Q3)

A) $f(t) = \begin{cases} 4, & 0 \leq x < 3 \\ 0 & x > 3 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$= \int_0^3 4 \cdot e^{-st}\, dt + 0$$

$$= 4\left[\frac{e^{-st}}{-s}\right]_0^3$$

$$= 4\left[-\frac{e^{-3s}}{s} + \frac{1}{s}\right]$$

$$= 4\left[\frac{1-e^{-3t}}{s}\right]$$

$$\mathcal{L}\{f'(t)\} = -f(0) + s\mathcal{L}\{f(t)\}$$

$$= -4 + s \cdot 4\left[\frac{1-e^{-3s}}{s}\right] = 4[1-e^{-3s}] - 4$$

$$\mathcal{L}\{f'(t)\} = -4e^{-3s}$$

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c) $f(x) = e^{-x}$

Let $f(x) = \sum_{n=1}^{\infty} b_n \sin\left[\frac{n\pi x}{l}\right]$

$l = 1$

$$b_n = \frac{2}{l}\int_0^l f(x) \sin\left[\frac{n\pi x}{l}\right] dx$$

$$= \frac{2}{1}\int_0^1 e^{-x} \sin(n\pi x) dx$$

$$= 2\left[\frac{1}{1^2 + n^2\pi^2} \cdot e^{-x}\left[(-1)\sin n\pi x - n\pi \cos n\pi x\right]_0^1\right]$$

$$= 2\left[\frac{e^{-1}}{1+n^2\pi^2}\left[-\sin(\pi n) - n\pi \cos n\pi\right] - \frac{1}{1+n^2\pi^2}\left[0 - n\pi(1)\right]\right]$$

$$b_n = 2\left[\frac{e^{-1}}{1+n^2\pi^2}\left[-(-1)^n n\pi\right] + \frac{n\pi}{1+n^2\pi^2}\right] = \frac{2n\pi}{1+n^2\pi^2}\left[1 - e^{-1}(-1)^n\right]$$

$$f(x) = 2\sum_{n=1}^{\infty} \frac{n\pi}{1+n^2\pi^2}\left[1 - e^{-1}(-1)^n\right] \sin\left(\frac{n\pi x}{1}\right)$$

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D) $u = r^2 \sin 2\theta$

$u_r = 2r \sin 2\theta$

$u_{rr} = 2 \sin 2\theta$

$u_\theta = 2r^2 \cos 2\theta$

$u_{\theta\theta} = -4r^2 \sin 2\theta$

$$u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta} = 0$$

$$LHS = 2\sin 2\theta + \frac{1}{r} 2r \sin 2\theta - \frac{1}{r^2} \cdot 4r^2 \sin 2\theta$$

$$= 4\sin 2\theta - 4\sin 2\theta$$

$$= 0$$

$\therefore$ Harmonic

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C-R eqⁿ

$$u_r = \frac{1}{r} v_\theta \qquad u_\theta = r v_r$$

$$2r\sin 2\theta = \frac{1}{r} v_\theta$$

$$v_\theta = 2r^2 \sin 2\theta$$

~~v_θ~~ = Integrating v_θ

$$\int v_\theta \, d\theta = 2r^2 \int \sin 2\theta \, d\theta$$

$$v = 2r^2 \left[\frac{-\cos 2\theta}{2}\right] + c$$

$$v = -r^2 \cos 2\theta + c$$

$$v(r,\theta) = -r^2 \cos 2\theta + c$$

$$w = \cancel{r^2} \; r^2 \sin 2\theta - i r^2 \cos 2\theta + c$$

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F)

$$E(x) = \sum_{i=1}^{4} p_i x_i$$

$$= 0 \times \frac{1}{16} + 1 \times \frac{4}{16} + 2 \times \frac{6}{16} + 3 \times \frac{4}{16} + 4 \times \frac{1}{16}$$

$$E(X) = 2$$

$$E(x^2) = \sum_{i=0}^{4} p_i x_i^2$$

$$= 0 \times \frac{1}{16} + 1^2 \times \frac{4}{16} + 2^2 \times \frac{6}{16} + 3^2 \times \frac{4}{16} + 4^2 \times \frac{1}{16}$$

$$= 5$$

$$\sigma = \sqrt{E(x^2) - (E x)^2}$$

$$= \sqrt{5 - 2^2}$$

$$V(x) = 1$$

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