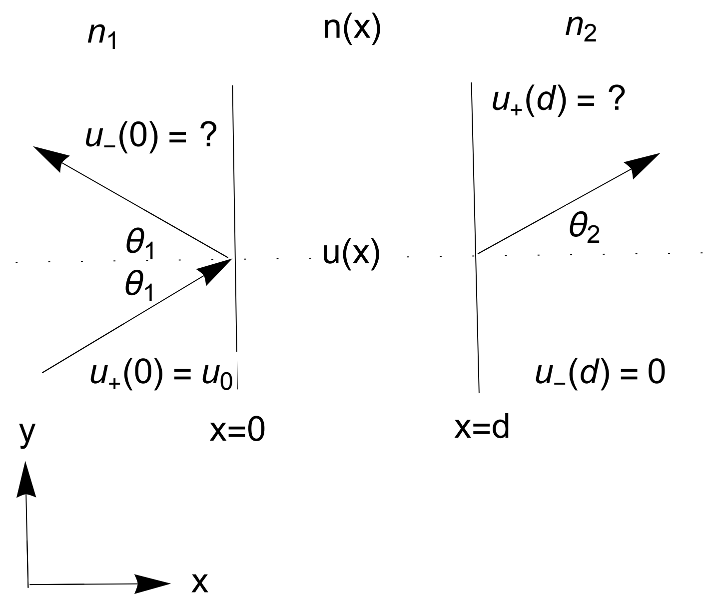
Homework 1: (Hand-out: 2025.3.4 Hand-in: 2023.04.8)

Consider the following optics problem illustrated below:



A TE plane wave is obliquely incident on a graded-index slab with the thickness d and the refractive-index profile n(x). You need to write a program to calculate the reflection and transmission coefficients of the setup as well as the optical field distribution inside the slab. The problem can be modelled by the 1D wave equation:

$$\frac{d^2}{dx^2} u(x) + \left(n^2(x)\, k_0^2 - k_y^2\right) u(x) = 0$$

Here u(x) is the complex electric field function at the frequency ω and $k_0 = 2\pi/\lambda = \omega/c = \omega\,(\mu_0 \epsilon_0)^{0.5}$ is the propagation constant in the vacuum. k_y is the propagation constant in the y direction. We will require $u(x)$, $\frac{d}{dx} u(x)$ to be continuous across the interfaces (assuming $\mu = \mu_0$ for all layers).

When n(x) is constant, $n(x) = n$, the solution of u(x) is given by

$$(1) \quad u(x) = u_+ e^{-j k_x x} + u_- e^{j k_x x}$$

with

$$k_x = \sqrt{n^2 k_0^2 - k_y^2} \qquad (\text{assuming } k_y < n k_0)$$

Here u_+ is the travelling wave field component in the +x direction while u_- is the travelling wave field component in the -x direction. From (1), they are related to u by:

$$u_+ e^{-j k_x x} = \frac{j k_x u - \frac{du}{dx}}{2 j k_x}, \quad u_- e^{j k_x x} = \frac{j k_x u + \frac{du}{dx}}{2 j k_x}$$

Therefore the boundary conditions at x=0 and x=d for the problem can be written as:

$$(2) \qquad \frac{j k_{x,1} u(0) - \frac{du}{dx}(0)}{2 j k_{x,1}} = u_0, \quad \frac{j k_{x,2} u(d) + \frac{du}{dx}(d)}{2 j k_{x,2}} = 0$$

Here

$k_{x,1} = \sqrt{n_1^2 k_0^2 - k_y^2}$, $k_{x,2} = \sqrt{n_2^2 k_0^2 - k_y^2}$

The power reflection and transmission coefficients are then given by:

$$R = \left| \frac{u_-(0)}{u_+(0)} \right|^2, \quad T = \frac{n_2 \cos(\theta_2)}{n_1 \cos(\theta_1)} \left| \frac{u_+(d)}{u_+(0)} \right|^2$$

Here θ_1 is the incident angle and θ_2 is the transmission angle of the plane wave.

Please do the following things and hand-in both your solutions and programs. You should also explain briefly what kind of methods you have used in your program.

1. 50%

Write a program to solve the above boundary value problem with the given mixed boundary condition.

2. 20%

Assume n_1=1.0, n_2=1.0, n(x)=3.0 for d/4<x<3d/4 and n(x)=1 otherwise. Also assume θ_1=0 (k_y=0, normal incidence) and d=5.0 μm. Use your program to calculate and plot the power transmission coefficient T as a function of the inverse wavelength 1/λ (Note that $k_0 = \omega / c = 2\pi/\lambda$). You can choose a suitable range for 1/λ to show the resonance peaks in the spectrum (assuming the center wavelength is 1.5 μm). Also choose the units of length to be 1.0 μm for normalization. Check with the analytic results to verify that your obtained results are correct.

3. 10%

Assume n_1=1.0, n_2=3.5, $n(x) = 1 + 1.25 * \left(1 - \cos\left(\frac{\pi x}{d}\right)\right)$ for 0<x<d. Also assume θ_1=0 (k_y=0, normal incidence) and d=5.0 μm. Use your program to calculate and plot the power transmission coefficient T as as a function of the inverse wavelength 1/λ for the same range as in problem 2. Explain why you think the obtained results are correct.

5. 10%

Estimate the accuracy of your obtained results.

6. 10%

Explain what will be different if the incident light is a TM plane wave.