Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
B		2109173

Australia Fires: How Drones In The Area Overall Planning

Summary

The Australian bushfires have always been a major concern around the world.To assist the Victorian Government in developing procurement plans for SSA and Radio Relay UAVs for frequent fires, we developed mathematical models to explore drone planning strategies for fire response.

For the first problem, we first established a calculation model for the characteristics of the fire site to quantify the fires in the region, and directly linked the fire intensity with the number of fire extinguishing points in a single fire site.Therefore, through this model, we can carry out the simulation of the fire in any area.Then, the UAV overall planning model is established, which can make reasonable planning according to the existing fire-extinguishing point information, and solve the number matching and specific location information of SSA UAV and radio relay UAV in the region.In the constraint conditions of the model, we mainly consider the fireman's safety requirements, fireman's signal propagation requirements, the necessary connection requirements between repeater and EOC, and the establishment of repeater group propagation network. Carry out the relevant restriction according to these four aspects.The strategy of radio signal propagation selection and repeater network connectivity is established. In the process of solving the problem, the improved particle swarm optimization algorithm is used to divide the solving process into two parts: the main global optimal solution and the main local optimal solution.

For the second problem, we processed the collected Australian fire data over the years and obtained four levels of fire intensity.The ARMR time series model is used to predict the future fire scale.The data samples are divided into sample training set and simulation verification set.Under the condition of error less than 2%, the possible fire grade in the future is predicted.Through prediction, we get a cycle of about five years in the future, and give the curve of fire intensity change.We also provide a time frame in which another large fire season is likely to occur within the next decade, and a formula for calculating the increased cost of drone purchases to cope with extreme fire events over the next decade.

For the three problem, In order to better adapt to the impact of terrain on UAV position planning, we improved and optimized the model established in the first question.A detailed analysis of vegetation in Victoria was added, and the simulation of real fire was further realized from the vegetation coverage of the terrain. The Cellular Automata model was used to generate wildfires, which was more closely related to the actual situation.At the same time, the network connectivity model of the repeater cluster established in Problem 1 is upgraded and optimized.Considering the height difference of the actual terrain which is too undulating, the closed-loop network in the non-horizontal plane of the UAV in three-dimensional space is studied.Finally, the improved UAV overall planning model which can be reasonably planned according to the terrain is obtained.

Finally, summarizing the results of our model,We provided a detailed budget request to the Victorian Government,The budget request includes the respective number of SSA and Radio Relay UAVs to be purchased in a given region, as well as a budget estimate to be rolled out across Victoria.In addition, some practical suggestions for the future are given based on our prediction of fire level changes over the next ten years.

Contents

1 Introduction

1.1 Problem Background

Australia is the largest island country in the world. Forest fires are a very common ecological phenomenon in the continent of Australia, especially in the summer, late spring and early autumn season of each year. The dry, hot and windy climate often causes a fire danger period of 3-5 months. On 28 July 2020, the World Wide Fund for Nature (WWF) released a report showing that bushfires in Australia between 2019 and 2020 (Figure 1) have killed or displaced nearly 3 billion animals, killed 33 people and destroyed more than 3,000 homes. The fires that lasted for such a long time and affected such a wide range reflect the tragedy of the Commons in global climate change governance, which is also a question of global climate governance policies, and also has a great impact on the global economy.

(a) Australian bushfire (b) Total fires in Australia over a month

Figure 1: Australia under fire

Because of the relatively small size, at the same time, has the very strong flexibility in the operation process, and battery life is long, can real-time imaging and data transmission, easy to spy on special fire and top target to close up, so the drones in fire fire fighting and rescue work has good information reconnaissance and transmission application base (Figure 2). The application of unmanned aerial vehicle system in forest fire prevention monitoring can help to detect forest fires in advance, grasp fire information in time, deploy fire prevention forces quickly, and change disaster into no disaster and major disaster into minor disaster. In order to achieve the purpose of automatic early warning, carry out comprehensive monitoring of the forest, improve the ability of fire prevention and disaster relief, and reduce the harm caused by forest fires, strengthen the research and application of UAV system is imperative and urgent.

1.2 Problem Analysis

The most important factors the Victorian Government looks for when purchasing UAVs are capability, safety and economy.Understandably, the drone's capabilities are reflected in its overall coverage of the fire scene, with SSA drones in the same area paired with radio repeater drones to work together on missions.The main function of the SSA UAV is to connect to the firefighters at each fire fighting point to ensure the personal safety of the firefighters.The main

(a) Put wings on the firefighters (b) Fire-fighting drone

Figure 2: Application of UAV in fire fighting

function of the radio repeater UAV is to expand the transmission of the signal to solve the shortcoming of the short and unstable transmission of the handheld radio signal.

In order to ensure the effectiveness of fire fighting, we think it is very necessary for the model to meet the actual use of the fire site.Under the premise of normal, safe and efficient work, the design of optimal planning is carried out so that the government needs to pay less capital. This is the basic principle of our model establishment.

The team analyzed the whole human-machine fire linkage process. In order to maximize the functional value of SSA UAV and repeater UAV, the following complete operation mode was proposed in three application scenarios:

- The first state is "Daily Patrol Mode",Patrol in key periods (Australian fire season, high temperature weather period) and key areas according to the plan,In order to reduce the cost, we plan to arrange the UAV to fly in a circular route in the area, so as to achieve the monitoring in a larger area, and can detect the fire outbreak in a short time and upload to EOC.
- The second is "fire human-machine interaction",Through man-machine cooperation, to achieve the function of joint command of various fire sites, This paper focuses on the establishment of the model in such a scene, through the simulation of the fire situation, the number and position of the two UAV planning.
- The third state is "Patrol Guard",After extinguishing the fire, there is still a certain risk of reignite. After extinguishing the fire, firefighters still need to be left for inspection. At this time, the thermal imaging camera of SSA UAV can continue to play its function, detect abnormal areas of high temperature, help firefighters clean up the dark fire, and guarantee the personal safety of firefighters.

2 Preparation of the Models

2.1 Assumptions

1. Through the analysis of the question, we believe that both SSA UAV and radio repeater UAV are essentially based on the same basic UAV. By carrying SSA equipment or repeater equipment on the UAV, a hybrid UAV can be formed to achieve more powerful functions.
2. It is assumed that each fire site can only set fire points from the outside of the fire circle,

without considering the fire points that exist inside the fire site due to the actual terrain.

3. Assume that the extinguishing points are evenly distributed around the ring of fire.
4. It is assumed that the power consumption of the UAV in the hover operation state is very small, which is basically negligible compared to the power consumption required for flight.
5. Assume that the radio video and telemetry equipment carried by the SSA UAV is still powered in the fire.
6. The high temperature and other adverse conditions caused by fire have no effect on the normal operation of the repeater.
7. The standard range of radios in high altitude mountains is 2km.
8. The SSA UAV has a coverage radius of 2km.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Definition
c	Fire perimeter
R	Ring of fire spread speed
t_0	Burning time
s	Fire area
t	Time required to extinguish a fire
n	Number of extinguishing points
R_1	Line of fire speed
R_2	Extinguish speed of fire line
C	The number of SSA drones
p_{ci}	Position coordinates of the SSA UAV
D	The number of relay drones
p_{di}	The position coordinates of the relay drone
p_{ik}	Position coordinates of the extinguishing point
F	Range of repeater radio
p_{ci}	Position coordinates of the SSA UAV
D	The number of relay drones
p_{di}	The position coordinates of the relay drone

3 UAV hybrid planning model

3.1 Characteristic model of a single fire site

Firstly, we study the specific situation of a single fire site and get the characteristic state value of each fire site. The number of fire extinguishing points plays an important role in the overall planning of fire extinguishing.

The calculation formula of the perimeter and area of the fire ring is:

$$c = 3Rt_0 \tag{1}$$

$$s = \frac{3}{4}(Rt_0)^2 \tag{2}$$

where

- c is the perimeter of the fire circle
- R is the spreading speed of the ring of fire
- t_0 is the burning time
- s is the fire area.

Based on the calculation model of the perimeter and area of the fire circle, the speed of advancing fire line and the speed of extinguishing fire line are introduced.It is known that only when the fire line extinguishing speed is greater than the fire line advancing speed, the fire can be gradually extinguished;When the extinguishing speed of the fire line is less than the advancing speed of the fire line, the circle of fire will become larger and larger, the uncontrollability will increase, and the danger level will increase.Fire extinguishing is a dynamic process, so the calculation model of the time required to extinguish a fire and the number of points required to extinguish a fire is as follows:

$$t = \frac{c}{NR_2 - 3R_1} \tag{3}$$

$$n = \frac{c + 3R_1 t}{tR_2} \tag{4}$$

where

- t is the time required to extinguish the fire
- n is the number of extinguishing points
- R_1 is the advance speed of the line of fire
- R_2 is the extinguishing speed of the fire line

The calculation model of fire site features we proposed takes full account of fire size and fire intensity, as shown in Figure 3. It is worth noting that the fire severity level of each fire site is directly considered in the above model, and the most intuitive response is proportional to the area of the fire site and the speed of advancing the fire line, and inversely proportional to the speed of extinguishing the fire line.The larger the area of the fire, the more intense the fire burns and the more uncontrollable it is.The greater the advance speed of the fire line, the smaller the extinguishing speed of the fire line, the more serious the overall fire, it is necessary to increase more fire fighting points, in order to better deal with the fire.The speed of advancing the fire line, the speed of extinguishing the fire and the total area of the fire site determine the difficulty of putting out the fire.

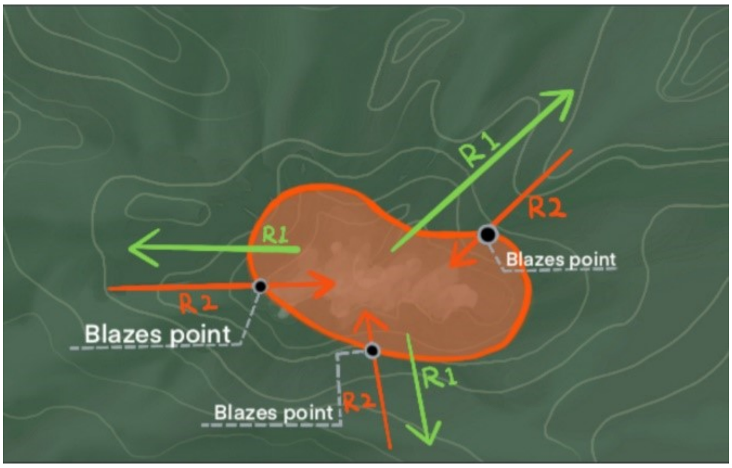

Figure 3: Single fire site feature model

3.2 Standardized treatment of multiple fire sites

In order to make the model applicable to any area that is not fixed (fire fighting operations are usually based on administrative district planning), we have carried out some standardized treatment for fires within the area.

According to Hypothesis 2 and Hypothesis 3 in the second part of the paper, it is determined that the extinguishing points are evenly distributed on the outside of the fire ring. Therefore, the required extinguishing points can be calculated through the above fire site feature model when simulating each small fire.

It is assumed that there are a total of N fire sites in this region, and each fire site is marked as $N_i(0 \le i \le N)$, and the i fire site N_i has m_i extinguishing points.Therefore, the coordinate set of the extinguishing point of the fire site N_i is defined as $\{p_{i1}, p_{i2}, p_{i3}, ..., p_{imi}\}$, the three-dimensional coordinates of all fire-fighting points are expanded as follows:

$$\begin{cases} \{(x_{11}, y_{11}, z_{11}), (x_{12}, y_{12}, z_{12}), \dots (x_{1m1}, y_{1m1}, z_{1m1})\} \\ \{(x_{21}, y_{21}, z_{21}), (x_{22}, y_{22}, z_{22}), \dots (x_{2m2}, y_{2m2}, z_{2m2})\} \\ \dots \\ \{(x_{i1}, y_{i1}, z_{i1}), (x_{i2}, y_{i2}, z_{i2}), \dots (x_{imi}, y_{imi}, z_{imi})\} \\ \dots \\ \{(x_{N1}, y_{N1}, z_{N1}), (x_{N2}, y_{N2}, z_{N2}), \dots (x_{Nmn}, y_{Nmn}, z_{Nmn})\} \end{cases} \tag{5}$$

Frontline firefighters at each fire fighting point should work together, which can be simplified to a single ground fire signal end.Fires can be large or small, and different sites dedicated to extinguishing the same fire may be assigned to different drone areas depending on the region.A frontline firefighter on the same spot must belong to the same UAV area.Therefore, when the emitter points are determined, in the process of UAV domain allocation, we believe that the emitter points are no longer disturbed by other emitter points in the same ring of fire, and all the emitter points are independent of each other.

As for the site selection of EOC, we preliminarily believe that it is randomly generated, and the initial quantity is 1.The site selection of EOC will be optimized in the following paper. However, the site selection of EOC should meet the following restrictions as far as possible:

- Can't be surrounded by fire to avoid bad effects on the command center.
- Keep a safe distance from the fire, lest the fire get too intense and spread to the vicinity of the EOC. An evacuation of the EOC midfield could have a negative impact on the

overall UAV layout.

The number of SSA UAVs is defined as C, and its coordinate set is $\{p_{c1}, p_{c2}, p_{c3}, \cdots, p_{cc}\}$.

The number of repeater drones is defined as D, and its coordinate set is $\{p_{d1}, p_{d2}, p_{d3}, \cdots, p_{dd}\}$.

The repeater radio range is defined as F.

3.3 Objectives of the UAV overall planning model

The core objective of the program is defined as the minimum cost of the two types of UAVs:

$$\min\{10000C + 10000D\} \tag{6}$$

Simplified as

$$\min\{C + D\} \tag{7}$$

3.4 Four constraints and their auxiliary models

Under the joint detection of satellite and patrol drones,If there is a fire in a certain area, the danger information will be reported to the fire department in the area, and the front-line command EOC will be established immediately. In addition, the SSA UAV and the radio relay UAV will arrive at the area designated by EOC and start to work after control. When multiple fire rings appear, the coordination function between them will be greatly played.Repeaters can transmit signals over a much wider range than low-power handheld radios,At the same time, in order to ensure the personal safety of firefighters at the front of each fire fighting point, SSA UAV is needed for real-time monitoring.The front-line firefighter's wearable sends a signal to both the SSA and the radio relay drone, which in turn can relay multiple signals to each other at great distances, and then to the EOC.The specific process is shown in Figure 4.

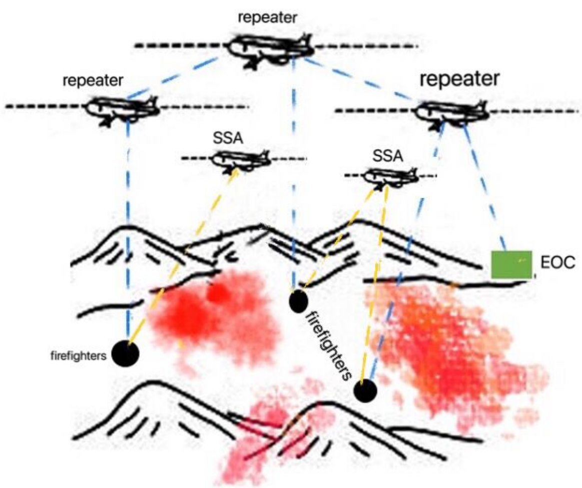

Figure 4: The SSA UAV and the repeater drone work together

Fire Safety Requirements

In real life, it often happens that forest firefighters are in danger because of the sudden change of wind direction and other reasons, so the personal safety of each frontline firefighter is a factor we must consider.According to Hypothes is 8, at least one SSA UAV is less than 2 km away from each fire extinguishing point, and the distance between multiple SSA UAV and the fire extinguishing point can reach the upper limit.Direct operation with two address vectors can be obtained:

$$\min_{\substack{1 \le j \le C, \\ 1 \le i \le N, \\ 1 \le k \le m_i}} \{||p_{cj} - p_{ik}||\} \le 2 \tag{8}$$

where

- p_{cj} represents the position coordinate of the SSA UAV
- p_{ik} represents the position coordinates of the flame extinguishing point
- R_1 is the advance speed of the line of fire
- R_2 is the extinguishing speed of the fire line

The needs of firefighters to transmit signals

Hand-held radios used by frontline firefighters are heavily influenced by the topography of the Australian forest, which is very complex, so it is necessary to take topography into account in the model.The range of effect of terrain on hand - held radio signal transmission is shown in Table 2 ,Each firefighter has to make sure they can communicate to the EOC, and we think there are two different paths for communication.

Table 2: Effects of terrain on hand-held radios

Influencing factors	Radio propagation range
The rural flat terrain	5km
The urban topography	2km
The rural undulating terrain	2km
Weather	No influence

When selecting the connection mode of each firefighting point, we assume that we should first judge the longitude and latitude information of the point, and classify the firefighting points according to the longitude and latitude, Figure 5:

If the longitude and latitude of this point belong to the plain rural area, first judge whether the direct distance between this point and EOC is less than 5km.If the conditions are met, it can be directly connected to the EOC, without the need for a radio relay drone to act as an intermediary; If the distance exceeds the limit, it can only arrange a radio repeater drone to connect to it.

If the site is not a plain rural area, it proves that the topography of the site is complex.The EOC must be within 2 kilometers of the fire site to be able to directly contact the front line firefighters.This is true in urban areas as well as in mountainous areas.

Required connection requirements for Repeaters and EOC

This part starts from the point of view of the relay UAV, ignoring the network distribution between the repeater UAV. Using the black box principle, the connection relationship inside the box is unknown, but there is a fixed and unique "branch" reaching out of the box to communicate

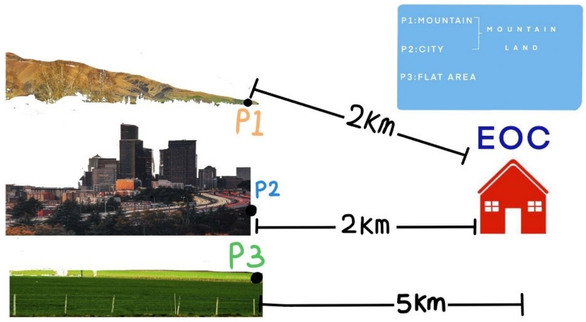

Figure 5: Selection of connection mode between fire fighting point and EOC

with the outside world. It is assumed that after a series of propagations through the repeater network, there will eventually be an outlet, a single repeater drone acting as a "branch", directly connected to the EOC.

In order to express this process more strictly, we find that it only needs to traverse all the distances from all radio Repeaters to EOC so that the minimum value is less than the range F, so the expression is as follows:

$$\min_{1 \le i \le D} \{||p_{di} - p_{eoc}||\} \le F \tag{9}$$

Construction of repeater group propagation network

In the previous constraint, it is not accurate to consider the repeater network as a black box only, because in the application process, only considering the coverage of the extinguishing point will cause the distance between the repeater drones to exceed the transmission range of the repeater signals, resulting in the signal transmission interruption, or even the failure of the entire model building. In front of the constraint condition, in order to be able to cover all the base point, we have already has carried on the limits to the location of the radio repeater unmanned aerial vehicle (uav), in this case, only from the entire network to consider: how to add the least relay uav, to guarantee the radio signals from EOC along the effective communication repeater, eventually reaching each repeater, so as to realize the bidirectional transmission, as shown in Figure 6.

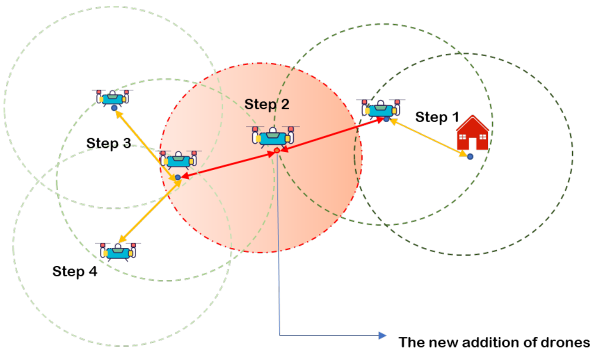

Figure 6: Distribution diagram of drone

We propose a network expansion algorithm for radio relay UAV. The main ideas are as follows:

Algorithm 1: a network expansion algorithm for radio relay UAV

Data: $S = \{p_{eoc}\}$, $U = \{p_{di}\}$

Result: $\{p_{add}\}$

Sort points in set U from near to far according to distance EOC; i = 0;
while $p_{di} \notin S$ **do**
    while $p_{di} \notin S$ **do**
        $u \leftarrow$ the point closest to p_{di} in set S ;
        $v \leftarrow$ the point at which a circle intersects with the line segment up_{di};
        if $vp_{di} \leq 20$ **then**
            $S \leftarrow v$;
            $S \leftarrow p_{di}$;
        else
            break;
        end
    end
    i++;
end

3.5 Solving UAV overall planning model

Our model has very complex constraints, so it is of great significance to find the most suitable solution method for the application of the model.

In Victoria we got a very detailed geographical information, the region scope of longitude east longitude 147.15° and 147.5° east longitude, 37.2° south equator to 37.6°. With the detailed longitude, latitude and altitude data of the region, the 3D model of the region can be simulated by Matlab for topographic map, Figure 7.

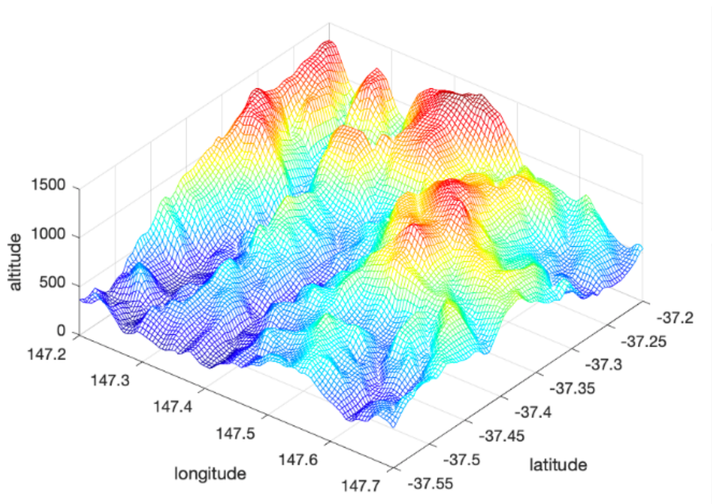

Figure 7: Simulation area 3D topographic map

Our next solution relies on the simulation experiment in this region, which can then be extended to the whole Victoria State. As long as the geographical data of a certain region is known, it can be brought into our UAV overall planning model for simulation experiment.

Firstly, the fire feature calculation model is used to simulate the random fire in this area.

After getting 7 independent fire sites at random, the coordinate of fire extinguishing points of each fire site was calculated, and a total of 22 fire extinguishing points were generated.

After the information of the emulation point is obtained, we can apply it to the UAV overall planning model to solve the addressing and number selection of SSA drones and radio repeater drones.

Particle swarm optimization (PSO) is a kind of evolutionary algorithm, which has a very good performance in finding the global optimal solution.

His basic idea is to start from the random solution, to find the optimal solution through constant iteration, and also to evaluate the quality of the solution through fitness. In the process of implementation, the global optimal value is found by following the optimal value currently searched. It will constantly update the individual optimal value and the group optimal value, and then continue to converge, and finally achieve convergence.

Our solution steps (Figure 8):

In the first iteration, a higher initial inertia weight is set. The higher the inertia weight, the stronger the trend of global optimization search. Through the initialized random particle swarm, it keeps getting closer to the fire fighting point of our fire ring, gets closer to the concentrated area of the fire field in the whole region, and gets closer to the global optimal solution.

In the process of the second step, after a certain iteration process, we modify the weight through the linear decreasing weight strategy. The global search is weakened and the local optimization is strengthened, so that a large number of particles are concentrated into different local optimal solutions. The optimal solution is the location and number of drones, which eventually aggregate into a few particle sets, which prove how many drones we need.

We choose to initialize 500 particles as our random particle swarm, and the particles move in a three-dimensional target search space.

The position of each particle is expressed as $x_i = (x_1, x_2, x_3)$, the flight velocity is expressed as $v_i = (v_1, v_2, v_3)$. Each particle has an adaptive value determined by the objective function.

We call the optimal position of the i particle so far as the individual extreme value $p_i = (p_{i1}, p_{i2}, p_{i3})$. The optimal position of the whole particle swarm is called the global extremum $g_i = (g_1, g_2, g_3)$.

In the iteration of the first step, the velocity and position of the particle will constantly change, and the change formula is shown as follows:

$$v_i(t+1) = \omega \times v_i(t) + c_1 r_1 (p_i(t) - x_i(t)) + c_2 r_2 (g_i(t) - x_i(t)) \tag{10}$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \tag{11}$$

The $i = 1, 2, \cdots, 500$ representing each particle. r_1 and r_2 are random numbers between 0 and 1. c_1, c_2 stands for learning factor, which we define as $c_1 = 0.4, c_2 = 0.6$. Speed is initialized to $v = [-1, -1; 1, 1]$. w is the initial inertia factor, which is defined as 0.8 optimal through debugging, and tends to search for the global optimal solution.

After the constant iteration of the first step, most particles have completed the preliminary process of approaching the optimal solution at this time. Next, we can find the answer by strengthening the local optimal solution. The linear decreasing weight strategy is expressed as follows:

$$w_t = \frac{(w_0 - w_{end}) \times (g_{\max} - g)}{g_{\max}} + w_{end} \tag{12}$$

Where w_0 represents the initial inertia weight of 0.8. w_{end} represents the inertia weight at the maximum iteration, which we set as 0.4. g_{max} represents the maximum number of iterations and g represents the current number of iterations.

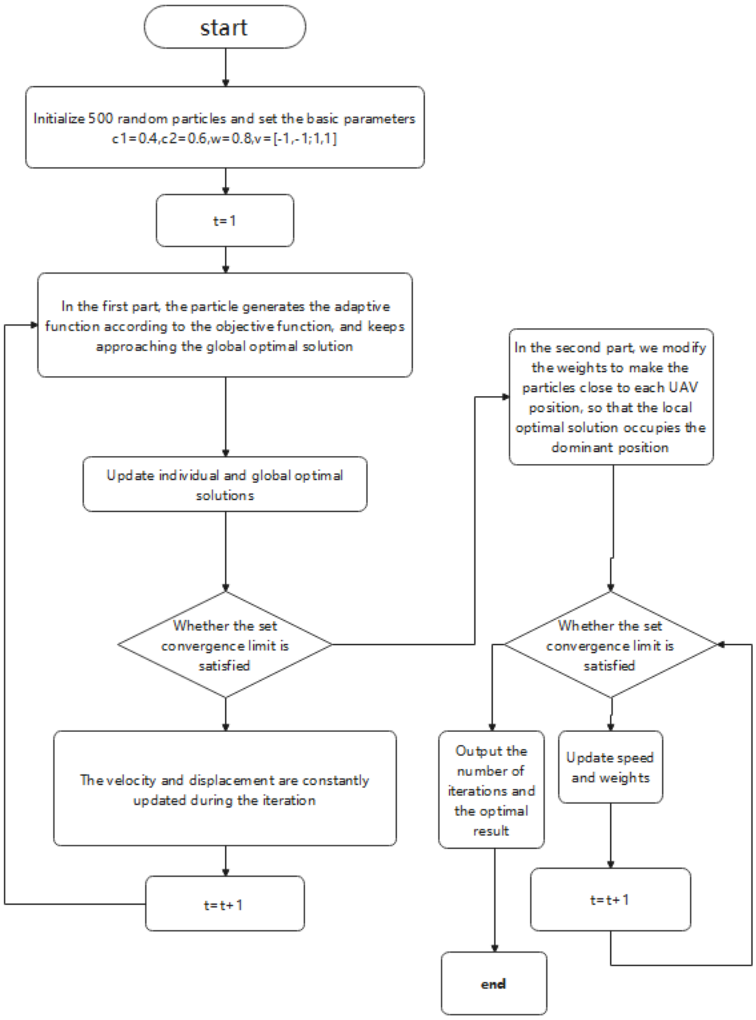

Figure 8: PSO algorithm

According to the characteristics of the fire site, the coordinate of the extinguishing point obtained by the model was calculated. The number of SSA drones and radio relay drones and their coordinate information are obtained by simulation, Table 3.

Table 3: SSA drone and radio repeater drone model results

number	longitude	latitude	altitude(meter)
1	147.2517	-37.4213	893
2	147.2315	-37.4032	913
3	147.2825	-37.3213	1025
4	147.2763	-37.3128	1073
5	147.2923	-37.3823	987
6	147.2947	-37.3746	1013
7	147.3021	-37.2812	1423

8	147.3823	-37.2634	1523
9	147.3814	-37.3843	945
10	147.4323	-37.3324	965
11	147.2632	-37.3982	875
12	147.2714	-37.2923	983
13	147.3613	-37.3154	1123
11	147.3923	-37.3678	1034
12	147.4503	-37.4398	923

Expansion to the entire Victoria region will require an analysis of the overall topographical characteristics of Victoria. It is known that Victoria covers a total area of 237,629 square kilometers. The northeastern part of Victoria is mainly mountainous, the southeastern part is heavily forested, and the western part is mainly grassland and hilly.

Coincidentally, the area we chose as our study area is both alpine, plain and forested, so there is some scientific justification for extending this area to the whole of Victoria.

According to our model, this 1,170 square kilometer area would cost about $\$150,000$. The total area of Victoria is 237,629 sq km. Using a linear scaling relationship, we predict that the government will need to prepare a budget of $30.5 million to pay for drones. Assignment is based on the overall 2-to-1 relationship between the SSA drone and the Radio Repeater drone.

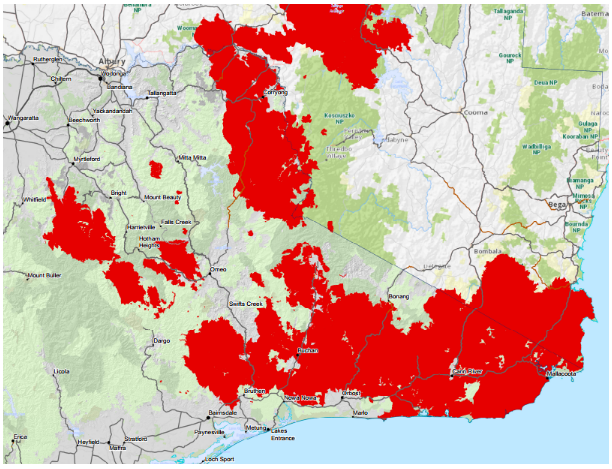

Figure 9: Victorian Fires

4 Autoregressive moving average fire prediction model

4.1 Material and Methods

4.1.1 ARMA model

An ARMA model, or Autoregressive Moving Average model, is used to describe weakly stationary stochastic time series in terms of two polynomials. The first of these polynomials is for autoregression, the second for the moving average.

Often this model is referred to as the ARMA(p,q) model, where:

- p is the order of the autoregressive polynomial.

- q is the order of the moving average polynomial.

The equation is given by:

$$X_t = c + \varepsilon_t + \sum_{i=1}^{p} \varphi_i X_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} \tag{13}$$

Where:

- φ = the autoregressive model's parameters.
- θ = the moving average model's parameters.
- c = a constant.
- $\sum$ = summation notation.
- ε= error terms (white noise).

4.1.2 Daniel Trend Test

Daniel's test for trend is based on Spearman's rank correlation coefficient, r_s. The hypothesis test is as follows:

H_0: The time series does not exhibit trend.

H_A: The time series does exhibit trend.

To perform this test, the observations are ranked in ascending order. (If two or more observations have the same value, they are each given a rank equivalent to their average ranking.) After ranking the observations, r_s, the rank correlation coefficient, is calculated as follows:

$$r_s = 1 - \frac{6\sum d_t^2}{n(n^2-1)} \tag{14}$$

where

- $d_t = t$ - (rank of observation y_t)
- n = the number of observations in the time series

If n is less than or equal to 30, the following procedure is used to test H_0 at a significance level of α:

Reject H_0 if $|r_s| > r_{\alpha/2}$.

In cases where the number of observations is greater than 30, the test statistic is calculated by $z = \frac{r_s}{\sqrt{n-1}}$.

H_0 is rejected if $|z| > z_{\alpha/2}$.

4.1.3 Ljung-Box Test

One of the major problems that a researcher is likely to encounter in fitting time series models is serial correlation. That is, temporal dependency between successive values of the model residuals. In this study, the Ljung-Box test proposed by Ljung and Box (1978) was used for testing the assumption that the residuals contain no serial correlation up to any order k. The test procedure is as follows;

H_0: There is no serial correlation up to order k.

H_1: There is serial correlation up to order k.

The test statistic is given by;

$$Q_m = T(T+2)\sum_{k=1}^{m}(T-k)^{-1}r_k^2 \tag{15}$$

where

- r_k^2 represent the residual autocorrelation at lag k
- T is the number of residuals
- m is the number of time lags included in the test When the p-value associated with Q_m is large, the model is considered adequate else the whole estimation process has to start again in order to get the most adequate model.

4.1.4 Parameter Estimation

Suppose that the stationary sequence of continuous observation samples is $\{x_1, x_2, \cdots, x_n\}$, the corresponding white noise sequence is $\{a_1, a_2, \cdots, a_n\}$. And the mean value of the white noise sequence is 0, independent of each other and obeying normal distribution.

The parameter calculation of AR(p) model is relatively simple. For the time series $\{X_t\}$ of length N, the AR(p) model is as follows:

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \varphi_3 x_{t-3} + + \varphi_p x_{t-p} + a_t \tag{16}$$

Substituting the time series $\{x_t\}\,(t = 1, 2, \cdots, N)$ into the model, the following linear equations can be obtained:

$$\begin{cases} x_{p+1} = \varphi_1 x_p + \varphi_1 x_{p+1} + \ldots + \varphi_p x_1 + a_{p+1} \\ x_{p+2} = \varphi_1 x_{p+1} + \varphi_2 x_{p+2} + \ldots + \varphi_p x_2 + a_{p+2} \\ x_N = \varphi_1 x_{N-1} + \varphi_2 x_{N-2} + \ldots + \varphi_p x_{N-p} + a_N \end{cases} \tag{17}$$

The matrix form is as follows:

$$y = x\varphi + a$$

According to the least squares estimation principle of multiple regression, the following results can be obtained:

$$\varphi = (x^T x)^{-1} x^T y$$

The ARMA(n, m) model is defined as follows:

$$x_t = \varphi_1 x_{t-1} + \varphi_2 x_{t-2} + \varphi_3 x_{t-3} + + \varphi_n x_{t-n} + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - - \theta_m a_{t-m}$$

Due to the unknown white noise $\{a_i\}$, the estimation process of ARMA model parameters is a nonlinear regression process, so it is much more complicated than that of AR model parameters.

In this paper, a relatively simple ARMA model parameter estimation method is used. Because this method transforms the nonlinear estimation process of ARMA model parameters into the linear estimation process of AR model parameters, it is simple and clear in concept and avoids a lot of complicated calculations in ARMA modeling.

For the time series $\{x_t\}$, the AR(p) $(p \geq n+m)$ model was first fitted, and the model parameter $\varphi_i(i=1,2,\cdots,p)$ was estimated. Then the residual series $\{a_t\}$ $(t=p+1,p+2,\cdots,N)$ was calculated from the AR(p) model, and the $\{a_t\}$ was substituted into the ARMA(n, m) model. The equation can be obtained as follows:

$$Y = X\beta + A$$

According to the least squares estimation principle of multiple regression, the following equation can be obtained:

$$\beta = (X^T X)^{-1} X^T Y$$

The fundamental condition for the establishment of ARMA model is that $\{a_t\}$ is a white noise sequence following $N(0,\sigma_a^2)$. Therefore, the most fundamental criterion for determining the n,m of ARMA(n, m) (i.e. the order determination of ARMA model) is to test $\{a_t\}$. AIC criterion is chosen here, and the function of AIC criterion is defined as:

$$AIC(p) = N \ln \sigma_a^2 + 2p$$

Where $p = n+m$. The order with the smallest $AIC(p)$ value is selected as the order of the applicable model, while the values of n and m are generally less than 2 in actual engineering.

4.2 Results and Discussion

The dataset we used contains data from the NASA satellite instrument MODIS C6 and VIIRS 375m for fires in Australia, compiled by Carlos Paradis and posted on Kaggle with source files from the NASA website.

Firstly, the fire intensity data of Australia in recent 5 years (61 months) were statistically analyzed, and the established model was used to estimate the parameters of the first 30 months and forecast the next 31 months. The results are shown in Figure 10.

To demonstrate the reliability of the model, the error of the 31 months was calculated. It can be seen from Figure 11 that the accuracy of the model is about 98% in the case of a small amount of data.

It can be seen from Figure 10 and Figure 11 that the fire prediction ability of ARMA model is relatively reliable. Therefore, all (61 months) known data are now considered as samples to forecast fire intensity in Australia in the next five years, as shown in Figure 12.

It can be found that fire intensity changes in Australia in the next five years (2021-2024) and the past five years (2016-2020) are basically the same. As Mark Twain said, "History does not repeat itself，but it does often rhyme."

To sum up, due to the limited living space of animals and plants, the weak regeneration ability of combustible materials, and the long recovery cycle of the ecosystem, the fire intensity in Australia changes periodically, about every five years. The fire intensity always rises first and then falls, and so on. As can be seen from the analysis of the last question, the intensity of fire

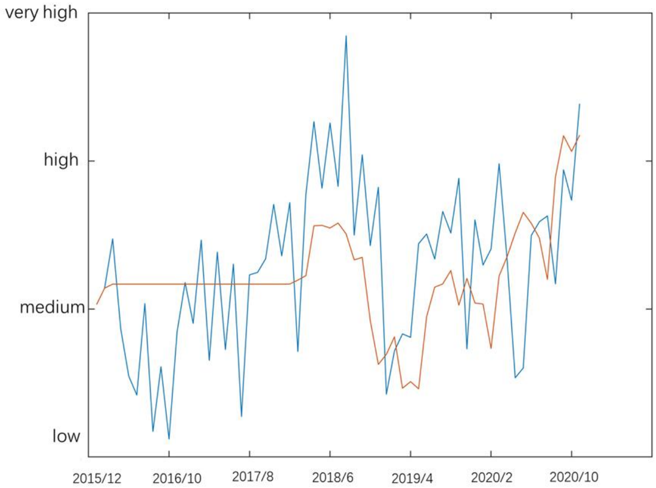

Figure 10: Forecasts for the 31 months

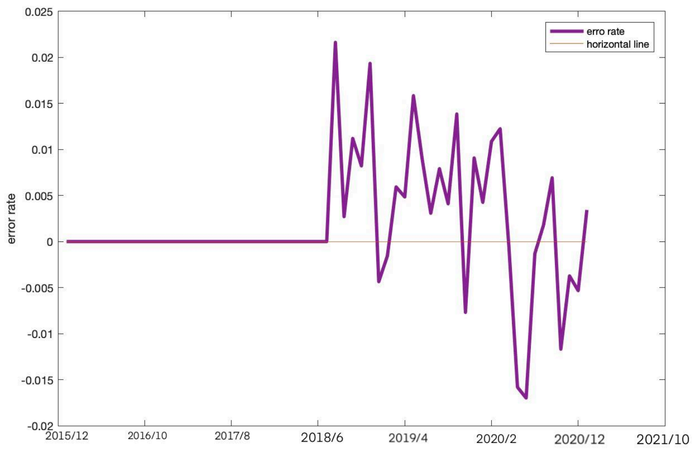

Figure 11: The error of the 31 months

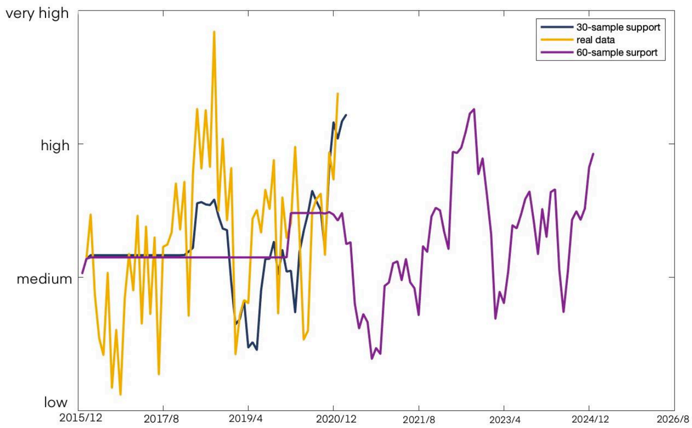

Figure 12: Forecasts for the next five years

determines the number and distribution of UAVs. Since the area is fixed, the increased intensity of the fire will inevitably lead to an increase in the number of fire fighting points, which will require more SSA drones to keep the firefighters safe and more repeater drones to keep the front line personnel communicating with the EOC. Assuming that the cost of UAS remains constant, the projected increase in equipment costs over the next decade is as follows:

$$\Delta S = \sum_{x=1}^{120} 10000(\theta_1 + \theta_2)p(x) \quad (\theta_1, \theta_2 \propto p(x)) \tag{18}$$

where

- x: the month you want to predict
- $p(x)$: the level of fire predicted by the model
- θ_1: the growth rate of the repeater drone population
- θ_2: the growth rate of the SSA drone population

5 Establishment and solution of the model

5.1 Cellular automata simulate fire

In our model of the first question, we generated the ring of fire randomly.The speed of advancing the fire line,the speed of extinguishing the fire line and so on are all determined by our experience. So there may be some deviation from the actual situation. Using cellular automata to simulate forest fires in Victoria is more realistic.

Cellular automata has the ability to simulate the evolution of complex systems.The grid represents the different states.The rules for generating fires in Australian forests are defined to simulate real-world conditions.Don't have to worry about complicated formulas.It mainly simulates the scale of development in physics and society through the rules between grids.

In the cellular automata model, space is discrete into grids, and each grid is called a cell.Forest fire cells have three states:Tree, fire (burning tree) and empty (vacant) states.According to the calculation model of fire site characteristics in the first question, we have known the eigenvalues of fire site and its development process.

Figure 13 shows the area divided into 64 blocks, with the green block being the "tree" in the model.We set the distribution density of the tree as 0.25, and carried out penetration analysis on all the trees in this area.The penetration probability represents the probability that the fire in the area will basically burn from one end to the other end.The greater the penetration probability is, the greater the possibility of a fire with a higher fire grade will occur in .

In the MATLAB simulation, we set a series of gradient area sizes.We hope to get the scale comparison of fire in different size range.In a different range of areas,We set up a series of vegetation densities.The change rule of penetration probability is shown in the Figure 14.

By studying the change curve, we can draw the following conclusion:When the area size range is fixed, the penetration probability increases with the increase of vegetation density, and the possibility of complete fire in the area is higher.With the increase of vegetation density in different regional sizes, the penetration probability is generally increasing. Where the vegetation density is different, the limit is 0.6.When the vegetation density is equal to 0.6, it can be

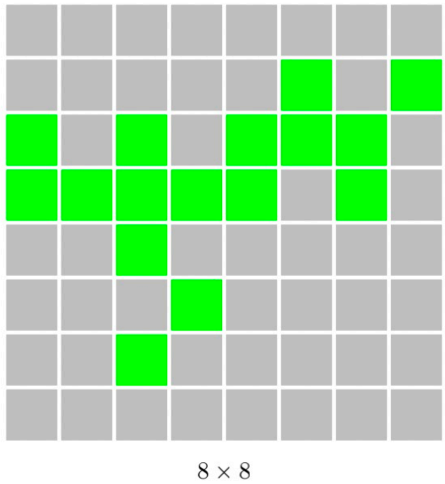

Figure 13: the area divided into 64 blocks

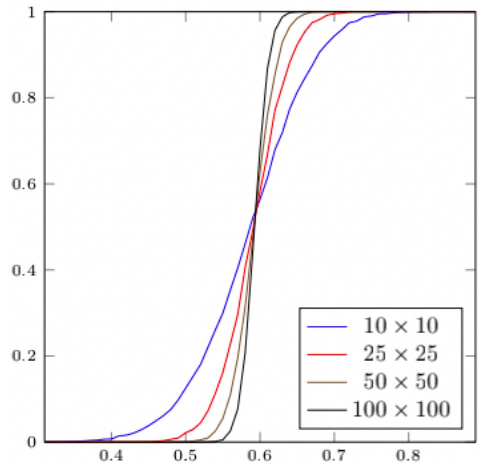

Figure 14: The change rule of penetration probability

found that the regional penetration probabilities of any size are equal. When the vegetation density is less than 0.6, the penetration probability of small size increases faster, while it is opposite when the vegetation density is greater than 0.6.

So far, we have basically studied the working principle of cellular automata and simulated the whole state of Victoria, so we can get the simulation map of the fire situation in Victoria, Figure 15.

Through cellular automata, we have completed the optimization and upgrading of the simulation fire model, eliminated the negative influence brought by randomness in the calculation model of fire site features, and applied it to our UAV overall planning model is more suitable for the actual terrain.

5.2 Spherical repeater group propagation network model

In the UAV overall planning model, the auxiliary model established by us, the third repeater group propagation network model, can be optimized and improved to meet the requirements of the third question for terrain with higher complexity.In the first question, the use of this model mainly considers the construction of the entire repeater connection network in two-dimensional coordinates, and does not consider the more detailed scene in three-dimensional coordinates.In

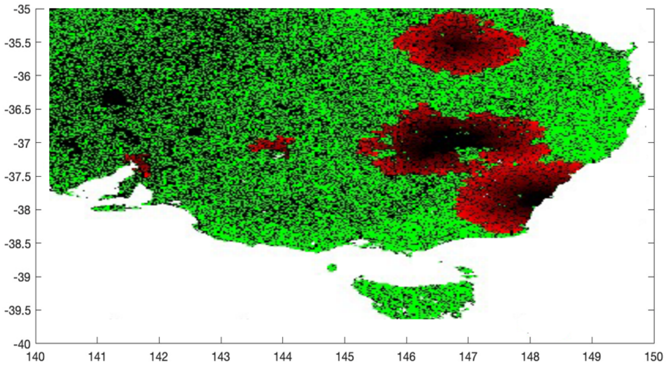

Figure 15: the simulation of the fire situation in Victoria

practice, affected by terrain in three-dimensional space, the distribution of UAVs will also form height difference, and the distribution simulation of UAVs is no longer a simple plane network.We assume that the drone stays at the same height from the vertical ground regardless of the area.Therefore, according to the longitude, latitude and altitude information of this point, we can get a three-dimensional UAV network distribution diagram.

Our model was scanned from EOC and made a sphere area with a radius of 20 km, which may cover any radio repeater UAV, Figure 16.

- When there is only one UAV in the region, we take the UAV as the new core and make a new spherical region with a radius of 20km to continue to look for the effective radio repeater UAV in the region.
- When the number of effective UAVs in the area exceeds 2, we choose the UAV point farther from the center of the ball as the new core and repeat the above steps.
- When there are no drones in the area, it means that there will be a certain interruption of the transmission signal. The only way to solve this problem is to increase the number of radio repeater drones to connect the interrupted signal. This involves the addition of radio repeater drones in the same way as in the first question.

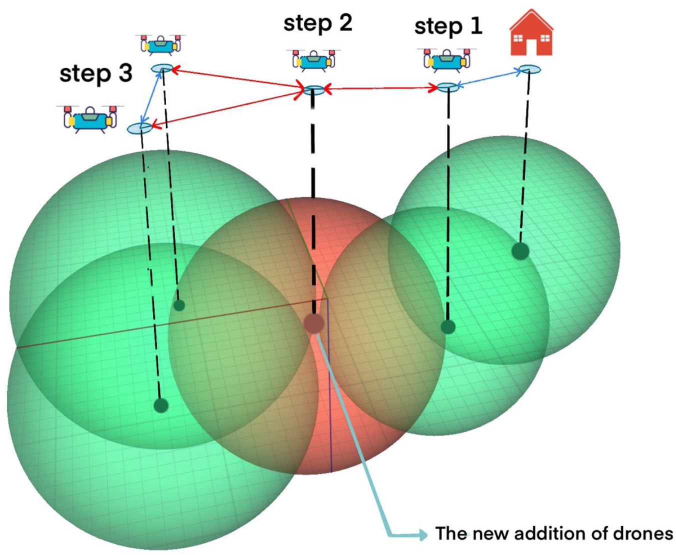

Figure 16: Spherical repeater group propagation network

6 Strengths and Weaknesses

6.1 Strengths

- In our calculation model of fire features, the severity of fire is skillfully combined with the calculation process of extinguishing point and extinguishing time. This means that the influence factors such as fire intensity and scale have been taken into account in the overall planning model of the UAV.
- Simple constraints do not satisfy the constraints necessary for this scenario. Therefore, for each constraint requirement, we have built auxiliary models, which use more complex nested models to achieve more powerful constraint functions.
- In our repeater group propagation network model, the calculation in two-dimensional space is mainly considered in problem 1. In the third question, the model is optimized and upgraded to make it fully consider the specific terrain, and the repeater network can be built in the three-dimensional space, forming a three-dimensional network. After improvement, it can satisfy the practical application in space.
- In our paper, we applied the idea of process visualization more and drew a lot of working model diagrams of UAV, which can help readers better understand our model idea.
- Due to the influence of climate change, road reconstruction, tourism development, forest aging, lack of forest management and other factors, the occurrence probability of forest fire is related to the long-term overall trend in the past few years. Conventional linear regression analysis is difficult to reflect both the periodic change of forest fire and the overall trend, but the ARMA model can solve this problem.

6.2 Weaknesses

- In the process of looking for data and solving the model, we found that it was difficult to find the topographic data in many areas. So it's a pity that there are no more data for us to better explore the model.
- Because forest fire is affected by site conditions and forest indexes, forest fire prediction is a complex and systematic problem. This model for the black box model, if based on this, for different site conditions and the growth of forest classification modeling, the choice of classification variables can include: age, tree species, distance from the road, is apart from the residents to distance, elevation, gradient, slope direction, the daily average temperature, wind direction, wind, etc., so that we can achieve higher simulation and prediction accuracy.

Budget Justification

Dear Government of Victoria,

This is a budget request from the Country Fire Authority (CFA) regarding the acquisition of drones in Victoria.

Our team has carried on the simulation, to an area of Victoria this area covers an area of 1170 square kilometers. With 7 fire sites and 22 fire suppression sites in the fire simulation, a mix-and-match scheme using 10 SSA drones and 5 radio repeater drones was finally obtained, and the government was due to pay $\$150,000$.

So the overall value of the estimate the state of Victoria area can be roughly. When a fire breaks out in each region, as long as the specific distribution of the fire site is known through satellite cloud images or forest patrol guards, we can bring the topographic data of the region into our drone overall planning model, so that we can get real-time drone matching and combination according to different actual data.

The general trend is that when the area increases, the number of fire sites increases, and the severity of fire is higher, the number of both types of drones we need will increase linearly, and the total cost will also increase. Considering the Australian strong fire frequency in recent years, we think the government fully prepared to deal with the fire danger. Victoria covers an area of $237,629$ square kilometers, nearly 30% of which is covered by forests. The mountainous areas in the northeast, the vast forests in the southeast, and the vast grasslands and hills in the west are all areas where fires are extremely prone to occur.

Based on the planning model we have developed, we recommend that the Victorian Government set a target of $30.5 million for drone purchases to respond to severe fire scenarios. Of this, $20.3 million was allocated to the acquisition of SSA drones and $10.2 million to the acquisition of Radio Repeater drones.

In addition, we forecast fire developments in Victoria over the next decade. It should be noted that climate has become increasingly unpredictable in recent years with the emergence of global warming and various extreme weather phenomena, and there is a strong possibility of some sudden adverse and extreme natural conditions. Our fire forecast was primarily based on historical data, also considered some sudden factors might affect.

After our forecast, we believe that in the next five years (2021-2024) of Victoria fire intensity change trend and over the past five years (2016-2020), the change trend of the overall agreement. According to some natural and human causes analyzed in the model, we judge that the intensity of fire in Australia presents periodic changes, with a cycle of about five years.

Therefore, we give the formula of expected equipment cost increase in the next ten years:

$$\Delta S = \sum_{x=1}^{120} 10000(\theta_1 + \theta_2)p(x) \quad (\theta_1, \theta_2 \propto p(x))$$

The specific explanation of the formula is explained in more detail in the model. It's about the growth rate of both types of drones and the use of fire intensity.

If you want to know more about our model, please read our paper!

References

[1] Stephens, Scott L. "Forest fire causes and extent on United States Forest Service lands." International Journal of Wildland Fire 14.3 (2005): 213-222.

[2] Blodgett, David, et al. "Mainstems: A logical data model implementing mainstem and drainage basin feature types based on WaterML2 Part 3: HY Features concepts." Environmental Modelling & Software 135 (2021): 104927.

[3] Zhichong, Zou, and Leng Hong. "State-of-the-art of Australian Bushfire Risk Researches and Its Enlightenment." Urban Planning International (2018): 03.

[4] https://www.statisticshowto.com/arma-model/

[5] https://www.quantstart.com/articles/Autoregressive-Moving-Average-ARMA-p-q-Models-for-Time-Series-Analysis-Part-3/

[6] Emmanuel Kojo Amoah. Predictive Model for Fire Outbreaks: A Case Study in the Ashanti Region, Ghana