

Name:- Srinaman Iyer

Q3) $L^{-1} \left(\frac{s+2}{s(s+3)} \right)$

$$\frac{s+2}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$s+2 = A(s+3) + B(s)$$

a) $s=0, A = \frac{2}{3}$

b) $s = -3$
 $\therefore -3+2 = -3B$

$$B = 1/3$$

$$L^{-1} \left(\frac{s+2}{s(s+3)} \right) = L^{-1} \left(\frac{2}{3s} + \frac{1}{3(s+3)} \right)$$

$$= \frac{2}{3} + \frac{1}{3} e^{-3t}$$

$$4) \phi(s) = \frac{4}{s-2} \cdot \frac{4}{s+2}$$

$$\phi_1 = \frac{4}{s-2}, \quad \phi_2 = \frac{4}{s+2}$$

$$\mathcal{L}^{-1}(\phi_1) = 4e^{2t}, \quad \mathcal{L}^{-1}(\phi_2) = 4e^{-2t}$$

$$\mathcal{L}^{-1}[\phi(s)] = \int_0^t 4e^{-2t} \cdot \cancel{e^{2(t-u)}} \cdot 4 \, du$$

$$= 16 \int_0^t e^{2t-4u} \, du$$

$$= \frac{16}{-4} \left[e^{2t-4u} \right]_0^t$$

$$= -4 \left[e^{2t-4u} \right]_0^t$$

$$= -4 \left[e^{2t-4t} - e^{2t-0} \right]$$

$$= \frac{4}{2} \cdot \frac{e^{2t} - e^{-2t}}{2}$$

$$= 8 \sinh 2t$$

$$Q6 \quad L^{-1} \left[\frac{1}{2} \log \left(\frac{s^2+4}{s^2+9} \right) \right]$$

$$\phi(s) = \frac{1}{2} \log \frac{s^2+4}{s^2+9}$$

$$= \frac{1}{2} \left(\log(s^2+4) - \log(s^2+9) \right)$$

$$\begin{aligned}\phi'(s) &= \frac{1}{2} \left(\frac{2s}{s^2+4} - \frac{2s}{s^2+9} \right) \\ &= \frac{s}{s^2+4} - \frac{s}{s^2+9}\end{aligned}$$

$$\begin{aligned}L^{-1}(\phi(s)) &= -\frac{1}{t} \cdot L^{-1}(\phi'(s)) \\ &= -\frac{1}{t} L^{-1} \left(\frac{s}{s^2+4} - \frac{s}{s^2+9} \right) \\ &= -\frac{1}{t} (\cos 2t - \cos 3t) \\ &= \frac{1}{t} (\cos 3t - \cos 2t)\end{aligned}$$