Application of the Kalman Filter to a Satellite Simulator to Predict Orbital Position

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$$
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\
a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\
a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\
a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66}
\end{bmatrix}
$$

Symbol	**Description**	**Dimensions**	**Role**
x	State variable	$n \times 1$ column vector	Output
P	State covariance matrix	$n \times n$ matrix	Output
z	Measurement	$m \times 1$ column vector	Input
A	State transition matrix	$n \times n$ matrix	System Model
H	State-to-measurement matrix	$n \times n$ matrix	System Model
R	Measurement covariance matrix	$m \times m$ matrix	Input
Q	Process noise covariance matrix	$n \times n$ matrix	System Model
K	Kalman Gain	$n \times m$	Internal

Table 1: System variables, matrices, and their roles in the Kalman Filter

Symbol	Name	Description	Example
x_k	State Vector	The current estimate of the system's state at time k.	$x_k = [px, py, pz, vx, vy, vz]^{\mathrm{T}}$ (6x1 column: position in meters, velocity in m/s).
$\hat{x}_k$	Predicted State	The predicted state at time k before incorporating sensor measurements.	Same 6x1 vector, e.g., $[0, 0, 0, 0, 0, 9]$ after predicting with $\text{accel}_z = 9\,\text{m/s}^2$ for 1s.
z_k	Measurement Vector	The sensor's observation at time k.	$z_k = [vx, vy, vz]$ (3x1), approximated from accel (e.g., $[0, 0, 0]$ if stationary).
F	State Transition Matrix	Describes how the state evolves over time without external inputs. $\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	6x6 matrix: (position grows with velocity).
B	Control Input Matrix	Maps the control input to the state. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix}$	6x3 matrix: (accel affects velocity).
u_k	Control Vector	The external input affecting the system at time k.	$u_k = [ax, ay, az]^T$ (3x1), e.g., $[0, 0, 9]$ from MPU6050 in m/s^2.
H	Observation Matrix	Relates the state to the measurement. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	3x6 matrix: (picks velocities from state).
P_k	Covariance Matrix	Represents the uncertainty in the state estimate at time k.	6x6 matrix, e.g., diagonal starts at 1000 (high uncertainty), adjusts with each step.
Q	Process Noise Covariance	Uncertainty added during prediction due to model imperfections.	6x6 matrix, e.g., diagonal $[0.01, 0.01, 0.01, 0.1, 0.1, 0.1]$ (small noise for pos, vel).
R	Measurement Noise Covariance	Uncertainty in the sensor measurements.	3x3 matrix, e.g., diagonal $[1.0, 1.0, 1.0]$ (noise in accel-derived velocity).
K	Kalman Gain	Determines how much to trust the measurement vs. the prediction.	6x3 matrix, computed to balance P and R, e.g., 0.5 means half-trust in sensor.
w_k	Process Noise	Random errors in the system model (assumed Gaussian).	Handled by Q, e.g., accel jitter or unmodeled effects.
v_k	Measurement Noise	Random errors in the sensor data (assumed Gaussian).	Handled by R, e.g., noise in MPU6050 accel readings.

Table 2: Kalman Filter Variables and Examples