Q2) A)

1) $R = \{(a,b) \mid (a+b) \text{ is even}\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,3), (1,5), (3,1), \cancel{(2,4)}, (5,1), (2,4), (4,2), (3,5), (5,3)\}$

Reflexive:- $(1,1), (2,2), (3,3), (4,4), (5,5) \in R$
$\therefore$ Reflexive

Symmetric:-
$a+b = b+a$
$\therefore (a,b) \in R$ & $(b,a) \in R$
$\therefore$ Symmetric

Transitive:-
If $a+b$ = even
$b+c$ = even
Then $a+c$ has to be even
$1+3=4$ (even)
$3+5=8$ (even)
$1+5=6$ (even)

$3+5=8$ (even)
$5+1=6$ (even)
$3+1=4$ (even
$\therefore$ Transitive
$\therefore$ It is reflexive, symmetric & transitive, hence it is equivalenc

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iii) a) Ring:-
The algebraic structure (R, +, ·) which consists of a non empty set R along with 2 binary operation like addition & multiplication (·), then it is called a ~~ring~~ ring
An algebraic system (R, *, o) consisting of non empty set R any 2 binary operations like *(*) and o defined on R such that
i) (R, *) is an algebraic group & (R, o) is semigroup
ii) The operation o is distributive over operation * is said to be ring

b) Cyclic group
A cyclic group is cycloid & monoid in which every element in the set has an inverse element
eg: The set of complex nos. {1, -1, i, -i} under multiplication operation is cyclic group

c) Monoid
An algebraic structure <A, o>, where the o is a binary operation is called a monoid if:
o is associative & there exists an identity element.
eg: S = {1, 2, 3, 4 . . .} with multiplication

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d) Normal Subgroup
A subgroup is called a normal subgroup if for any $a \in G$ $aH = Ha$
Consider alg system $<H, \ >$ where $H = \{0, 2, 3\}$

$$0 * H = 0H = \{0 \times 0, 2 \times 0, 3 \times 0\} = \{0, 2, 3\}$$

e) Planner graph
A graph G is called Plannar graph if it can be drawn in a plane without any edge ~~crosesed~~. crossed

B)

i) a) Euler Path
→ An eulers Path is a simple path that contains every edge of G

b) Eulers Circuit
→ It is in a graph G such that its a simple circuit containing every edge of G

c) Condition of eulers path
→ A connected multigraph with at least 2 vertices has eulers circuit if all its vertices have even degree
→ If G is connected and has exactly 2 vertices that are odd then eulers path begins from 1 odd to 2^{nd} odd vertex

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Hamiltons Path

→ A simple path in G graph that passes through every vertex exactly once is called Hamiltonian path

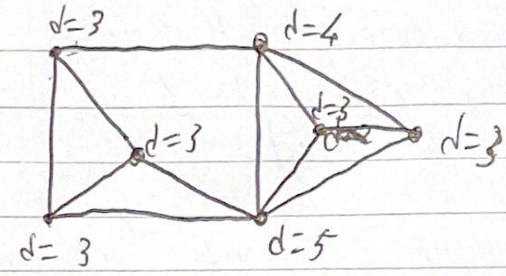
Hamilton Circuit

→ A circuit in graph ~~G~~ G that ~~passes~~ passes through every vertex is called Hamilton circuit.

In the Figure

Eulers Path:- Doesnot have Euler Path because it has odd degree vertices

Eulers circuit:



∴ There are exactly 3 vertices with odd degree hence euler circuit is not possible

→ Hamilton Path → :- 1 2 3 4 5 6 7

→ Hamilton Circuit → :- 1 2 3 4 5 6 7 1

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