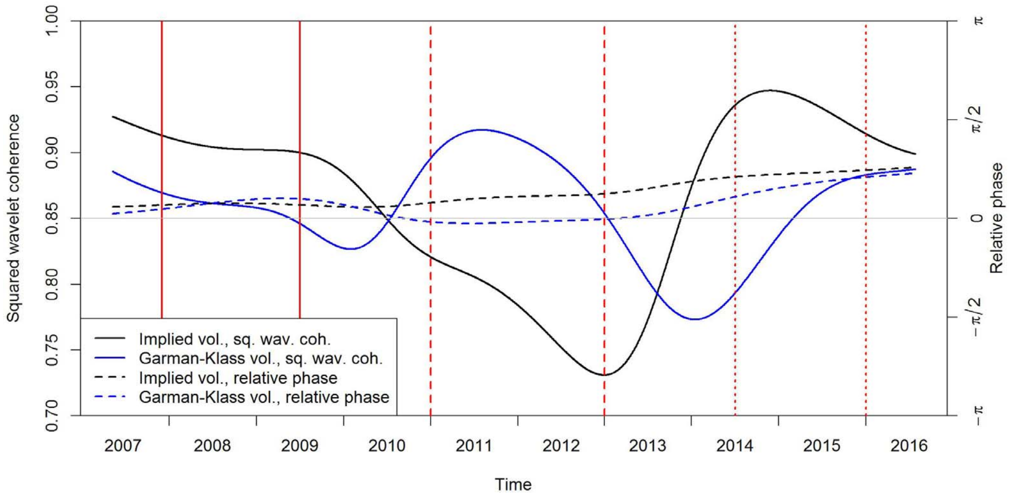

Fig. 6. Squared wavelet coherence and relative phase between LVIX and LOVX and between the LGK S&P 500 and LGK USO for the Fourier period of 252 days. Note that the left vertical axis ranges from 0.7 to 1. The Great Recession, the Arab spring and the Oil price fall are depicted by the solid, dashed and dotted red vertical lines. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

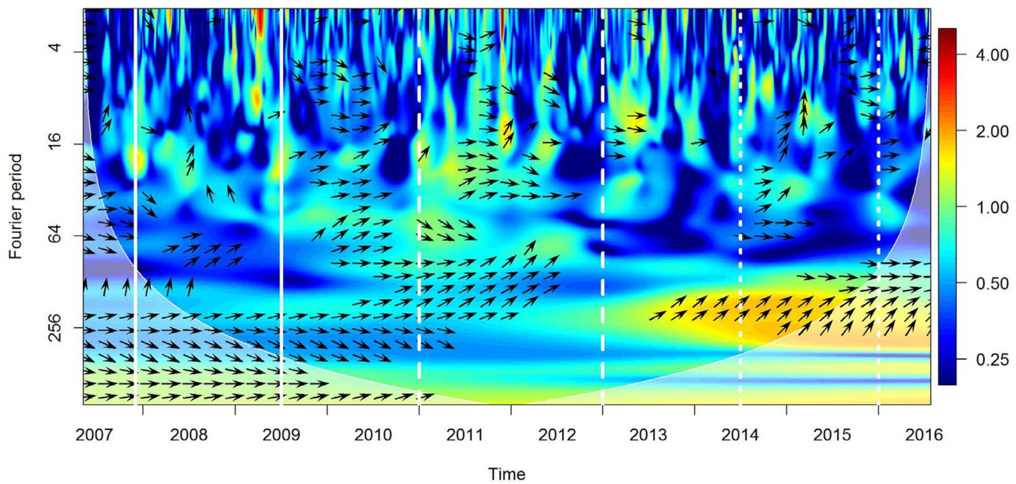

Fig. 7. Cross-wavelet gain between LOVX (response variable) and LVIX (explanatory variable). The cross-wavelet gain is depicted in colour. The value of relative phase is depicted by arrows in exactly the same regions as in Fig. 3, i.e. in the regions with a high value of squared wavelet coherence. The semi transparent region at the left and right boundary of the figure separated by the white U-shaped curve is the cone of influence. The Great Recession, the Arab spring and the Oil price fall are depicted by the solid, dashed and dotted white vertical lines. (For interpretation of colours in this figure, the reader is referred to the web version of this article.)

slightly larger/much larger change in LOVX. Analogously to Fig. 3, relative phase is depicted by arrows in the regions where the squared wavelet coherence is larger or equal to the 70th percentile of the distribution of the squared wavelet coherence - the value of the cross-wavelet gain will be interpreted and discussed only in these regions since this helps us to understand the nature of the comovement in the regions where the time series are strongly correlated.

Since changes in LVIX and LOVX can be directly interpreted as percentage changes of VIX and OVX, the interpretation of Fig. 7 is straightforward. Specifically, during the dates from the start of the year 2010 till the end of the year 2012 (which covers the start of the recovery from the Great Recession, and also the Arab spring) percentage changes of VIX in regions with large squared wavelet coherence (which correspond mostly to Fourier periods of 16 days and larger) are accompanied by percentage changes of OVX of a similar size. This is in agreement with Fig. 1 where medium and long-run changes in LVIX and LOVX are correlated and comparable in size during these dates. During the Great Recession, percentage changes of VIX in regions with large squared wavelet coherence (and especially those regions where Fourier periods are larger than 16 days) are generally accompanied by slightly smaller percentage changes of OVX. This is also in agreement with Fig. 1 where long-run changes of LVIX are correlated with long-run changes of LOVX of smaller amplitudes. Approximately one year before the Oil price fall, percentage changes of VIX in regions with large squared wavelet coherence and Fourier periods of 64 days and larger are accompanied by percentage changes of OVX with larger amplitudes.

We present the cross-wavelet gain only for the case where VIX is the independent and OVX the dependent variable and not vice versa since, as follows from Eqs. (5) and (9), the cross-wavelet gain for the latter case is equal to the squared wavelet coherence divided by the cross-wavelet gain for the former case. Consequently, the two cross-wavelet gains are complementary, the latter cross-wavelet gain bringing no new information which would not be included in the former cross-wavelet gain. Our choice of VIX being the independent and OVX the dependent variable is natural in the view of the previous results where VIX was seen to lead OVX.

5. Conclusions

We have employed wavelet analysis to study the relationship between the stock market and oil market volatility. We used the logarithm of volatility of the S&P 500 equity index and USO oil fund and of their implied volatilities VIX and OVX.[8] Wavelet analysis allowed us to study both the frequency as well as the temporal aspect of the relationship.

As expected, our findings show that the implied volatility of the equity market (VIX) and the implied volatility of the oil market (OVX) are highly correlated. This result is in accordance with Liu et al. (2013) who also find strong relationship between the OVX and VIX. However, the correlation between the stock and oil market volatility, whether measured as implied or realized volatility, is time-varying and depends on the time scale. It is strongest at Fourier periods round 210 days and gets weaker as we decrease or increase the Fourier period. Moreover, the VIX index slightly leads the OVX index, while this feature is weaker between realized volatilities.

We also have several more specific findings. Namely, during the Great Recession LVIX was correlated with LOVX especially at Fourier periods larger than 64 days. LVIX is suggested to have slightly led LOVX. A one percentage change of VIX was generally accompanied by a percentage change of OVX of a *smaller* size. This suggests that the Great Recession primarily affected the stock market, the impact on the oil market happening simultaneously or with a slight delay, and with lower amplitude.

Further, from the middle of 2009 till 2012 (which covers the start of the recovery from the Great Recession and the Arab Spring) LVIX generally exhibited strong correlation with LOVX at most of the explored Fourier periods, while slightly leading LOVX at some events in the time-frequency plane. A one percentage change of VIX was generally accompanied by an approximately one percentage change of OVX at most Fourier periods. This suggests that the recovery from the Great Recession demonstrated itself very similarly in both the markets with the stock market slightly leading the oil market.

Further, the Oil price fall (and the overlapping epochs of the Russian financial crisis and the Chinese stock market crisis) resulted in the correlation of LVIX and LOVX at several Fourier periods and impacted the oil market more strongly than the stock market despite the fact that the stock market led the oil market at some events.

Our results have implications both for general understanding of financial markets as well as for traders and other market participants exposed to the volatility of the oil and stock market. We found that volatilities of the oil and stock market are more strongly correlated on yearly horizon than on horizons of several days. This implies that diversification benefits for traders or investors exposed to volatility of both the oil and the stock market in the horizon of a few days are higher than for traders or investors on the yearly horizon.

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[8] Realized volatility is estimated from Garman–Klass formula (Garman and Klass, 1980) and its logarithm is denoted as LGK S&P 500 and LGK USO, whereas logarithm of the VIX and OVX indices are denoted as LVIX and LOVX.

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