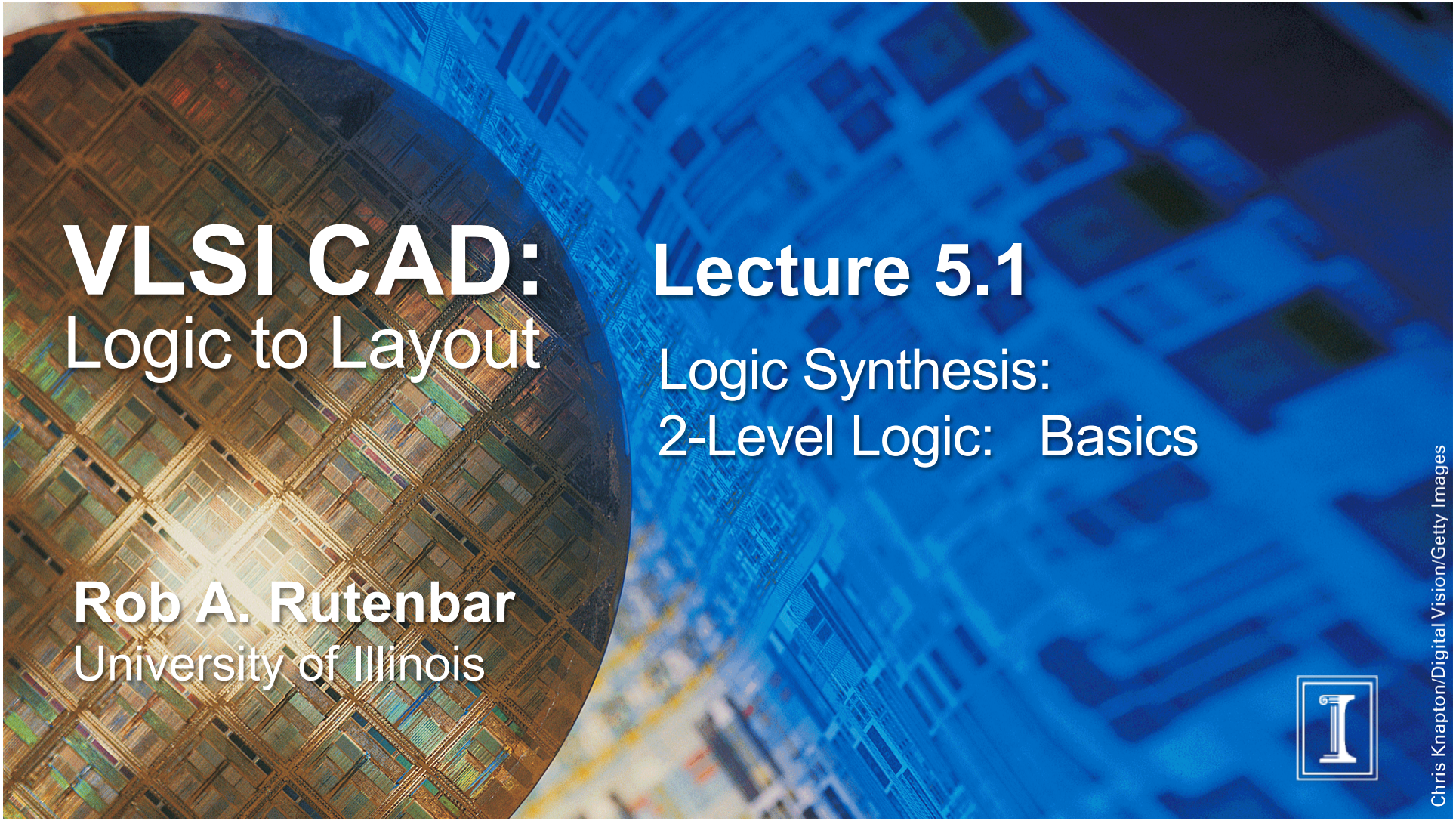

VLSI CAD: Logic to Layout

Lecture 5.1

Logic Synthesis:
2-Level Logic: Basics

Rob A. Rutenbar
University of Illinois

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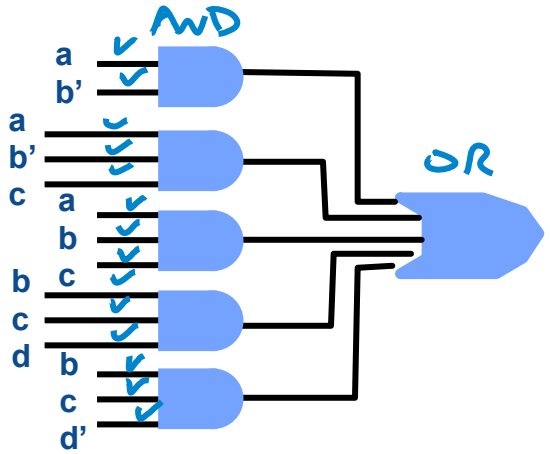
2-Level Minimization Means: This…

- **Trying to find minimal SOP logic: many AND gates, 1 OR gate**
  - Want: **fewest** AND gates, and among all such, one with **fewest input wires**.
  - These input wires called **literals**: 1 variable in true or complemented form in AND gate

f = ab’ + ab’c + abc + bcd + bcd’ **has** 14 **literals**



Good metric for success is to **minimize** the number of **literals** in this 2-level result

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2-Level Minimization

- **None of these methods you know is really effective, practical…**
  - **Boolean algebra:** Hard with many variables. Can't tell when have a good solution
  - **Kmaps**: Same. Hard with many variables, can't tell when you're really done
  - **Tabular solution:** E.g, Quine McCluskey. Exponential complexity to get best result

- **Need a better strategy**
  - **Big idea #1:** **Don't** try for the best, perfect answer. Just get a **good** answer.
  - **Big idea #2:** **Iterative improvement.** From one answer, **reshape** the solution to discover a (possibly better) answer. Continue until no more improvement.

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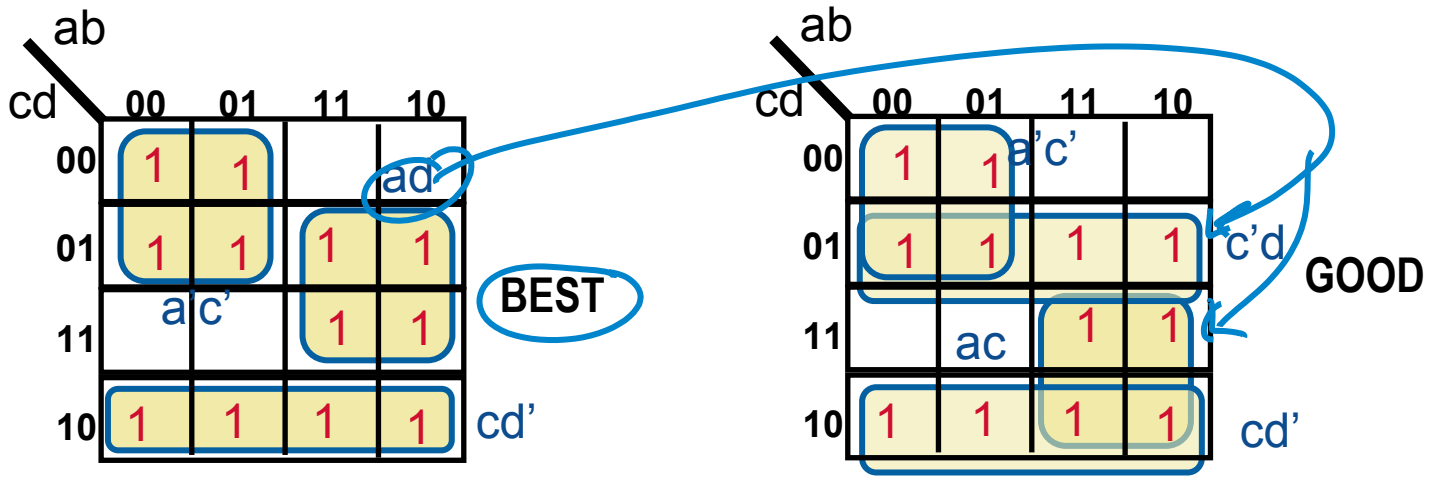
Example: Best vs "Good Enough"



- **Comparing the two solutions**
  - Both are made of product terms ("**cubes**") that are "as big as possible". We insist on this. These products/cubes are called "**Prime Implicants**" (or "primes").
  - **Famous result from 1950s:** Best solution is composed of *cover of primes*

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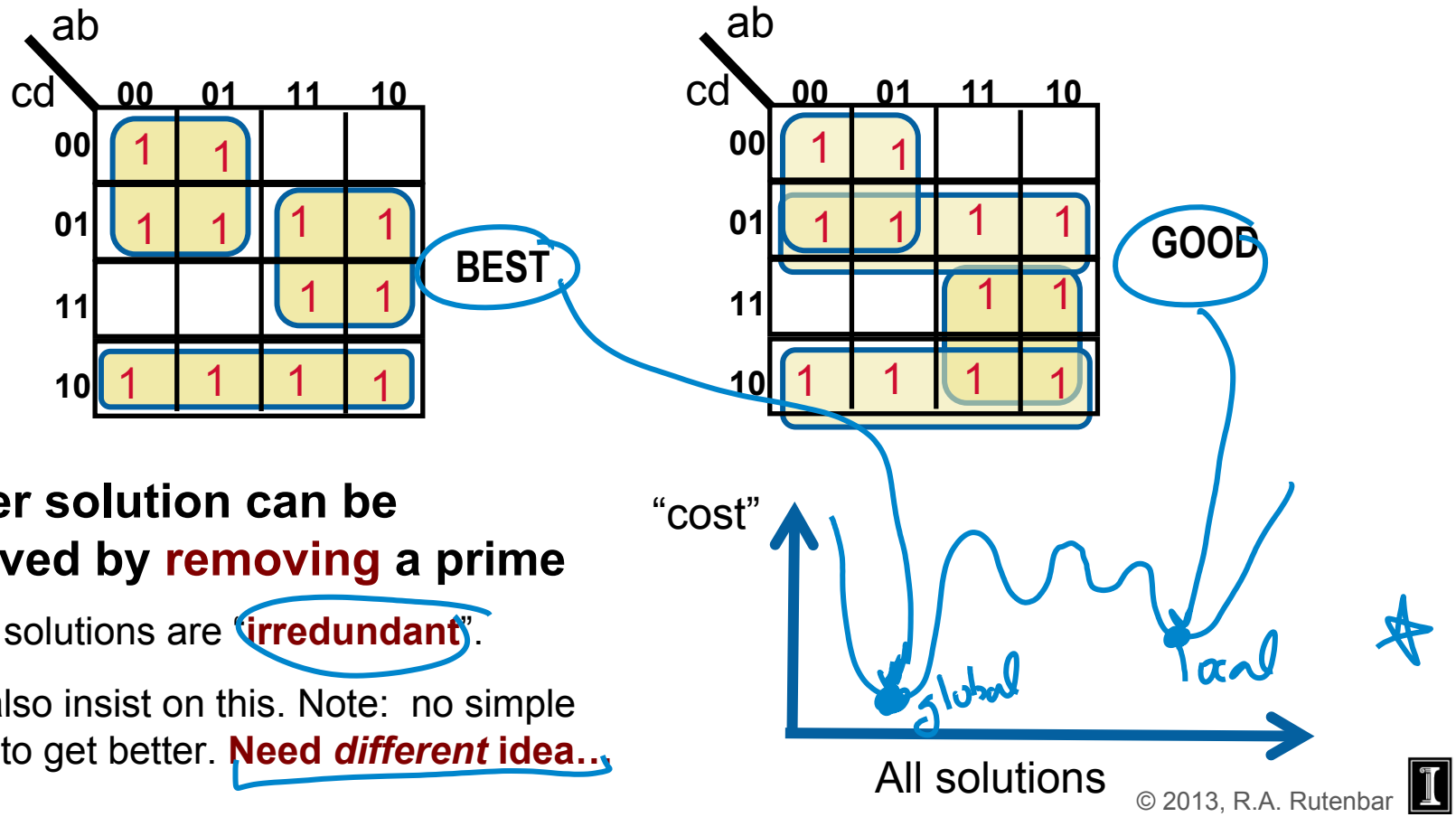
Example: Best vs "Good Enough"



- **Neither solution can be improved by removing a prime**
  - Both solutions are "**irredundant**".
  - We also insist on this. Note: no simple path to get better. **Need *different* idea…**

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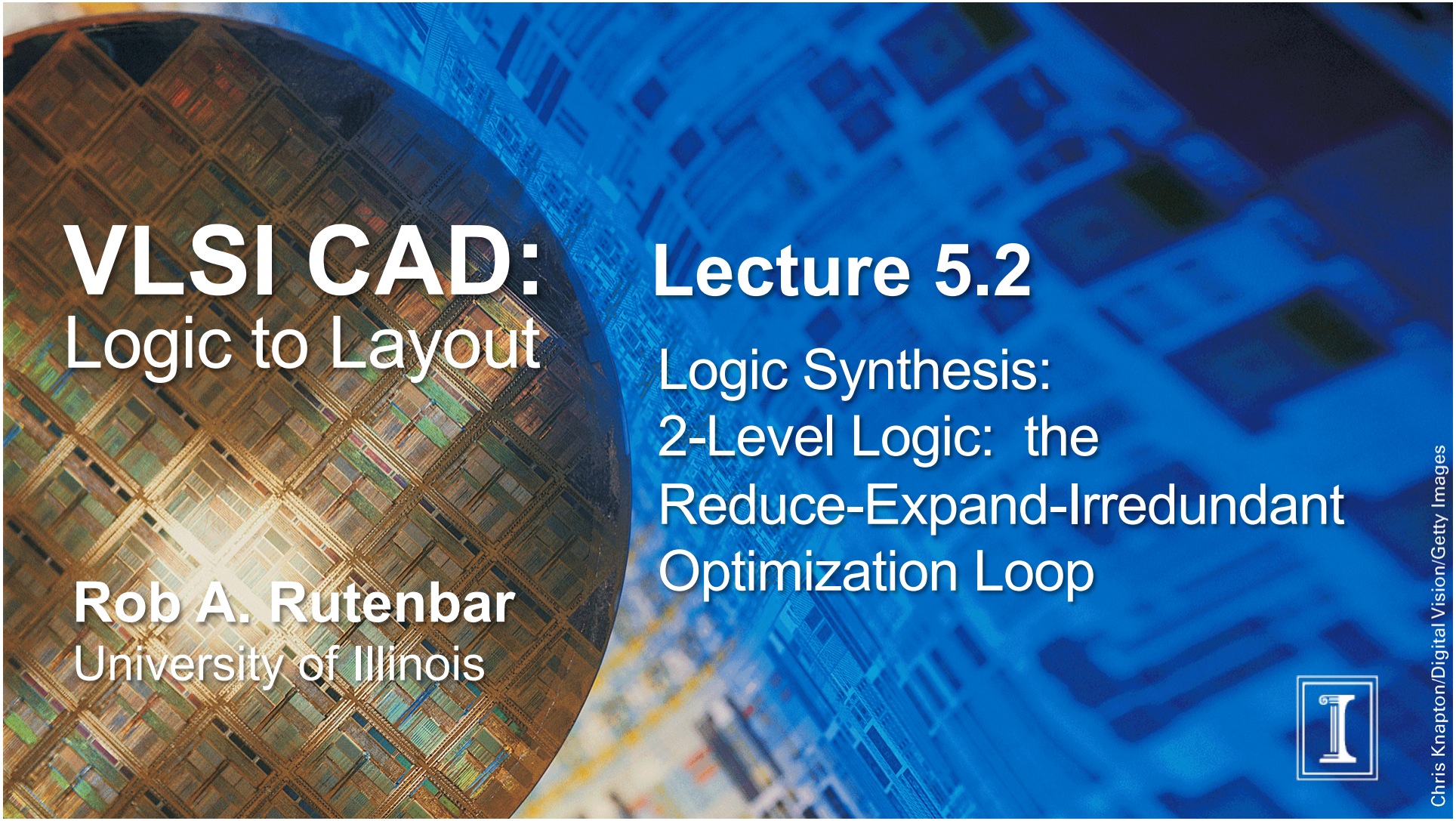

VLSI CAD: Logic to Layout

Lecture 5.2

Logic Synthesis:
2-Level Logic: the Reduce-Expand-Irredundant Optimization Loop

Rob A. Rutenbar
University of Illinois

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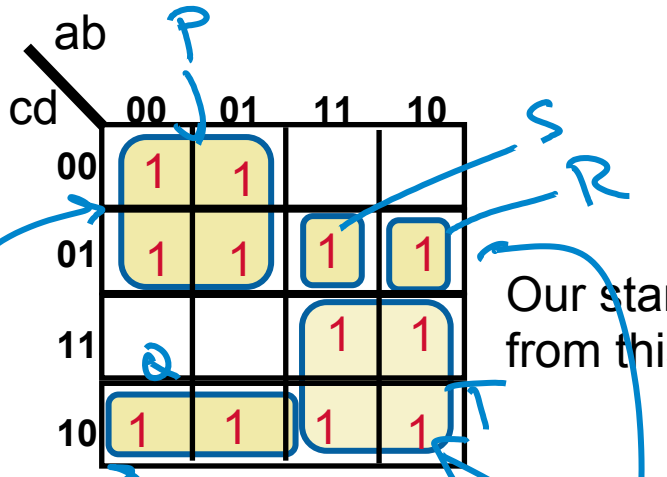
Assume Start With A Truth Table

- **But, might be a truth table (TT) with input Don't Cares**
  - Just means each TT row can match ***many*** rows in a full truth table
  - Only list where the function is **1**; assume all other rows are **0**
  - Makes it easy to specify a function of many variables
  - Each row in this input TT defines a **product (cube)**– might **not** be prime, but it surely covers all the 1s



abcd	F	cube-label
0*0*	1	P
0*10	1	Q
1001	1	R
1101	1	S
1*1*	1	T

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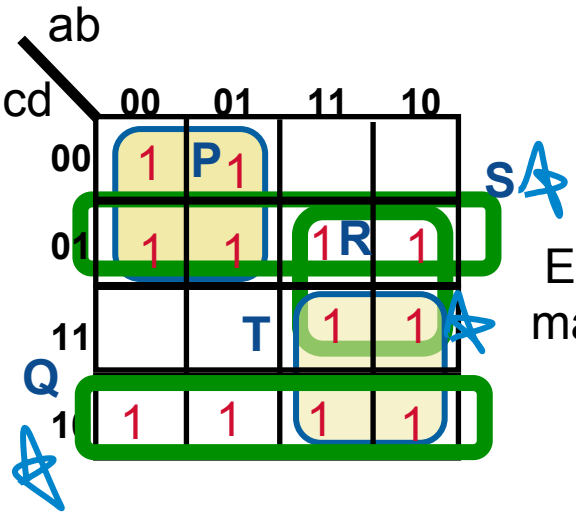
Next Step: *Expand* Each Cube to be Prime

- **"Expand" is a heuristic, done one cube at a time**
  - Make each cube as **big as possible**
    - Might be ***different*** ways to do this for any specific cube…
  - **3** of our cubes have now been grown
    - **Q, R, S** cubes expanded
  - This new solution is a prime cover
  - But it might **not be best** we can do…



Expand cubes, make them prime

abcd	F	cube-label
0*0*	1	P
0*10	1	Q
1001	1	R
1101	1	S
1*1*	1	T

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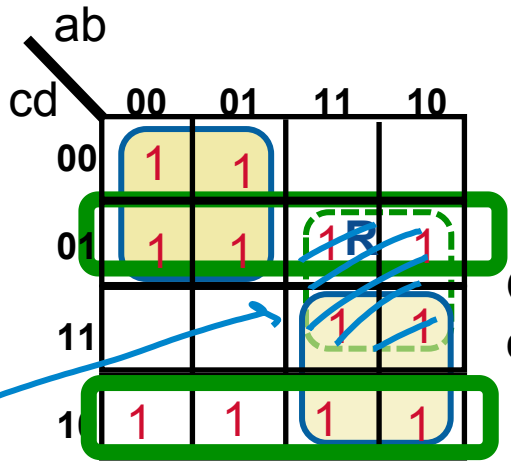
Next Step: Remove *Redundant* Cubes

- **"Irredundant" is a heuristic**
  - A cube is **redundant** if we can remove it, and all its **1's** are still covered by **other** cubes in the rest of the cover
  - *Irredundant* operation removes redundant cubes in our cover
- **Assume we remove cube R**
  - This new solution is a **prime cover**
  - And it is technically "**minimal**" –cannot remove another cube without breaking it, *uncovering* some **1s**
  - But maybe we can still do *better*…



One redundant cube – remove it

abcd	F
0*0*	1
0*10	1
1001	1
1101	1
1*1*	1

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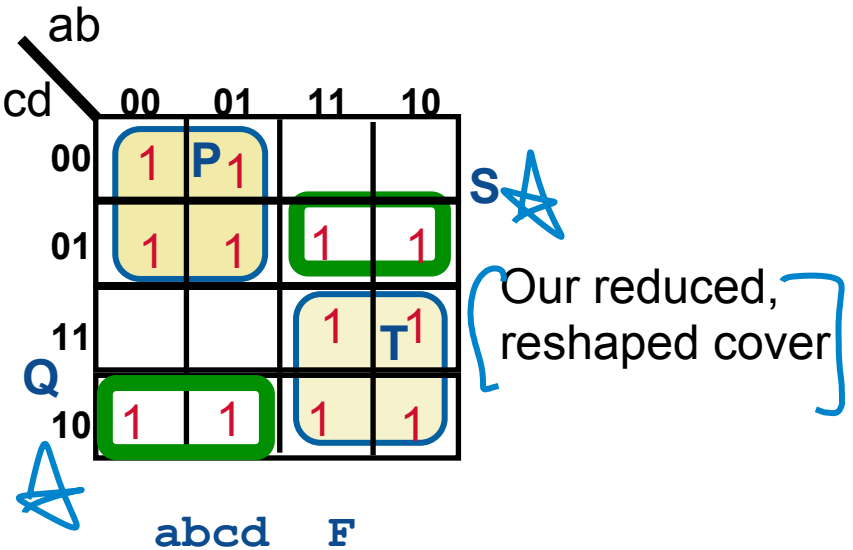
Next Step: *Reduce* the Prime Cover

- **"Reduce" is another heuristic**
  - Take each cube, **"shrink it"** as much as possible, but **do not uncover** any 1s.
  - These result cubes may **not** be prime; i.e. this is **not** necessarily a prime cover

- **Surprising, *essential* step!**
  - This new solution has different **shape**
  - **Big Idea:** When we expand it *again*, maybe we get a new, *better* solution
  - So, maybe we can still do *better*…



abcd	F
0*0*	1
0*10	1
1001	1
1101	1
1*1*	1

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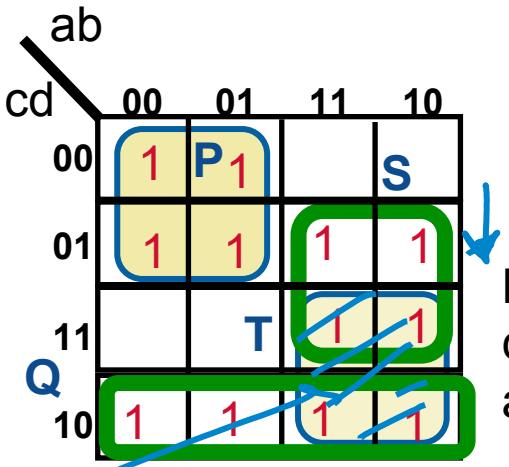
Next Step: *Expand* Cubes Again

- **Same "Expand" heuristic**
  - But it is starting from a **different cover**, so can get a different answer!
  - Take each cube and **"expand"** it to make it prime, and also…
  - …try to cover other cubes, to make them **redundant** (so we kill them later)

- **In example: look at T cube!**



New, expanded cover, all cubes are prime again

abcd	F
0*0*	1
0*10	1
1001	1
1101	1
1*1*	1

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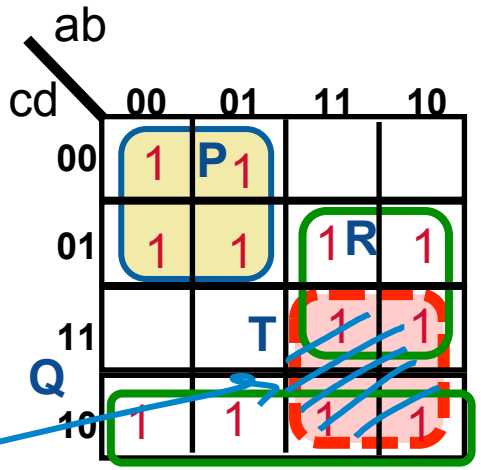
Next Step: Check *Redundant* Again

- **Same "Irredundant" heuristic**
  - But it is starting from a **different cover**, so can get a different answer!
  - Took each cube and **"expanded"** it to make it prime, but we also…
  - …tried to cover other cubes, to make them **redundant** (so we kill them later)



One newly redundant cube, remove it

- **In this example: we can kill another cube (T), it's redundant**
  - After this, the cover is again prime, and irredundant. Can't remove anything to make it better (smaller)

abcd	F
0*0*	1
0*10	1
1001	1
1101	1
1*1*	1

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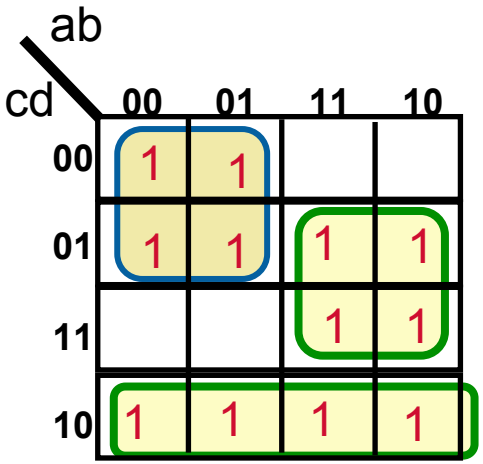
This Result: Is Really Good!

- **Got lucky: this is BEST answer**
  - This will **not** generally happen !!
  - But we can guarantee a **prime**, **minimal, irredundant** solution
  - And it turns out in practice, that this iterative improvement "reshaping" of the cover produces excellent solutions



Final, optimized, Prime, and Irredundant Cover

abcd	F
0*0*	1
0*10	1
1001	1
1101	1
1*1*	1

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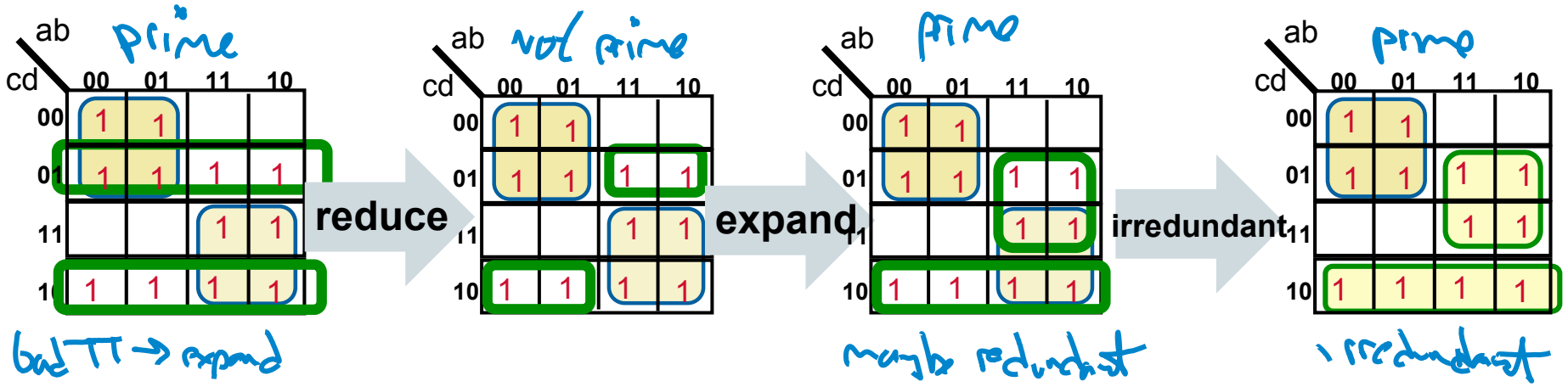
Famous: Reduce-Expand-Irredundant Loop



- **Famous tool: ESPRESSO for 2-level minimization**
  - Started at IBM, finished at Berkeley
  - Brayton, Hachtel, McMullen, Sangiovanni-Vincentelli, **Logic Minimization Algorithms for VLSI Synthesis**, Kluwer Academic Press, 1984, is the reference here
  - Richard L. Rudell, (1986-06-05), "Multiple-Valued Logic Minimization for PLA Synthesis", Memo No. UCB/ERL M86-65 (U. California Berkeley M.S. Thesis)
  - Also, Giovanni DeMicheli, **Synthesis and Optimization of Digital Circuits,** McGraw Hill, 1994

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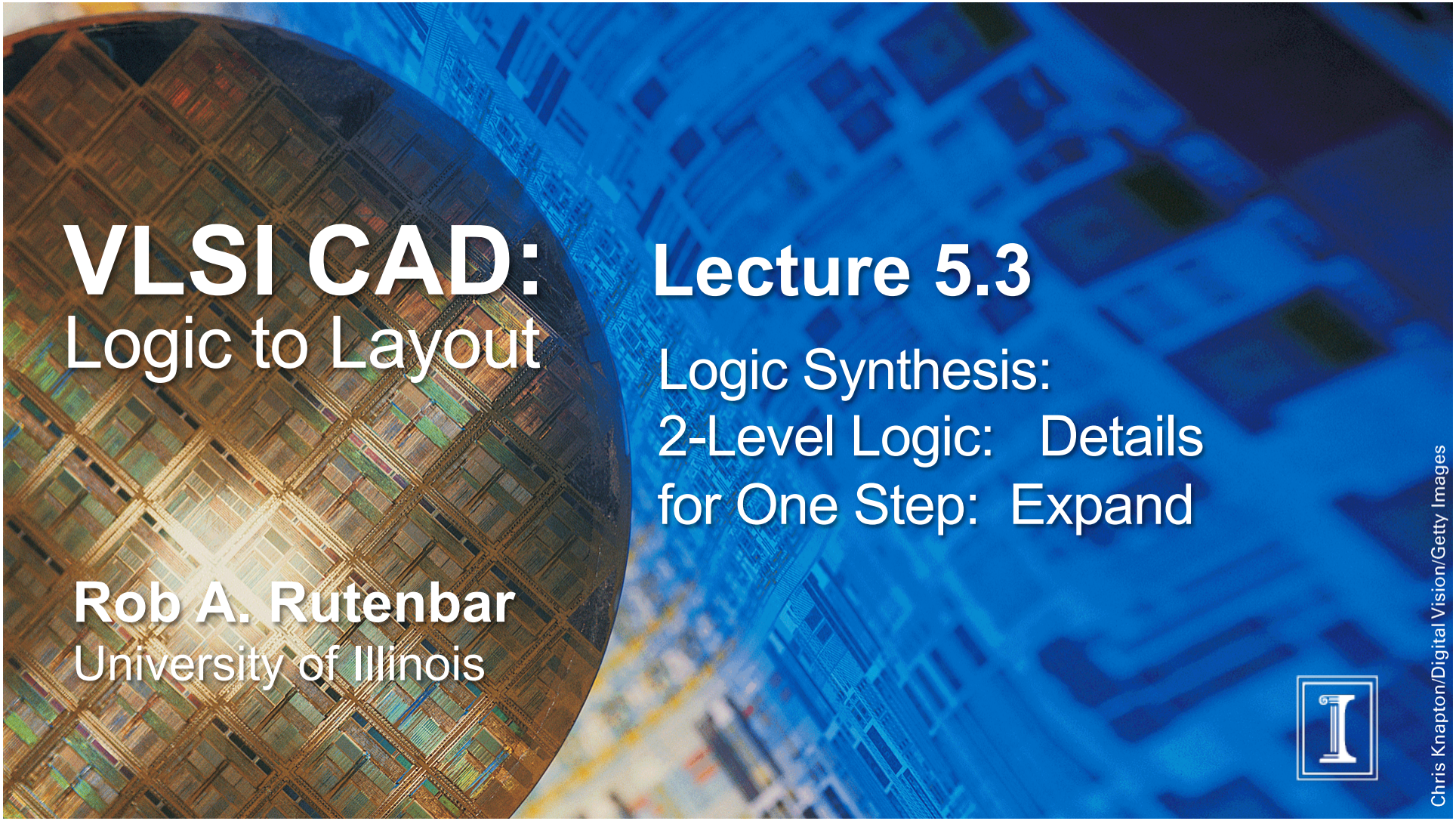

VLSI CAD: Logic to Layout

Lecture 5.3

Logic Synthesis:
2-Level Logic: Details
for One Step: Expand

Rob A. Rutenbar
University of Illinois

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How It Works: Interesting Mechanics

- **The core representation is PCN cube lists**
  - Just like **Week 1's** lectures on URP tautology!
- **Many heuristics use clever ordering of the cubes, and then operate on each cube in some ranked order**
  - Mostly doing bit-level PCN operations on the cube lists
- **Many of the rest of the operations are all done as URP**
  - Just like **Week 1's** lectures!
- **With some work, you could read much of primary source literature**
  - Congratulations – progress!

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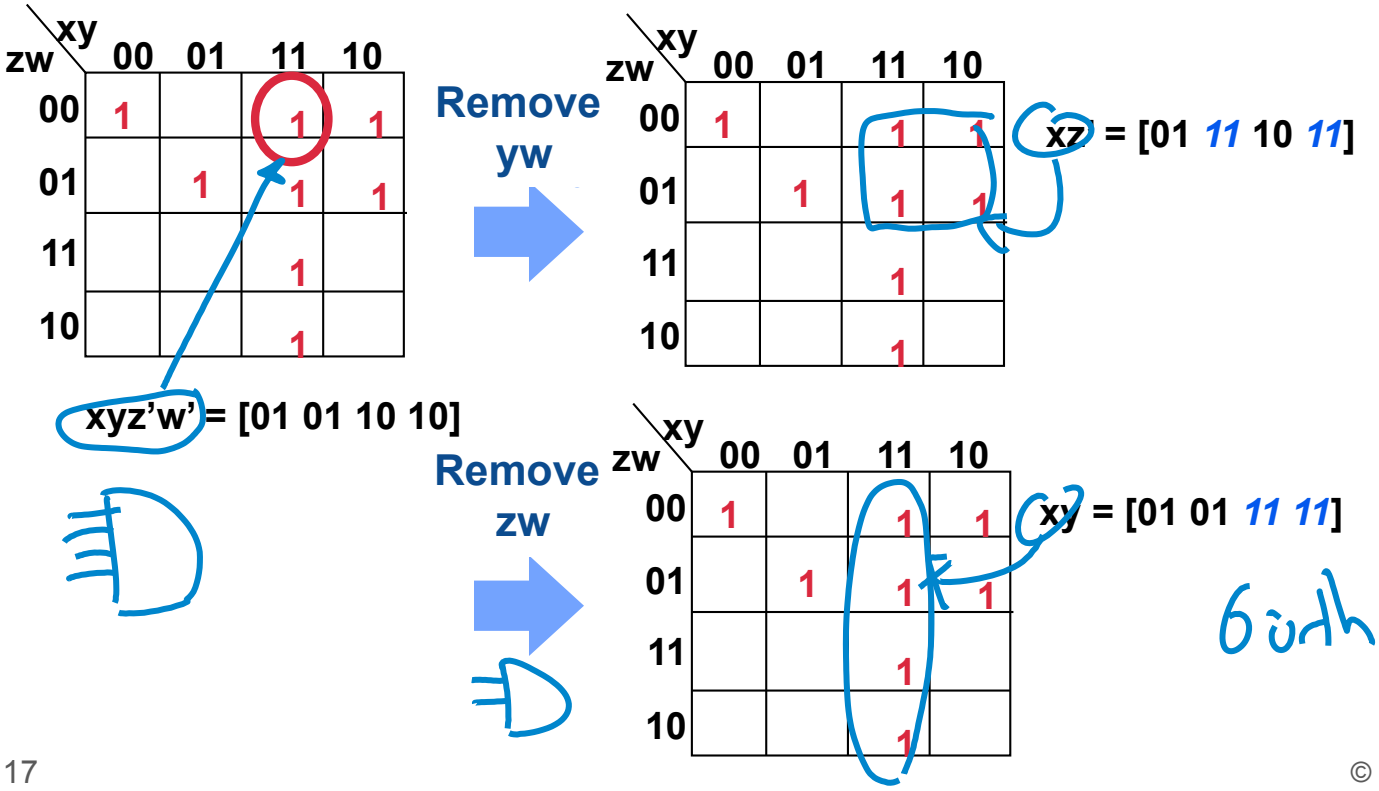
Lets Look (Briefly) At One Step: *Expand*

- **What does 'expand a cube' mean? Remove variables from cube**



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Expand: Transform into a *Covering Problem*

- **Here is the most basic Covering Problem**
  - Given matrix of **R** rows x **C** columns. Matrix has 1s & 0s in it. Only draw the 1s.
  - Chose **smallest set of rows** so that, using only these rows, every column has *at least* a single **1** in it – i.e., **every column is "covered" by the selected rows**
  - Very good **heuristics** to get decent, fast solutions for these

	C1	C2	C3	C4	C5
R1		1			
R2	1	1	1		
R3			1		1
R4		1		1	1

cover

	C1	C2	C3	C4	C5
R1		1			
R2	1	1	1		
R3			1		1
R4		1		1	1

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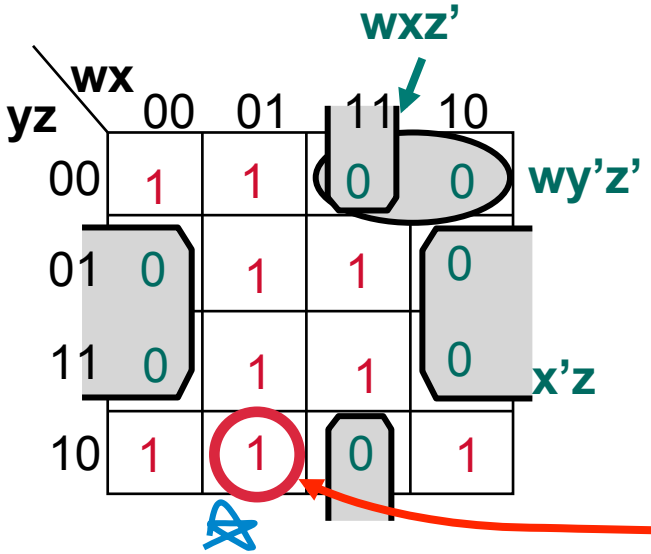
Expand: the Blocking Matrix

- **Expand = a Covering Problem on the Blocking Matrix**
  - **First**: Given function **F**, build a **cube cover** of the **0s** in **F** (called the **OFF Set**)
  - **Why**: We need to know what our cube **cannot touch** when it expands
  - How: **URP Complement** of the starting cover of the function! (Yes, this *exactly* our Programming Assignment #1, this is why we assigned it…)



F = w'y'z' + xz + x'yz' + w'xyz'

URP Complement

F' = x'z + wxz' + wy'z'

Expand this cube

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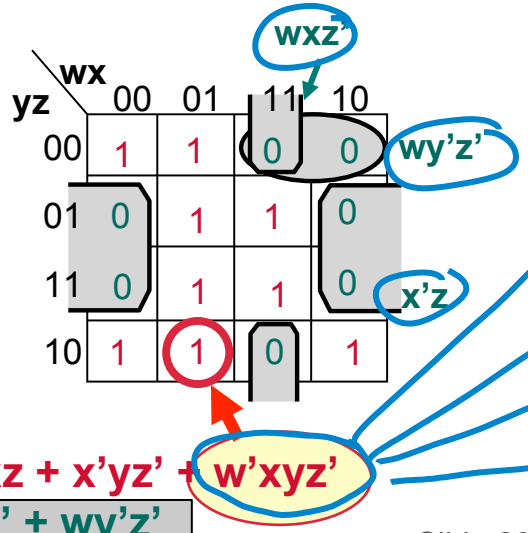
Next: Build the *Blocking Matrix*

- **This is a binary matrix structured as follows:**
  - One **row** for each variable in the cube you are trying to expand
  - One **column** for each cube in the cover of the **OFF** set
  - Put a **"1"** in the matrix if the cube variable (row) **≠ polarity** of variable in the cube (column) of the **OFF** cover; else **"0"**. If don't care, it's a **"0"** (draw as *blank*)



cubes in F' cover (may be a lot...)

vars in expand cube	x'z	wxz'	wy'z'
w'		1	1
x	1		
y			1
z'	1		

F = w'y'z' + xz + x'yz' + w'xyz'

F' = x'z + wxz' + wy'z'

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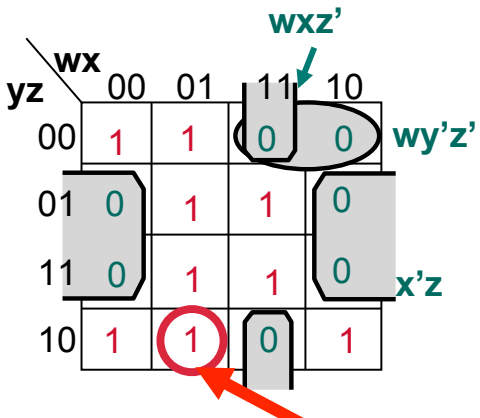
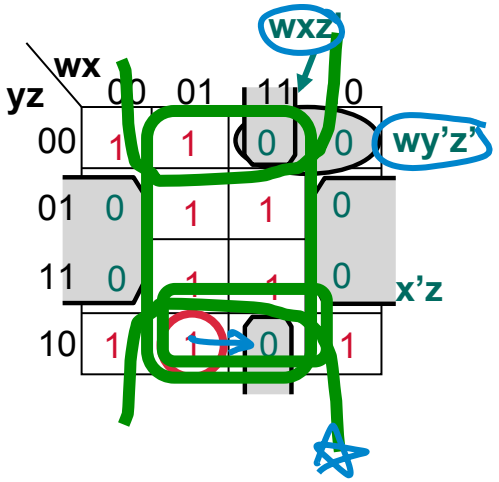
"1" in Blocking Matrix Row/Column Means…

- **Row**: If you **remove** this variable (to expand cube)…
- **Column**: …then you might **overlap** this **OFF** cube – depending on what *other* variables you remove from this cube to expand it. You cannot hit any **OFF** cubes.



	x'z	wxz'	wy'z'
w'		1	1
x	1		
y			1
z'	1		



F = w'y'z' + xz + x'yz' + w'xyz'

F' = x'z + wxz' + wy'z'

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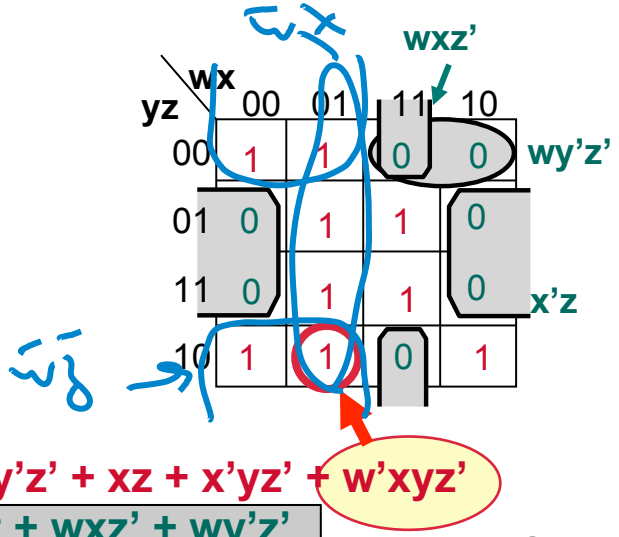
Next: Build the *Blocking Matrix*

- **Result: Find *smallest* set of rows that covers each column. Product of these row variables is a legal cube expansion**
  - If you **keep** just these variables, you "mutually avoid" **ALL** the **OFF** cubes!
  - (Also, try to **overlap** other cubes to make them **redundant**; not talking about this, but it turns out to be yet another connected covering problem…)



F = w'y'z' + xz + x'yz' + w'xyz'

F' = x'z + wxz' + wy'z'

cubes in F' cover (may be a lot...)

vars in expand cube	x'z	wxz'	wy'z'
w'		1	1
x	1		
y			1
z'	1		

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ESPRESSO: Collection of Elegant Heuristics

- **Reduce-Expand-Irredundant cycle**
  - **Reduce**
    - Rank cubes in a clever order, do PCN bit hacking to reduce them individually
  - **Expand**
    - Rank cubes in the opposite of this clever order, expand each individually as a pair of covering problems
  - **Irredundant**
    - A clever URP algorithm (like tautology) + a clever covering problem
  - And a bunch of **other** interesting steps we did not mention…

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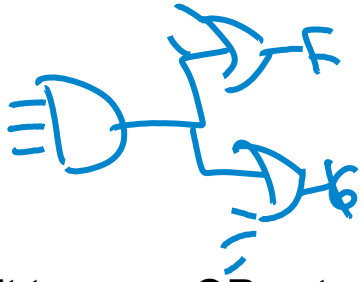
Aside: Other Things Method Can Do

- **Minimize several functions at the same time**
  - Each function will be reduced to a 2-level form
  - But some product terms (AND gates) will be **shared**
  - This means: make this AND product once in hardware, connect it to many OR gate outputs to sum it into other functions. Can save a lot of hardware this way



- **Handle conventional Don't Cares**
  - Can specify a row of the truth table (TT) as being a "Don't Care"
  - Means the hardware can make a **1** or a **0** as output for this input– you don't care
  - Let algorithm choose **0** vs **1** output to make better, more minimal hardware.

output = DC

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How Well Does All This Work...?

- **Fabulous: Very fast, very robust**
- **Where does ESPRESSO spend its time?** **[Brayton et al Kluwer book, 1984]**
  - **Complement** 14% (big if there are lots of cubes in cover)
  - **Expand** 29% (depends on of size of complement)
  - **Irredundant** 12%
  - **Essentials** 13% (some primes *must* be in answer; find them first)
  - **Reduce** 8%
  - **Various optimizations** 22% (special case optimizations)
- **How fast?**
  - Usually **< 5 reduce-expand-irredundant** iterations; often converges in just **1-2**
  - Thousands of cubes, tens of thousands of literals: **<< 1 CPU second**

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Summary

- **2-level logic synthesis uses heuristics to find good solutions**
  - Not "best", but instead "good enough"
  - Minimal (not minimum), prime, irredundant
  - Famous idea: iterative improvement – **reduce-expand-irredundant loop**
  - All done with PCN cubelists, covering matrices, and URP ideas

- **But… not every piece of logic is implemented in 2-level form**
- **Next: Multi-level logic**

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