Application of the Kalman Filter to a Satellite Simulator to Predict Orbital Position

Nathan Blackburn–blacknand

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$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}$$

Symbol	**Description**	**Dimensions**	**Role**
x	State variable	$n \times 1$ column vector	Output
P	State covariance matrix	$n \times n$ matrix	Output
z	Measurement	$m \times 1$ column vector	Input
A	State transition matrix	$n \times n$ matrix	System Model
H	State-to-measurement matrix	$n \times n$ matrix	System Model
R	Measurement covariance matrix	$m \times m$ matrix	Input
Q	Process noise covariance matrix	$n \times n$ matrix	System Model
K	Kalman Gain	$n \times m$	Internal

Table 1: System variables, matrices, and their roles in the Kalman Filter

Symbol	Name	Description	Example
x_k	State Vector	The current estimate of the system's state at time k.	$x_k = [px, py, pz,$ umn: position i m/s).
$\hat{x}_k$	Predicted State	The predicted state at time k before incorporating sensor measurements.	Same 6x1 vector, ter predicting wit 1s.
z_k	Measurement Vector	The sensor's observation at time k.	$z_k = [vx, vy, vz]$ from accel (e.g., [
F	State Transition Matrix	Describes how the state evolves over time without external inputs. $\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	6x6 matrix: (position grows w
B	Control Input Matrix	Maps the control input to the state. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix}$	6x3 matrix: (accel affects velo
u_k	Control Vector	The external input affecting the system at time k.	$u_k = [ax, ay, az]$ from MPU6050 i
H	Observation Matrix	Relates the state to the measurement. $\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$	3x6 matrix: (picks velocities f
P_k	Covariance Matrix	Represents the uncertainty in the state estimate at time k.	6x6 matrix, e.g., c (high uncertainty step.
Q	Process Noise Covariance	Uncertainty added during prediction due to model imperfections.	6x6 matrix, $[0.01, 0.01, 0.01, 0$ noise for pos, vel)
R	Measurement Noise Covariance	Uncertainty in the sensor measurements.	3x3 matrix, e.g., (noise in accel-de
K	Kalman Gain	Determines how much to trust the measurement vs. the prediction.	6x3 matrix, comp R, e.g., 0.5 means
w_k	Process Noise	Random errors in the system model (assumed Gaussian).	Handled by Q, e modeled effects.
v_k	Measurement Noise	Random errors in the sensor data (assumed Gaussian).	Handled by R, e. accel readings.

Table 2: Kalman Filter Variables and Examples