Extra Credit Problem

October 15, 2013

Problem 1

For tridiagonal matrices, such as those given in Problem 1, a fast algorithm was developed called the Thomas algorithm. Go here for a description of this algorithm:

`http://en.wikipedia.org/wiki/Tridiagonal_matrix_algorithm`

a. (10 pts) Implement the algorithm in `Matlab` for the system $At = b$ given as in equation (1) with high-level comments to show your understanding of the algorithm. Use your implementation to compute solutions for the system given in Problem 1 for $N = 400$ and $N = 1000$. Compare the 2-norm of the solutions found using Thomas and backslash `A\b` in `Matlab` for both $N = 400$ and $N = 1000$.

b*. (15 pts) A $n \times n$ matrix A is said to be strictly diagonally dominant if

$$|A_{ii}| > \sum_{j=1}^{i-1} |A_{ij}| + \sum_{j=i+1}^{n} |A_{ij}| \qquad \text{for all } i \text{ between 1 and } n$$

where A_{ij} denotes the entry in the i-th row and j-th column. Show that strict diagonal dominance guarantees that Thomas's algorithm will work.

c*. (5 pts) The matrix in equation (1) is not strictly diagonally dominant. Nevertheless, the Thomas algorithm works, which we hope you found out in part (a) of this problem. How can you explain that it does?