Inversion of 2x2 Matricies

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Notes on the inversion of 2x2 matrices, which can be applied to an arbitrary number of $n \cdot n$ matrices.

1 The inversion of a matrix

Just like every number has a reciprocal such as 8^{-1}, a matrix n also has a reciprocal: n^{-1}. When a matrix is multiplied by it's reciprocal $(n \cdot n^{-1})$ the result is I, also known as the identity matrix. The identity matrix is the matrix equivelance of the number 1 obtained from the reciprocal of a real number, in other words, the identity matrix is just 1, exactly the same way that the reciprocal of $\frac{1}{8} \cdot 8 = 1$. This information is not terribly important, but is nontheless useful to know.

A good example of an identity matrix is a 3×3 matrix, I:

$$i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

You can see here that it has got 1 going down diagonally and 0 in every other row and column. An identity matrix, as mentioned in the title can be $2 \cdot 2$, $3 \cdot 3$, $4 \cdot 4$... The inverse of a matrix a is a^{-1} only when $a \cdot a^{-1} = a^{-1} \cdot a = i$

2 2x2 matricies

There are a few easy steps to find the inverse of a 2x2 matrix. The first step is to determine if a matrix actually has an inverse, you do so by working out the detriment. If it does then you can find the inverse, and then check if the inverse is correct by mutliplying the inverse by the original matrix. We can break the steps further down into the following:

1. get the inverse
2. multiply the detriment by the inverse
3. multiply the result of the inverse * the detriment
4. subtract/add the values inside of the matrix of the result of the inverse * the detriment
5. check if the resulting matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2.1 The detriment * the inverse matrix

To workout the detriment, we do the following:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{a \cdot d - b \cdot c} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Here in the matrix on the right (the inverse of the original matrix) we have swapped matrix element a and d with each other, and then added a $-$ sign in front of b and c. We then times that matrix by $\frac{1}{a \cdot d - b \cdot c}$. If the detriment is non-zero then the 2x2 matrix *does* have an inverse. Otherwise, this 2x2 matrix does have an inverse, and is therefore incalculable. Here is an example:

$$\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}^{-1} = \frac{1}{4 \cdot 6 - 7 \cdot 2} \cdot \begin{pmatrix} 6 & -7 \\ -2 & 4 \end{pmatrix}$$
$$= \begin{pmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{pmatrix}$$

The best idea is to just enter $\frac{1}{4 \cdot 6 - 7 \cdot 2}$ onto a scientific calculator directly. Do not work out $4 \cdot 6 - 7 \cdot 2$ yourself as this can lead to an incorrect calculation, especially since there are so many decimal places involved.

2.2 Multiplying the matrix by the result from the inverse and detriment

Once we have worked out the detriment, we then use it to multiply the matrix by its inverse. The first thing to do is to just multiply the original matrix with the inverse we just worked out. We shall continue to use the same example from the previous subsection.

$$\begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix} \cdot \begin{pmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{pmatrix}$$

$$= \begin{pmatrix} 4*0.6+7*(-0.2) & 4*(-0.7)+7*0.4 \\ 2*0.6+6*(-0.2) & 2*(-0.7)+6*0.4 \end{pmatrix}$$

$$= \begin{pmatrix} 2.4-1.4 & -2.8+2.8 \\ 1.2-1.2 & -1.4+2.4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

When the resulting matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then it means you have correctly found the inverse. You can see this is true because we have ended up with the identity matrix, with a value of 1 and it's diagonal 1s and corresponding 0s.

3 Examples

This section contains 5 worked samples