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3)

R_1	R_2	$d_i = R_1 - R_2$	d_i^2
3	2	1	1
5	1	4	16
7	4	3	9
1	5	-4	16
2	7	-5	25
8	6	2	4
6	3	3	9
4	8	-4	16

$\Sigma d_i^2 = 96$

$$R = 1 - \left[\frac{6 \Sigma di}{n^3 - n}\right]$$

$$= 1 - \left[\frac{6 \times 96}{8^3 - 8}\right]$$

$$= 1 - 1.14285$$

$$= \cancel{0} - 0.14285$$

4) $f(x) = \frac{(\pi - x)^2}{4}$ $(0, 2\pi)$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} \frac{(\pi - x)^2}{4} = \frac{1}{4\pi} \int_0^{2\pi} (\pi - x)^2 \, dx = \frac{1}{4\pi} \int_0^{2\pi} (\pi^2 - 2\pi x + x^2) dx$$

$$a_0 = \frac{1}{4\pi} \left[\pi^2 x - \pi x^2 + \frac{x^3}{3}\right]_0^{2\pi} = \frac{1}{4\pi} \left[2\pi^3 - 4\pi^3 + \frac{8\pi^3}{3}\right]$$

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$$= \frac{1}{4\pi}\left[\frac{2}{3}\cdot\pi^3\right]$$

$$a_0 = \frac{\pi^2}{6}$$

6) $f(x) = x \qquad (0, l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\int_0^l x\,dx$$

$$= \frac{2}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{2}{l}\left[\frac{l^2}{2}\right] = l$$

$$a_n = \frac{2}{l}\int_0^l x \cos\left(\frac{n\pi x}{l}\right)dx$$

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$$a_n = \frac{2}{l}\left[\frac{x \sin n\pi x \cdot l}{n\pi} - \left(-\cos\left(\frac{n\pi x}{l}\right)\left(\frac{l}{n\pi}\right)^2\right)\right]_0^l$$

$$= \frac{2}{l}\left[\cos\left(\frac{n\pi x}{l}\right)\left(\frac{l}{n\pi}\right)^2\right]_0^l$$

$$= \frac{2}{l}\left(\frac{l}{n\pi}\right)^2\left[\cos n\pi - \cos 0\right]$$

$$= \frac{2l}{(n\pi)^2}\left[(-1)^n - 1\right]$$

for odd $a_n = -\frac{4l}{n^2\pi^2}$

even $a_n = 0$

Parsevals

$$\frac{2}{l}\int_0^l [f(x)]^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2$$

$$\Rightarrow \frac{2}{l}\int_0^l x^2\,dx = \frac{l^2}{2} + \sum_{\substack{n=1\\ odd}}^{\infty}\left[\frac{-4l}{(n\pi)^2\pi^2}\right]^2$$

$$\frac{2}{l}\left[\frac{l^3}{3}\right] = \frac{l^2}{2} + \sum_{\substack{n=1\\ \text{for odd}}}^{\infty}\frac{16 l^2}{n^4\pi^4}$$

$$\frac{2l^2}{3} - \frac{l^2}{2} = \frac{16l^2}{\pi^4}\left[\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right]$$

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$$\frac{\pi^2}{6} \times \frac{\pi^4}{(6)^2} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

$$\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

7)

$$f(x) = x - x^3$$

$$f(-x) = \cancel{f}(-x) - (-x)^3$$

$$= -x + x^3$$

$$= -(x - x^3)$$

$$= -(f(x))$$

$\therefore$ odd $\therefore$ $a_0 = a_n = 0$

$\therefore$ $\cancel{b_n = \sum_{n=1}^{\infty}}$

$$b_n = \frac{\cancel{1}}{2} \int_{-\cancel{1}}^{1} (x - x^3) \sin \frac{n\pi x}{2} \, dx$$

$$= \cancel{2} \int_{-1}^{1} (x - x^3) \sin n\pi x$$

$$= \cancel{2} \left[(x - x^3)\left(\frac{-\cos n\pi x}{n\pi}\right) - \left[(1 - 3x^2)\left(\frac{-\sin n\pi x}{(n\pi)^2}\right)\right] + \right.$$

$$\left. \left((-6x)\left(\frac{\cos n\pi x}{(n\pi)^3}\right) - (-6)\left(\frac{-\sin n\pi x}{(n\pi)^4}\right)\right)\right]_{-1}^{1}$$

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$\cancel{b_n = 2\left[\left(0\right) - \left(\right.\right.}$

$$b_n = \left[\left((0) - \left(\frac{3\sin n\pi}{(n\pi)^2}\right) - \left(\frac{6\cos n\pi}{(n\pi)^3}\right) - \frac{6\sin n\pi}{(n\pi)^4}\right) - \left[(-6)(-1)\left(\frac{-\cos}{n^3}\right.\right.\right.$$

$$= \left(\left(\frac{-6\cos n\pi}{(n\pi)^3}\right) - \left[\frac{-6\cos n\pi}{n^3\pi^3}\right]\right)$$

$$\cancel{b_n = \frac{-12(-1)^n}{(n\pi)^3}} \qquad b_n = 0$$

$$\cancel{b_2 = \frac{-12}{2^3}\frac{(-1)^2}{\pi}}$$

$\therefore b_2 = 0$

r) $6x + 3y = 31 \qquad 3x + 2y = 26$

$6x + 3y = 31$

$$x = \frac{31}{6} - \frac{3}{6}y$$

$$bx_y = \frac{-1}{2}$$

$3x + 2y = 26$

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8) $6x + 3y = 31$ $3x + 2y = 22$

$6x + 3y = 31$

$x = 31 - 3y$

$y = 13 - \frac{3}{2}x$

$b_{yx} = \frac{3}{2}$

$r^2 = b_{xy} * b_{yx}$

$= \frac{-1}{2} \cdot \frac{-3}{2} = \frac{3}{4}$

$r = -0.8660$ ∵ both are (-)ve

4)

x	y	x^2	xy
0	3	0	0
1	6	1	6
2	8	4	16
3	11	9	33
4	13	16	52
5	14	25	70
15	55	55	177

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$\Sigma y = Na + b\Sigma x$

$55 = \cancel{6a + b(15 \times 10)}\ (6a + 15b) \times 10$

$550 = 60a + 150b$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$177 = (15a + 55b) \times 4$

$708 = 60a + 220b$

$$\begin{aligned} 708 &= 60a + 220b \\ 550 &= 60a + 150b \\ - \quad & \quad - \quad\ - \\ \hline 158 &= 70b \end{aligned}$$

$b = 2.2571$

$a = 3.\cancel{25}71$

$y = 3.5241 + 2.2571x$

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1) $f(x) = x^2$

$f(-x) = x^2 \therefore$ even

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos nx + b_n \sin nx\right]$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx \, dx$$

$$= \frac{2}{\pi}\left[x^2 \frac{\sin nx}{n} - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[2x\left(\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{4}{\pi n^2}\left[x \cos nx\right]_0^{\pi}$$

$$= \frac{4}{\cancel{\pi} n^2}\left[\pi \cos n\pi - 0\right]$$

$$a_n = \frac{4}{n^2}(-1)^n$$

$a_1 = -4$

$b_1 = 0$

$\therefore a_1 + b_1 = -4$