CS3055_Q.3_A_C_D_F

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Q.3) A) Given $f(t) = \begin{cases} 4 ; & 0 \leq x < 3 \\ 0 & x > 3 \end{cases}$

To find:- $L[f(t)]$, $L[f'(t)]$

Solution:-

$$L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^3 4 \cdot e^{-st}\, dt + 0$$

$$= 4 \left[\frac{e^{-st}}{-s}\right]_0^4$$

$$= 4 \left[-\frac{e^{-4s}}{s} + \frac{1}{s}\right]$$

$$* \boxed{L[f(t)] = 4\left[\frac{1 - e^{-4s}}{s}\right]}$$

$$L[f'(t)] = -f(0) + sL[f(t)]$$

$$f(0) = 4$$

$$\therefore L[f'(t)] = -4 + \cancel{s} \cdot 4\left[\frac{1-e^{-4s}}{\cancel{s}}\right]$$

$$= -4 + 4[1 - e^{-4s}]$$

$$* \therefore \boxed{L[f'(t)] = -4e^{-4s}}$$

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Q.3).c) $f(x) = e^{-x}$, $0 < x < 1$

Soln :- Let, $f(x) = \sum_{n=1}^{\infty} b_n \sin\left[\frac{n\pi x}{l}\right]$ - {Half range Sine}

$l = 1$

$\therefore b_n = \frac{2}{l} \int_0^1 f(x) \sin\left[\frac{n\pi x}{l}\right] dx$

$b_n = \frac{2}{1} \int_0^1 e^{-x} \sin(n\pi x)\, dx$

$b_n = 2 \left[\frac{1}{1^2+n^2\pi^2} \cdot e^{-x} [(-1)\sin n\pi x - n\pi \cos n\pi x]\right]_0^1$

$b_n = 2 \left[\frac{e^{-1}}{1^2+n^2\pi^2} [-\underset{(0)}{\sin(\pi n)} - n\pi \underset{(-1)^n}{\cos n\pi}] - \frac{(1)}{1^2+n^2\pi^2} [0 - n\pi(1)]\right]$

$\therefore b_n = 2 \left[\frac{e^{-1}}{1+n^2\pi^2} [-(-1)^n n\pi] + \frac{n\pi}{1^2+n^2\pi^2}\right]$

$\cancel{\therefore b_n = \frac{2 n\pi}{1^2+n^2\pi^2} [n\pi - e^{-1}(-1)^n]}$

$\therefore b_n = \frac{2n\pi}{1^2+n^2\pi^2} [1 - e^{-1}(-1)^n]$

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$$\therefore \boxed{f(x) = 2 \sum_{n=1}^{\infty} \frac{n\pi}{1^2 + n^2\pi^2} \left[1 - e^{-1}(-1)^n\right] \sin\left[\frac{n\pi x}{1}\right]}$$

$$\therefore f(x) = 2\pi \left[\frac{\left[1+e^{-1}\right] \sin \pi x}{1^2 + \pi^2 1^2} + \frac{\left[1-e^{-1}\right] \sin 2\pi x}{1^2 + 2^2 \pi^2} + \frac{\left[1+e^{-1}\right] \sin 3\pi x}{1^2 + 3^2 \pi^2} + \cdots \right]$$

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Q. 3) a) $\underline{Sol^n}$:- $\omega = u + iv$

$$u = r^2 \sin 2\theta$$

$u_r = 2r \sin 2\theta$ — {Partially diff u w.r.t r

$u_{rr} = 2 \sin 2\theta$ {Partially diff u_r w.r.t

$u_\theta = 2r^2 \cos 2\theta$ {Partially diff u w.r.t

~~$u_{\theta\theta} = -4r^2 \cos$~~

$u_{\theta\theta} = -4r^2 \sin 2\theta$ {Partially diff. u_θ w.r.t θ}

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$$

{Laplace eq^n}

Substituting above values in Laplace eq^n.

$$L.H.S = 2\sin 2\theta + \frac{1}{\cancel{r}} \cdot 2\cancel{r}\sin 2\theta - \frac{1}{\cancel{r^2}} \cdot 4\cancel{r^2}\sin 2\theta$$

$$= 4\sin 2\theta - 4\sin 2\theta$$

$$= 0$$

$$L.H.S = R.H.S$$

$\because$ u satisfied Laplace eq^n

$\therefore$ It is an harmonic function.

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$\because$ u is harmonic it will satisfied C-R eq^n.

$\therefore$ $u_r = \frac{1}{r} v_\theta$; $u_\theta = -r v_r$ — {C-R eq^n}

$2r \sin 2\theta = \frac{1}{r} v_\theta$ — { $u_r' = 2r \sin 2\theta$ }

$v_\theta = 2r^2 \sin 2\theta$

Integrating v_θ w.r.t. to θ

$\therefore \int v_\theta \, d\theta = 2r^2 \int \sin 2\theta \, d\theta + c$

$v = \cancel{2} r^2 \cdot \left[\frac{-\cos 2\theta}{\cancel{2}} \right] + c$

$v = -r^2 \cos 2\theta + c$

$\therefore \boxed{v(r, \theta) = -r^2 \cos 2\theta + c}$

$\boxed{w = r^2 \sin 2\theta - i r^2 \cos 2\theta + c}$

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Q.3) F) Given:- $P(X=x) = \frac{1}{16} ({}^4C_x) \rightarrow (x = 0,1,2,3,4)$

Soln:- Probability mass function:-

$X=x$	0	1	2	3	4
$P(X=x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

$$E(x) = \sum_{i=0}^{4} p_i x_i$$

$$E(x) = 0 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2 \times \frac{6}{16} + 3 \times \frac{4}{16} + 4 \times \frac{1}{16}$$

$$E(x) = \cancel{2.75}\ 2$$

$$E(x^2) = \sum_{i=0}^{4} p_i x_i^2$$

$$E(x^2) = 0 \times \frac{1}{16} + 1^2 \times \frac{4}{16} + 2^2 \times \frac{6}{16} + 3^2 \times \frac{4}{16} + 4^2 \times \frac{1}{16}$$

$$\therefore E(x^2) = 5$$

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$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

$$\sigma = \sqrt{5 - 2^2}$$

$$\boxed{\sigma = 1}$$

$$\boxed{V(x) = 1}$$