# Some Comments on Assignment 3

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## Problem 1

(a) Basis for column space of a matrix $A$. There are 2 ways to find this.

- Perform GE on $A$ to get an upper-triangular matrix $U$. Then the basis of column space of $A$ is formed by the columns of ORIGINAL MATRIX $A$ that correspond to the columns with pivots of $U$. Many of you took the columns with the pivots from matrix $U$ which is wrong.
- Perform GE on $A^T$ to get an upper-triangular matrix $U_1$. Then the basis of column space of $A$ is formed by the rows with pivots of $U_1$.

(b) For this problem, when you get matrix $B$, you have to make sure that $B$ has rank 3. This is because if rank of $B$ is less than 3, then the null space of $B$ will be strictly larger than the row space of $A$. And if rank of $B$ is larger than 3, then the null space of $B$ is strictly less than row space of $A$. The problem asks you to find $B$ whose null space is IDENTICAL to row space of $A$.

For example

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 \end{bmatrix}$$

Note that rank of $B$ is 2. The null space of $B$ is spanned by

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

This is not identical to row space to $A$.

## Problem 2

For this problem, you have to show that the conditions on $m, n, r$ that you claim are both *sufficient* and *necessary* conditions.

In order to show that condition $a$ is a *sufficient* condition for statement $p$ to be true, you must show that condition $a$ implies statement $p$ ($a \implies p$).

In order to show that condition $a$ is a *necessary* condition for statement $p$ to be true, you must show that statement $p$ implies condition $a$ ($p \implies a$).

# Problem 5

This is a general comment on proving statement $p$ implies statement $q$ ($p \implies q$). There are many ways to do this. However, one thing that is wrong is starting with assume $q$ is true.

For example, problem 5 asks you to show that if $V$ and $W$ are 3 dimensional subspaces of $\mathsf{R}^5$, then $V$ and $W$ have a nonzero vector in common. In this case, $p$ is the statement: "$V$ and $W$ are 3 dimensional subspaces of $\mathsf{R}^5$". And $q$ is the statement: "$V$ and $W$ have a nonzero vector in common". I found that some people start with $q$ is true and try to show that $p$ is true, that is, start with "suppose $V$ and $W$ have a nonzero vector in common", and try to show that "$V$ and $W$ are 3 dimensional subspaces of $\mathsf{R}^5$". This is not correct.