Homework 2: (Handout: 2025/04/15 Handin: 2025/05/13)

In this homework, we will solve the following master equation for modelocked lasers:

$$\frac{\partial U(T,t)}{\partial T} = \frac{g_0}{1+\frac{\int |u|^2 dt}{E_s}}\left(1+d_r\frac{\partial^2}{\partial t^2}\right)U - l_0 U + i\, d_i \frac{\partial^2 U}{\partial t^2} + (k_r + i\, k_i)\,|U|^2 U$$

1. 30%

Write a program to solve the above master equation assuming U(0,t) is known.

2. 20%

Set d_i=0.5, d_r=0, k_i =1.0, k_r =0, g_0 =0, l_0=0 to reduce the problem into the nonlinear Schrodinger equation case. Check if you can correctly simulate the propagation of the fundamental soliton solution as described in the lecture notes. Let t=0 correspond to the time window center so that the pulse is centered in the time window.

3. 30%

Set d_i=0.5, d_r=0.05, k_i =1.0, k_r =0.1, g_0 =4, l_0=1.0, E_s =0.5. Solve the equation to find the steady state solution. You can use the soliton pulse in 2 as the initial condition at T=0 and solve the equation for T>0 till the pulse shape remains unchanged. After that, set k_r =0 and see if you can still get the steady state pulse.

4. 20%

Add one more term in the equation:

$$\frac{\partial U(T,t)}{\partial T} = \frac{g_0}{1+\frac{\int |u|^2 dt}{E_s}}\left(1+d_r\frac{\partial^2}{\partial t^2}\right)U - l_0 U + i\, d_i \frac{\partial^2 U}{\partial t^2} + (k_r + i\, k_i)\,|U|^2 U + i\, M \cos[\Omega_M t]\, U$$

Assume Ω_M=2π/100 and M=0.8. The rest of the parameters are the same as in 3. Use the steady state solution in 3 as the initial condition at T=0 and simulate the evolution of the pulse for T>0. Again, after that, set k_r =0 and see if you can still get the steady state pulse. Try to explain what you observe in the solutions.

To learn more about the modelocked laser physics, the following references can be used as the starting point.

1. W.-W. Hsiang, H.-C. Chang, and Y. Lai, “Laser dynamics of a 10 GHz 0.55 ps asynchronously harmonic modelocked Er-fiber soliton laser,” IEEE J. Quantum Electron. 46, 292 (2010).

2. S.-Y. Wu, W.-W. Hsiang and Y. Lai, “Synchronous-asynchronous laser mode-locking transition,” PHYSICAL REVIEW A 92, 013848 (2015).