Solving two simultaneous unkowns using matricies

Nathan Blackburn blacknand@github

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Notes on solving two simultaneous unkown equations using matricies. You must first understand the inversion of 2x2 matricies before reading these notes.

1 The steps

Throughout this article, we will be using the example equations: $3x + 2y = -3$ and $5x + 3y = -4$

1. Rewrite the equations using matricies:

   $$\begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

   You can see that the numbers from the first equation, $3x+2y$ are in the first row of the matrix, and the numbers from the second equation, $5x + 3y$ are in the second row of the equation. The dot product of these two matricies now result in the matrix of the results of both unkown equations, -3 and 4. We now clarify with the following:

   $$a = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix} x = \begin{pmatrix} x \\ y \end{pmatrix} b = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

2. Second item:

   Now we have $ax = b$. If $ax = b$ then we need to find x, to do this we can multiply both sides by the inverse of a so a^{-1}, also remeber that the inverse of $a \cdot a^{-1} = i$, the inverse matrix which is just 1.

$$a \cdot a^{-1} \cdot x = b \cdot a^{-1}$$
$$ix = b \cdot a^{-1}$$
$$x = b \cdot a^{-1}$$

Here we can see now that if we multiply b by the inverse of a we get x.

3. Workout the inverse of a: The next step is to workout the inverse of a so we can then multiply the inverse by b.

$$\begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}^{-1} = \frac{1}{3*5-2*5} \cdot \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix}$$

$$= \frac{1}{-1} \cdot \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix}$$

4. Multiply the inverse by b: Now that we have the inverse, we multiply it by b to find x:

$$x = a^{-1} \cdot b = \begin{pmatrix} -3 & 2 \\ 5 & -3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ -4 \end{pmatrix}$$
$$= \begin{pmatrix} -3*(-3) & 2*(-4) \\ 5*(-3) & -3*(-4) \end{pmatrix} = \begin{pmatrix} 9+(-8) \\ -15+(-12) \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

Here we can see that the x and y are $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$