CS2040S
Data Structures and Algorithms

Welcome!

Problem Set 3

Sorting Detective

- Six suspicious sorting algorithms
  - Investigate the mysterious sorting code.
  - Identify each sorting algorithm.
  - Find the criminal: Dr. Evil!

- Focus on the properties:
  - Asymptotic performance
  - Stability
  - Performance on special inputs

- Absolute speed is not a good reason…

Today: Sorting

Sorting algorithms

- BubbleSort
- SelectionSort
- InsertionSort
- MergeSort

Properties

- Running time
- Space usage
- Stability

Sorting

Problem definition:

Input: array A[1..n] of words / numbers

Output: array B[1..n] that is a permutation of A such that:

$B[1] \leq B[2] \leq \ldots \leq B[n]$

Example:

$A = [9, 3, 6, 6, 6, 4] \rightarrow [3, 4, 6, 6, 6, 9]$

Sorting

```java
public interface ISort{

    public void sort(int[] dataArray);

}
```

Aside: BogoSort

```
BogoSort(A[1..n])
```

Repeat:

a) Choose a random permutation of the array A.
b) If A is sorted, return A.

What is the expected running time of BogoSort?

Aside: BogoSort

`BogoSort(A[1..n])`

Repeat:

a) Choose a random permutation of the array A.

b) If A is sorted, return A.

What is the expected running time of BogoSort?

$\boxed{O(n \cdot n!)}$

Aside: BogoSort

```
QuantumBogoSort(A[1..n])
```

a) Choose a random permutation of the array A.
b) If A is sorted, return A.
c) If A is not sorted, destroy the universe.

What is the expected running time of Quantum BogoSort?

(Remember QuantumBogoSort when you learn about non-deterministic Turing Machines.)

Aside: MaybeBogoSort

MaybeBogoSort(A[1..n])

1. Choose a random permutation of the array A.
2. If A[1] is the minimum item in A then:

   MaybeBogoSort(A[2..n])

   Else

   MaybeBogoSort(A[1..n])

What is the expected running time of MaybeBogoSort?

Today: Sorting

Sorting algorithms
- BubbleSort
- SelectionSort
- InsertionSort
- MergeSort

Properties
- Running time
- Space usage
- Stability

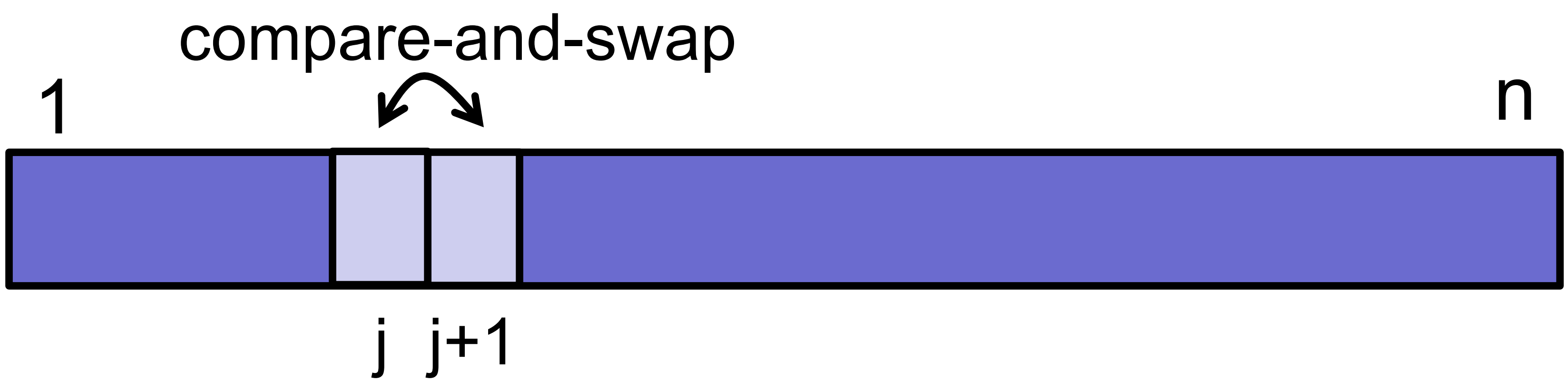
BubbleSort

BubbleSort(A, n)

repeat n **times:**

for j $\leftarrow$ 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])



BubbleSort

Example: 8 2 4 9 3 6

BubbleSort

Example: **8** **2** 4 9 3 6

2 **8** 4 9 3 6

BubbleSort

Example:	8	2	4	9	3	6
	2	**8**	**4**	9	3	6
	2	**4**	**8**	9	3	6

BubbleSort

Example:						
	8	2	4	9	3	6
	2	8	4	9	3	6
	2	4	**8**	**9**	3	6
	2	4	**8**	**9**	3	6

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	**9**	**3**	6
2	4	8	**3**	**9**	6

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	4	8	3	**9**	**6**
2	4	8	3	**6**	**9**

BubbleSort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	4	8	3	9	6
2	**4**	**8**	**3**	**6**	**9**

BubbleSort

Pass 2:

2	4	8	3	6	9
2	4	8	3	6	9
2	4	8	3	6	9
2	4	3	8	6	9
2	4	3	6	8	9
2	**4**	**3**	**6**	**8**	**9**

BubbleSort

Pass 3:

2	4	3	6	8	9
2	4	3	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	**3**	**4**	**6**	**8**	**9**

BubbleSort

Pass 4:

2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	**3**	**4**	**6**	**8**	**9**

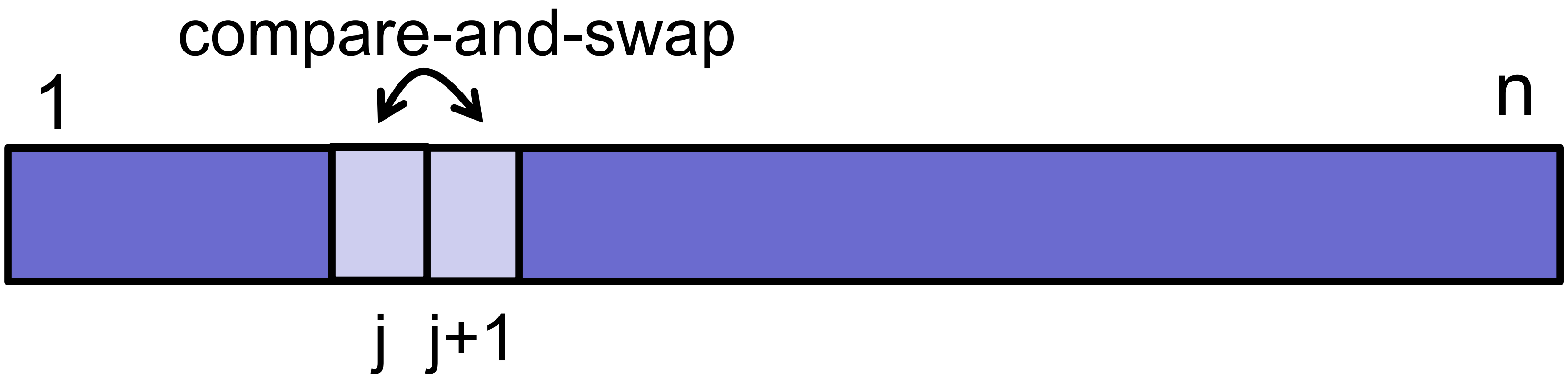
BubbleSort

BubbleSort(A, n)

repeat n **times:**

for j $\leftarrow$ 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])



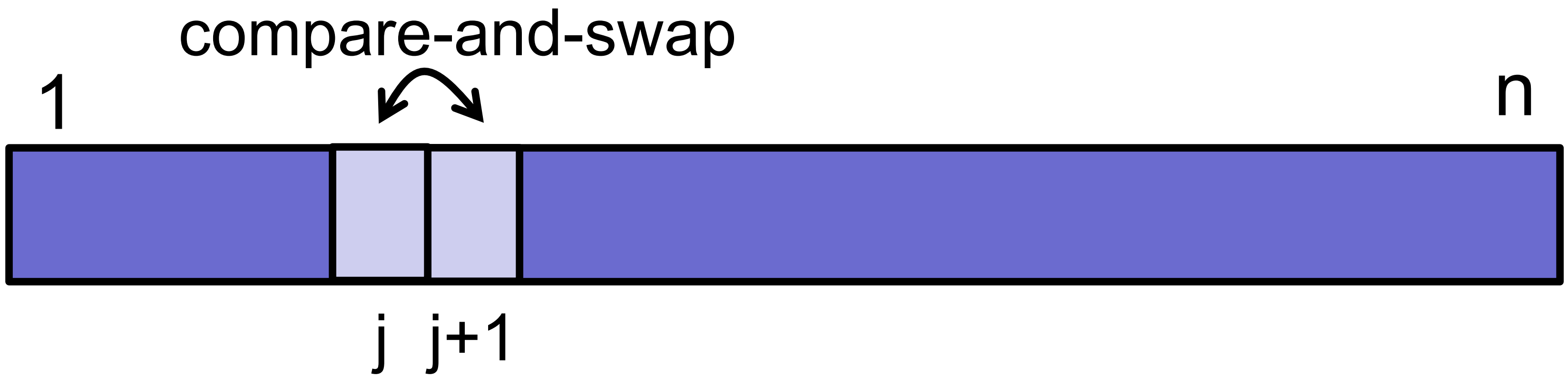
BubbleSort

BubbleSort(A, n)

repeat (until no swaps) **:**

for $j \leftarrow 1$ **to** $n-1$

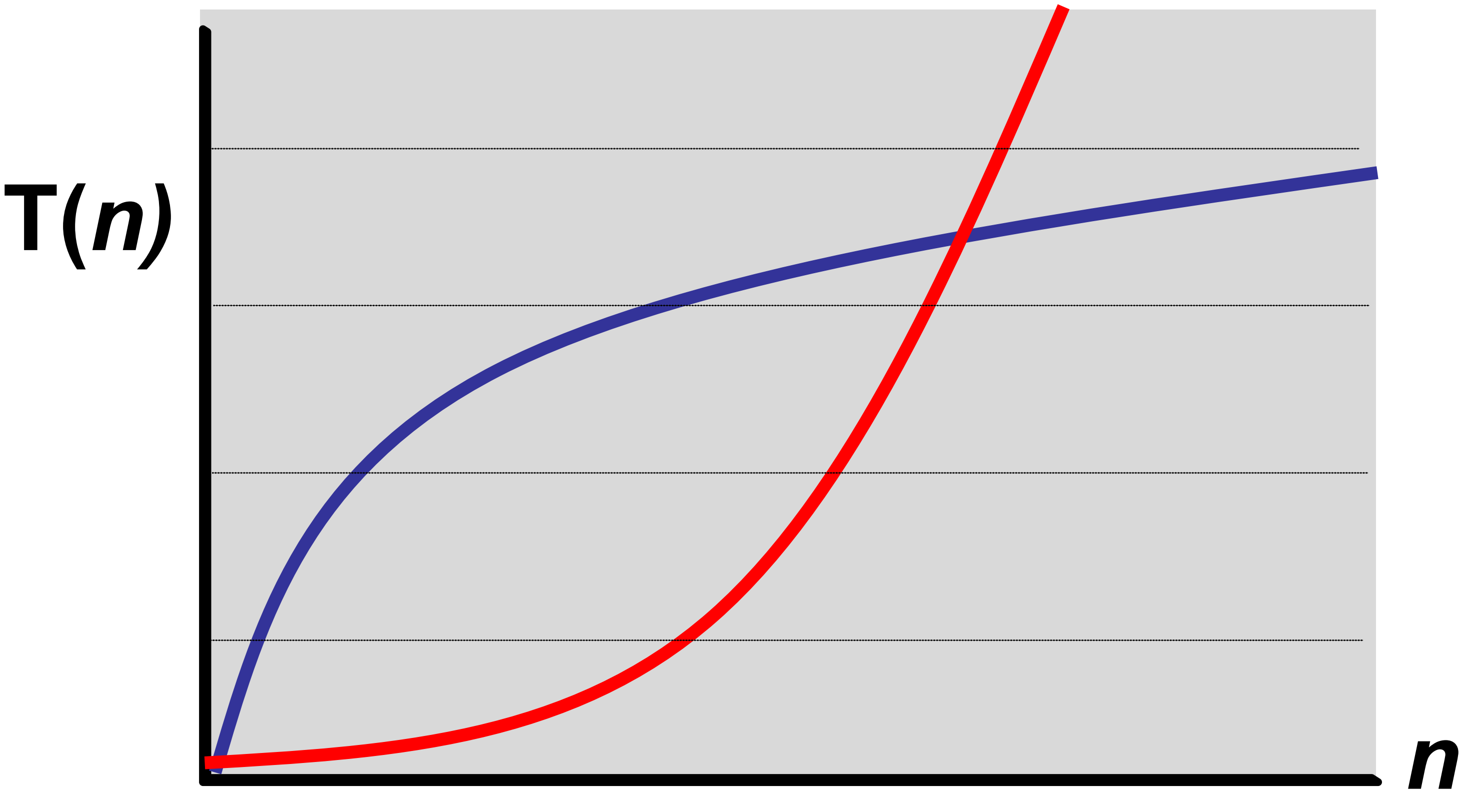
if $A[j] > A[j+1]$ **then** swap($A[j]$, $A[j+1]$)



Big-O Notation

How does an algorithm scale?

- For large inputs, what is the running time?
- T(n) = running time on inputs of size n



What is the running time of BubbleSort?

A. $O(n)$
B. $O(n \log n)$
C. $O(n\sqrt{n})$
D. $O(n^2)$
E. $O(2^n)$

BubbleSort

Running time:

- Depends on the input!

BubbleSort

Example:

2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	3	4	6	8	9
2	**3**	**4**	**6**	**8**	**9**

BubbleSort

Running time:

- Depends on the input!

Best-case:

- Already sorted: O(n)

BubbleSort

Best-case:

- Already sorted: $O(n)$

Average-case:

- Assume inputs are chosen at random.

Worst-case:

- Max running time over all possible inputs.

BubbleSort Analysis

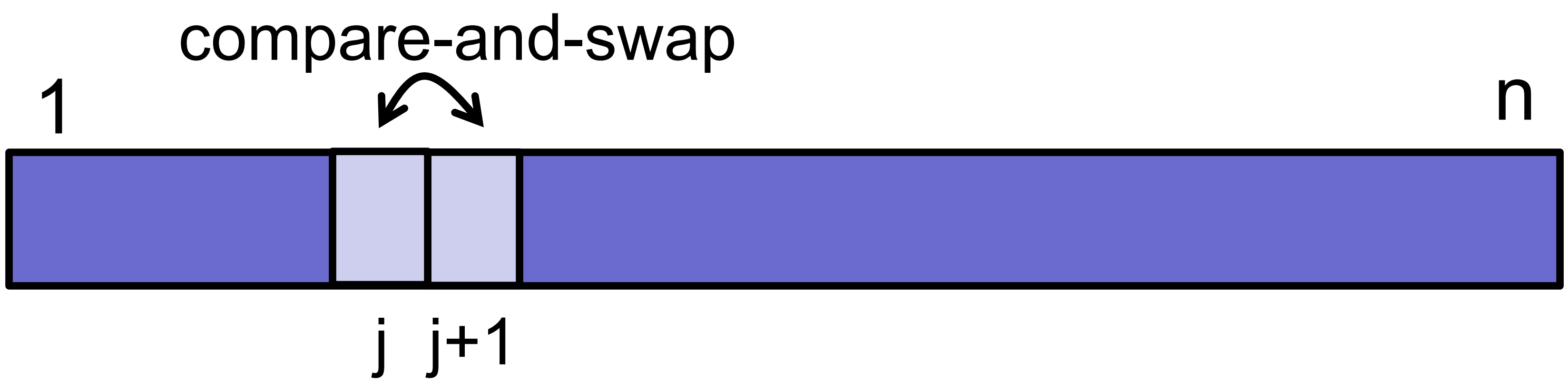
How many iterations do we need?

BubbleSort(A, n)

repeat (until no swaps) **:**

for j $\leftarrow$ 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])



BubbleSort Analysis

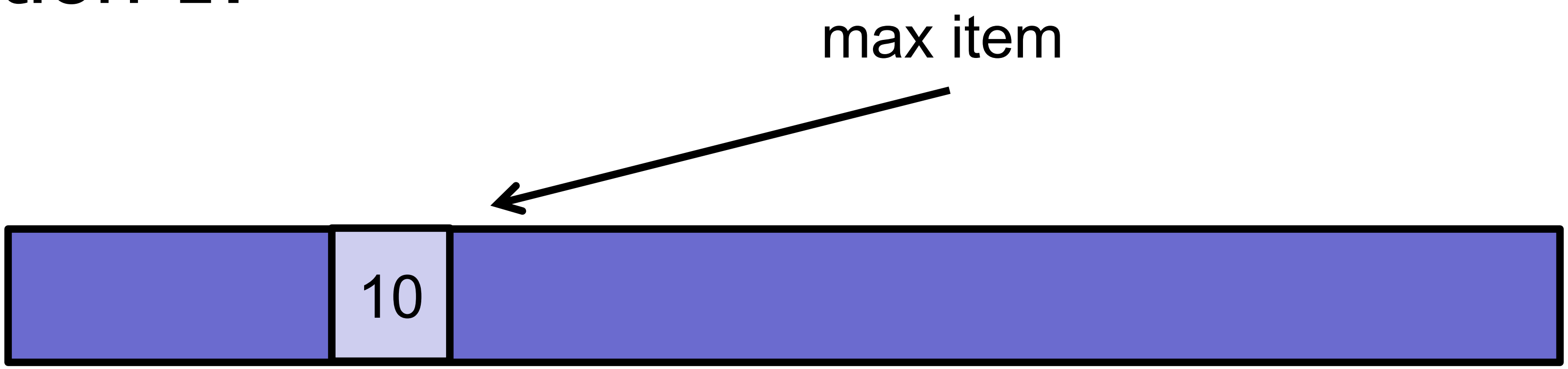
BubbleSort(A, n)

repeat (until no swaps) **:**

for j ← 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])

Iteration 1:



BubbleSort Analysis

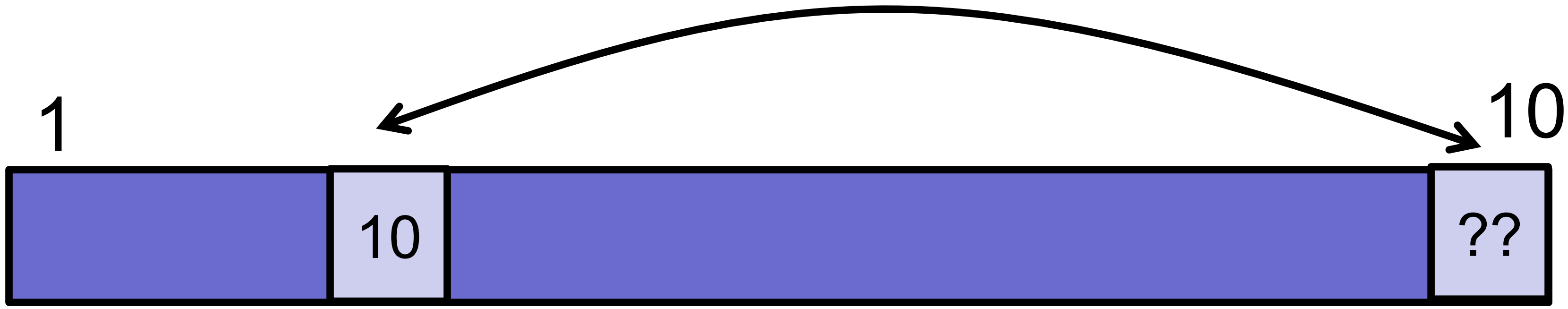
BubbleSort(A, n)

repeat (until no swaps) **:**

for j ← 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])

Iteration 1:



BubbleSort Analysis

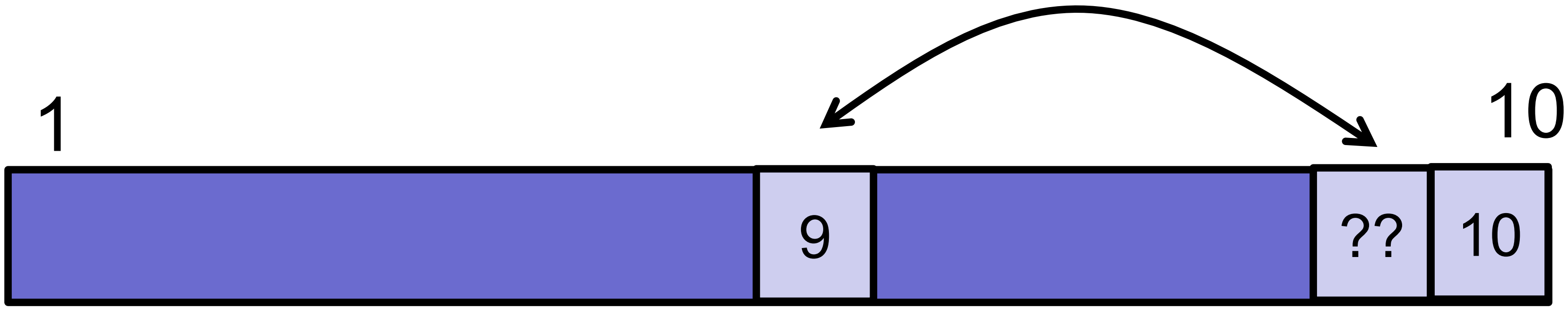
BubbleSort(A, n)

repeat (until no swaps) **:**

for j $\leftarrow$ 1 **to** n-1

if A[j] > A[j+1] **then** swap(A[j], A[j+1])

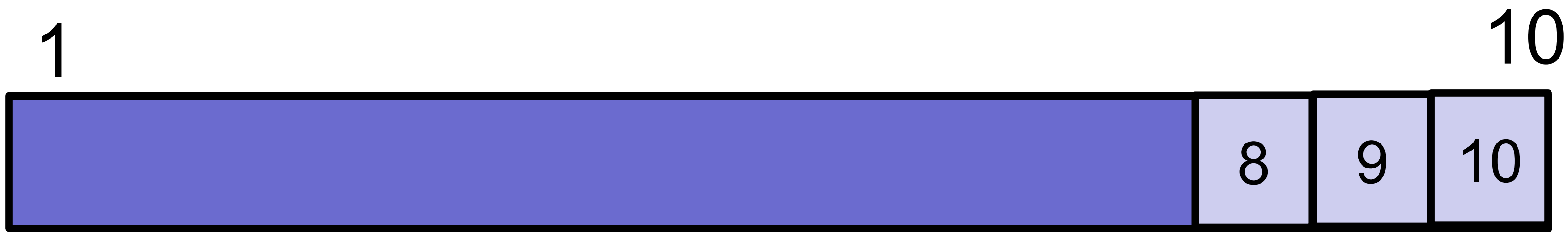
Iteration 2:



BubbleSort Analysis

Loop invariant:

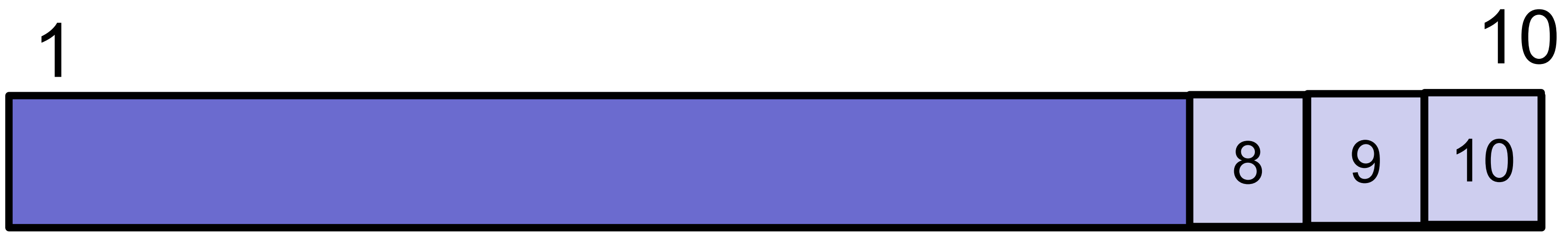
At the end of iteration j: ???



BubbleSort Analysis

Loop invariant:

At the end of iteration j, the biggest j items are correctly sorted in the final j positions of the array.

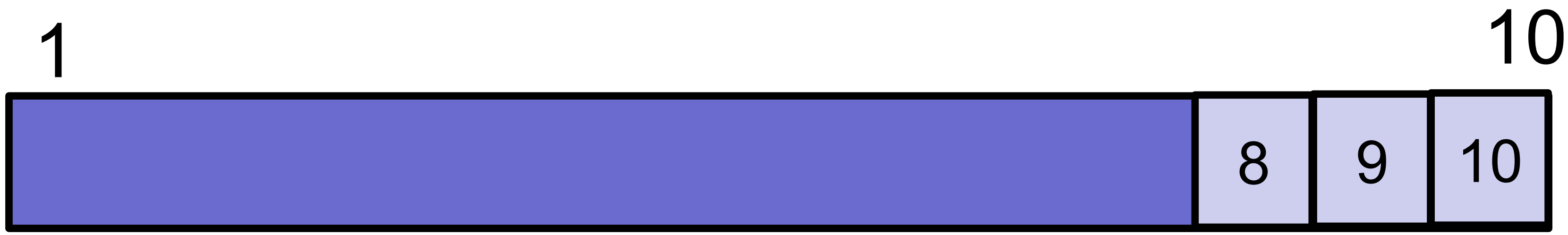


BubbleSort Analysis

Loop invariant:

At the end of iteration j, the biggest j items are correctly sorted in the final j positions of the array.

Worst case: n iterations → $O(n^2)$ time



BubbleSort

Best-case: $O(n)$

- Already sorted

Average-case: $O(n^2)$

- Assume inputs are chosen at random…

Worst-case: $O(n^2)$

- Bound on how long it takes.

Today: Sorting

Sorting algorithms

- BubbleSort
- SelectionSort
- InsertionSort
- MergeSort

Properties

- Running time
- Space usage
- Stability

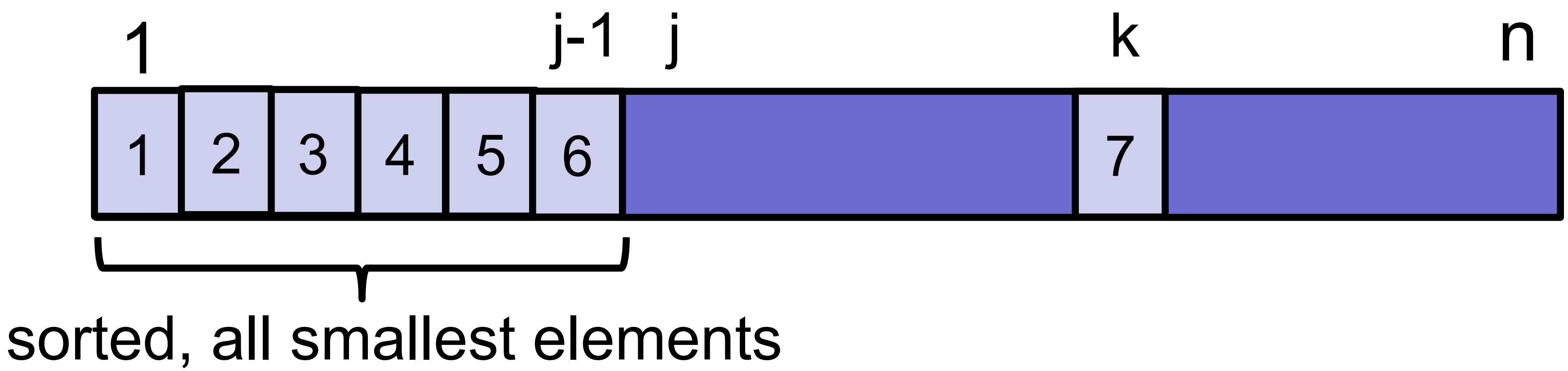
SelectionSort

SelectionSort(A, n)

for j ← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])



SelectionSort

Example: 8 2 4 9 3 6

SelectionSort

Example: 8 **2** 4 9 3 6

SelectionSort

Example:	8	**2**	4	9	3	6
	2	8	4	9	3	6

SelectionSort

Example: 8 **2** 4 9 3 6

2 8 4 9 **3** 6

SelectionSort

Example:	8	**2**	4	9	3	6
	2	8	4	9	**3**	6
	2	3	4	9	8	6

SelectionSort

Example:						
	8	**2**	4	9	3	6
	2	8	4	9	**3**	6
	2	3	**4**	9	8	6

SelectionSort

Example:

8	**2**	4	9	3	6
2	8	4	9	**3**	6
2	3	**4**	9	8	6
2	3	4	9	8	6

SelectionSort

Example:						
	8	**2**	4	9	3	6
	2	8	4	9	**3**	6
	2	3	**4**	9	8	6
	2	3	4	9	8	**6**
	2	3	4	6	8	9

SelectionSort

Example:					
8	**2**	4	9	3	6
2	8	4	9	**3**	6
2	3	**4**	9	8	6
2	3	4	9	8	**6**
2	3	4	6	**8**	9
2	3	4	6	8	9

What is the (worst-case) running time of SelectionSort?

A. $O(n)$
B. $O(n \log n)$
C. $O(n\sqrt{n})$
D. $O(n^2)$
E. $O(2^n)$

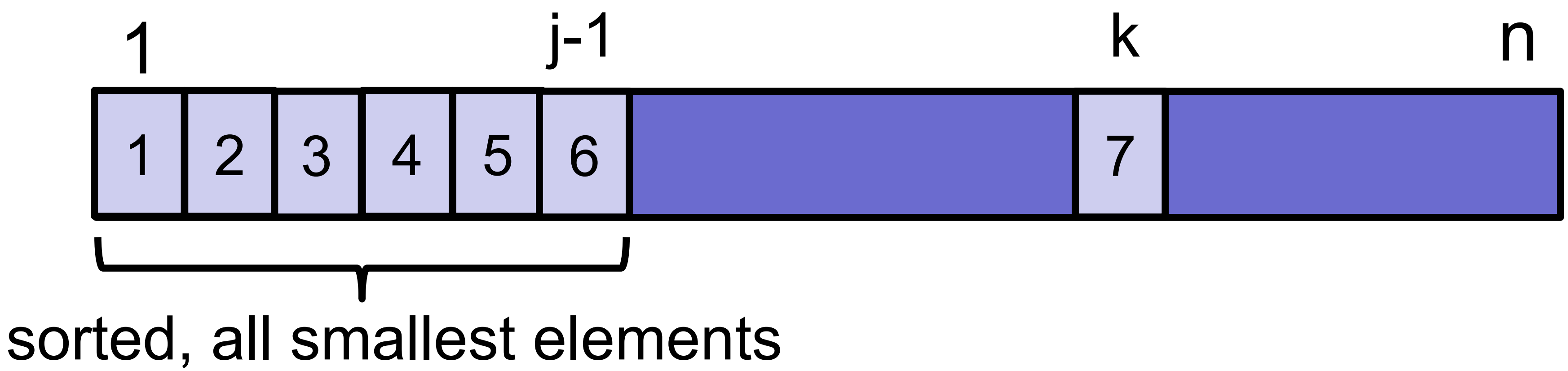
SelectionSort

SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])



SelectionSort

Time: $(n - j)$

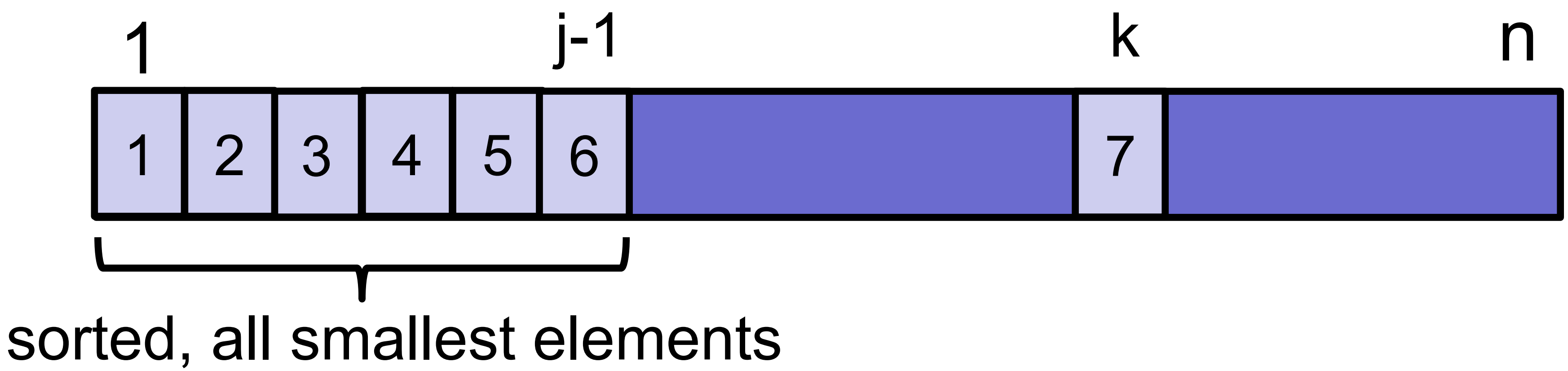
SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Running time: $n + (n-1) + (n-2) + (n-3) + \ldots$



SelectionSort

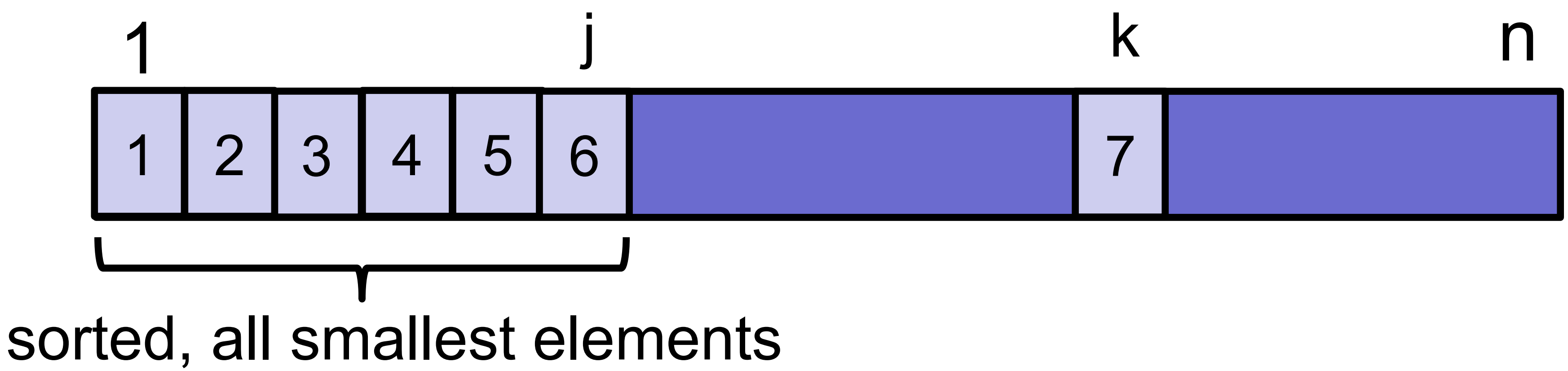
Time: $(n - j)$

SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])



Basic facts

$$n + (n - 1) + (n - 2) + (n - 3) + \ldots + 1 \qquad = (n)(n+1)/2$$

$$= \Theta(n^2)$$

SelectionSort

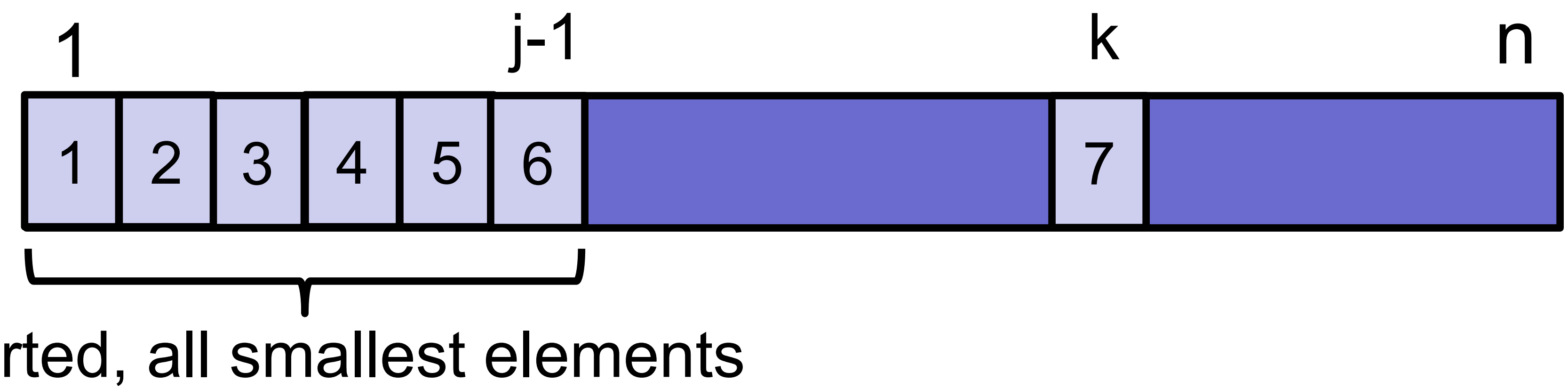
SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Running time: $O(n^2)$



What is the BEST CASE running time of SelectionSort?

A. $O(n)$

B. $O(n \log n)$

C. $O(n\sqrt{n})$

D. $O(n^2)$

E. $O(2^n)$

SelectionSort

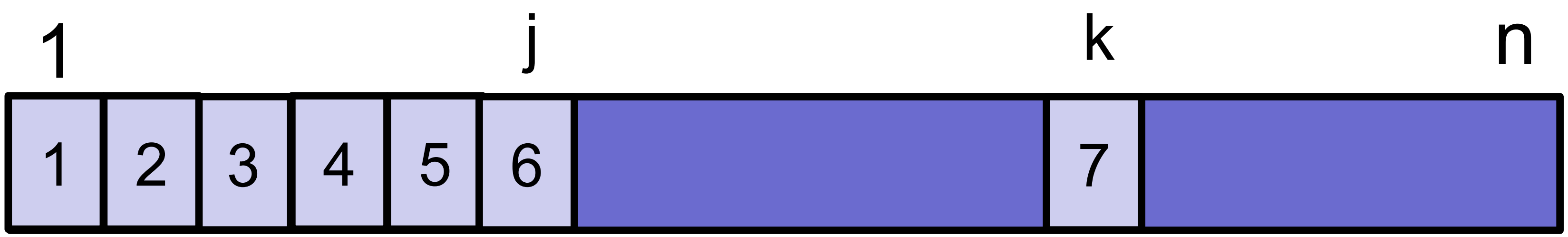
SelectionSort(A, n)

for j← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

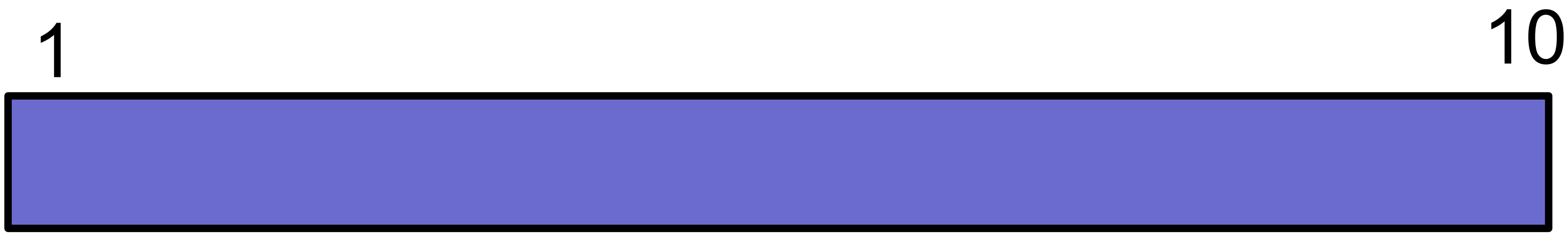
Running time: $O(n^2)$ and $\Omega(n^2)$



SelectionSort Analysis

Loop invariant:

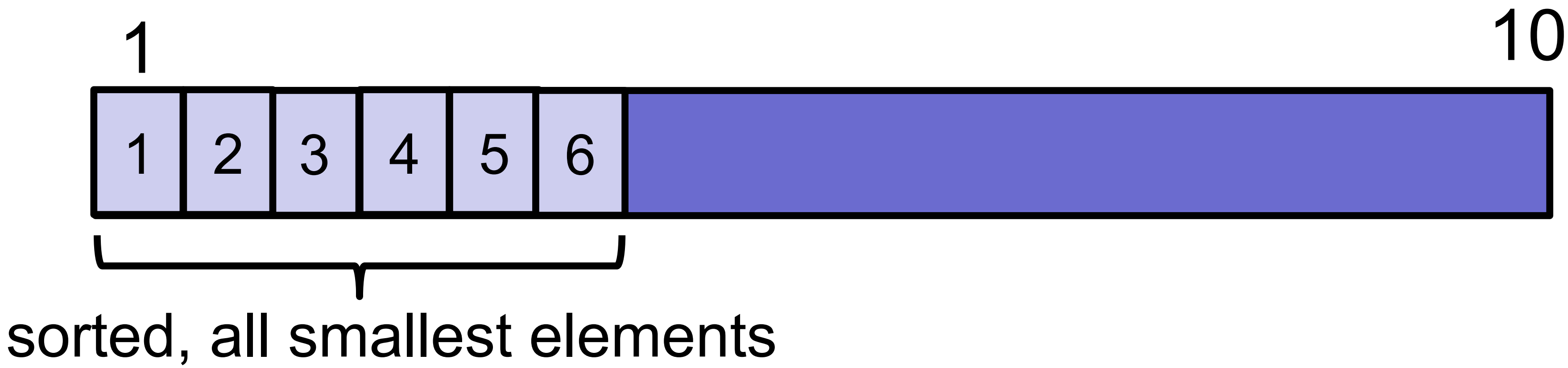
At the end of iteration j: ???



SelectionSort Analysis

Loop invariant:

At the end of iteration j: the smallest j items are correctly sorted in the first j positions of the array.



Today: Sorting

Sorting algorithms

- BubbleSort
- SelectionSort
- InsertionSort
- MergeSort

Properties

- Running time
- Space usage
- Stability

Insertion Sort

InsertionSort(A, n)

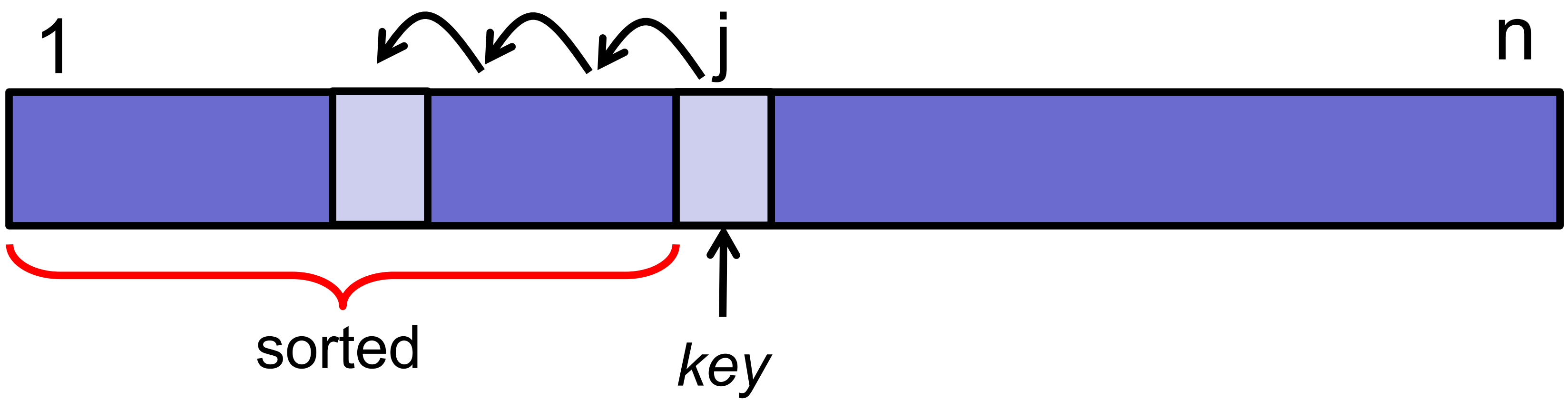
for j ← 2 **to** n

key ← A[j]

Insert key into the sorted array A[1..j-1]

Invariant: A[1..j-1] is sorted

Illustration:



Insertion Sort

InsertionSort(A, n)

for j ← 2 **to** n

Invariant: A[1..j-1] is sorted

```
InsertionSort(A, n)
    for j ← 2 to n
        key ← A[j]
        i ← j-1
        while (i > 0) and (A[i] > key)
            A[i+1] ← A[i]
            i ← i-1
        A[i+1] ← key
```

Insertion Sort

Example: **8 2 4 9 3 6**

Insertion Sort

Example: 8 2 4 9 3 6

2 8 4 9 3 6

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	**4**	**8**	**9**	**3**	**6**

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	**4**	**8**	**9**	**3**	**6**

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	**3**	**4**	**8**	**9**	**6**

Insertion Sort

Example:

8	2	4	9	3	6
2	8	4	9	3	6
2	4	8	9	3	6
2	4	8	9	3	6
2	3	4	8	9	6
2	**3**	**4**	**6**	**8**	**9**

What is the (worst-case) running time of InsertionSort?

A. $O(n)$
B. $O(n \log n)$
C. $O(n\sqrt{n})$
D. $O(n^2)$
E. $O(2^n)$
F. I have no idea.

Insertion Sort

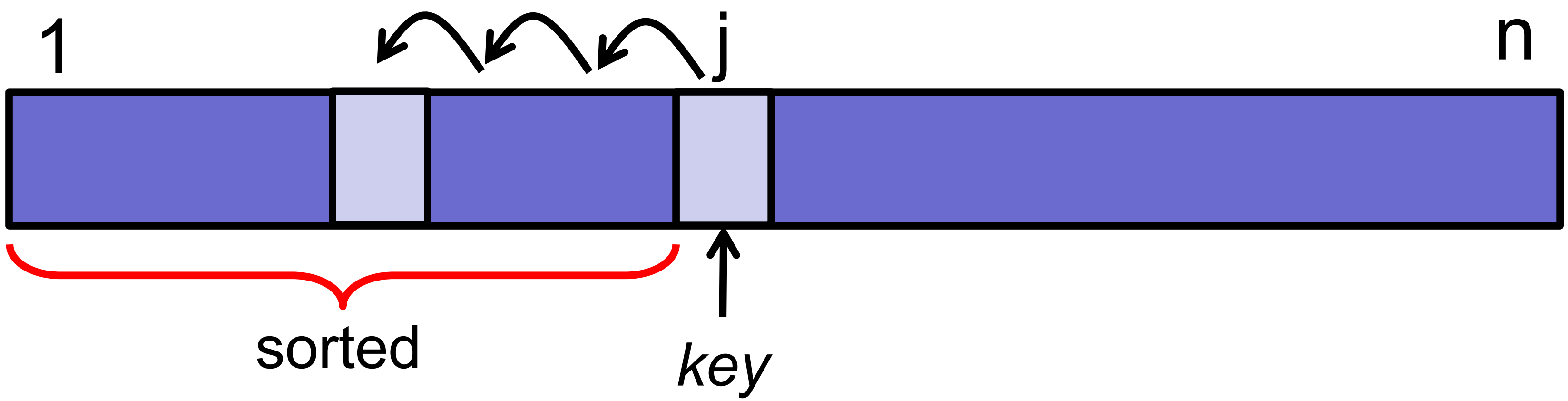
What is the max distance that the key needs to be moved?

Insertion-Sort(A, n)

for j ← 2 **to** n

key ← A[j]

Insert key into the sorted array A[1..j-1]



Insertion Sort Analysis

Insertion-Sort(A, n)

for j ← 2 **to** n

key ← A[j]

i ← j–1

while (i > 0) **and** (A[i] > *key*)

A[i+1] ← A[i]

i ← i–1

A[i+1] ← *key*

Repeat at most j times.

Basic facts

$$1 + 2 + 3 + \ldots + (n-2) + (n-1) + n \quad = (n)(n+1)/2$$

$$= \Theta(n^2)$$

Insertion Sort

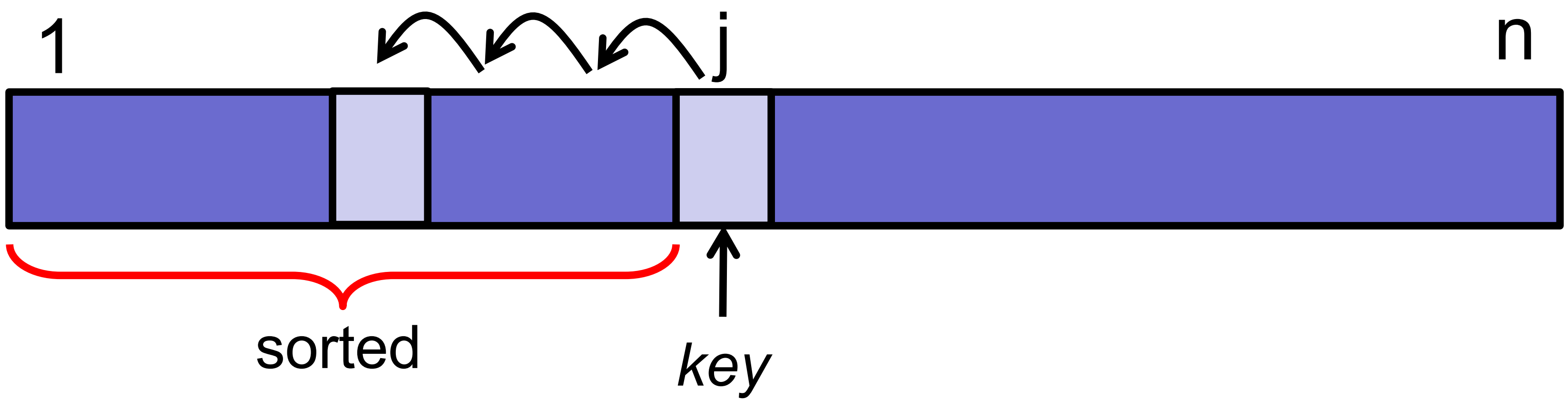
Insertion-Sort(A, n)

for $j \leftarrow 2$ **to** n

key $\leftarrow A[j]$

Insert key into the sorted array $A[1..j-1]$

Running time: $O(n^2)$



Insertion Sort

Best-case:

Average-case:

- Random permutation

Worst-case:

Insertion Sort

Best-case:

- Already sorted: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Average-case:

- Random permutation?

Worst-case:

- Inverse sorted: [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]

Insertion Sort

Very fast!

Best-case: $O(n)$

- Already sorted: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

Average-case:

- Random permutation?

Worst-case: $O(n^2)$

- Inverse sorted: [10, 9, 8, 7, 6, 5, 4, 3, 2, 1]

Insertion Sort Analysis

On average, a key in position j needs to move j/2 slots backward (in expectation).

Average-case analysis:

- Assume all inputs equally likely

$$\sum_{j=2}^{n} \Theta\left(j/2\right) = \Theta\left(n^2\right)$$

- In expectation, still **θ(n²)**

Today: Sorting

Sorting algorithms

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Properties

- Running time
- Space usage
- Stability

Properties of Sorting Algorithms

Time complexity

- Worst case: $O(n^2)$

- Sorted list:

- Almost sorted list?

Properties of Sorting Algorithms

Time complexity

- Worst case: $O(n^2)$
- Sorted list: BubbleSort

  SelectionSort

  InsertionSort

  $O(n)$

  $O(n^2)$

- Almost sorted list?

How expensive is it to sort:

[1, 2, 3, 4, 5**, 7, 6**, 8, 9, 10]

How expensive is it to sort:

[1, 2, 3, 4, 5, **7**, **6**, 8, 9, 10]

BubbleSort and InsertionSort are fast.

SelectionSort is slow.

Challenge of the Day:

Find a permutation of [1..n] where:

- BubbleSort is slow.
- InsertionSort is fast.

Or explain why no such sequence exists.

Properties of Sorting Algorithms

Moral:

Different sorting algorithms have different inputs that they are good or bad on.

All $O(n^2)$ algorithms are not the same.

Properties of Sorting Algorithms

Space complexity

How much space does a sorting algorithm need?

- Worst case: O(n)

Properties of Sorting Algorithms

Space complexity

- Worst case: O(n)

- In-place sorting algorithm:
  - Only O(1) extra space needed.
  - All manipulation happens within the array.

So far:

All sorting algorithms we have seen are in-place.

Properties of Sorting Algorithms

Stability

What happens with repeated elements?

Key	1	2	5	3	4	5	6	7	8	9
Value	a	b	C	g	h	D	j	k	l	m

Databases often contain (key, value) pairs.

The key is an index to help organize the data.

Properties of Sorting Algorithms

Stability

What happens with repeated elements?

Key	1	2	**5**	3	4	**5**	6	7	8	9
Value	a	b	**C**	g	h	**D**	j	k	l	m

Two values have the same key!

Properties of Sorting Algorithms

Stability

What happens with repeated elements?

Key	1	2	**5**	3	4	**5**	6	7	8	9
Value	a	b	**C**	g	h	**D**	j	k	l	m

UNSTABLE

Key	1	2	3	4	**5**	**5**	6	7	8	9
Value	a	b	g	h	**D**	**C**	j	k	l	m

Properties of Sorting Algorithms

Stability: preserves order of equal elements

What happens with repeated elements?

Key	1	2	**5**	3	4	**5**	6	7	8	9
Data	a	b	**C**	g	h	**D**	j	k	l	m

⇩ STABLE

Key	1	2	3	4	**5**	**5**	6	7	8	9
Data	a	b	g	h	**C**	**D**	j	k	l	m

Which are stable? Which are not stable?

A. BubbleSort

B. SelectionSort

C. InsertionSort

InsertionSort

Insertion-Sort(A, n)

for $j \leftarrow 2$ **to** n

$key \leftarrow A[j]$

$i \leftarrow j-1$

while $(i > 0)$ **and** $(A[i] > key)$

$A[i+1] \leftarrow A[i]$

$i \leftarrow i-1$

$A[i+1] \leftarrow key$

Stable as long as we are careful to implement it properly!

SelectionSort

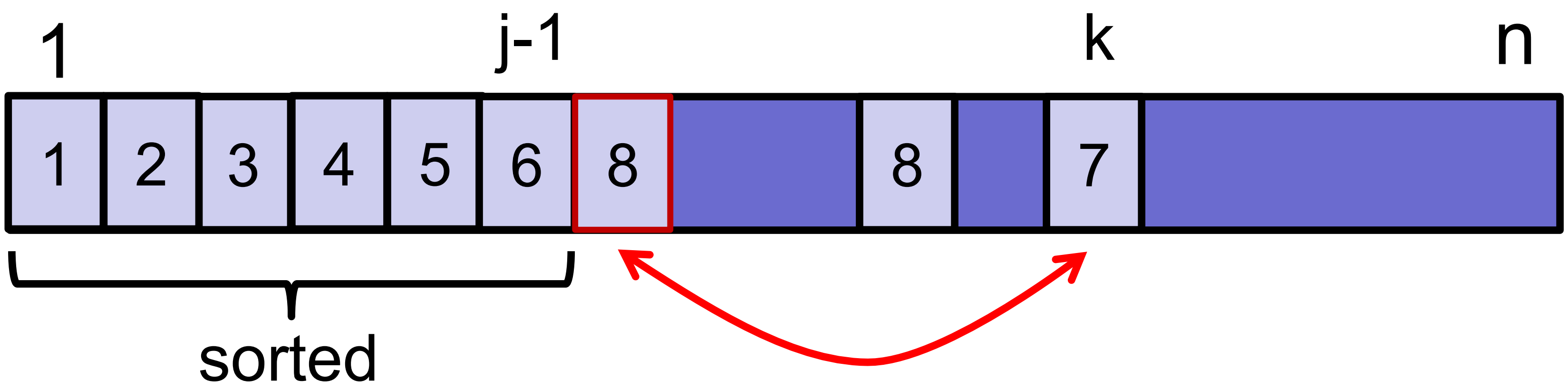
SelectionSort(A, n)

for j ← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Not stable: swap changes order



SelectionSort

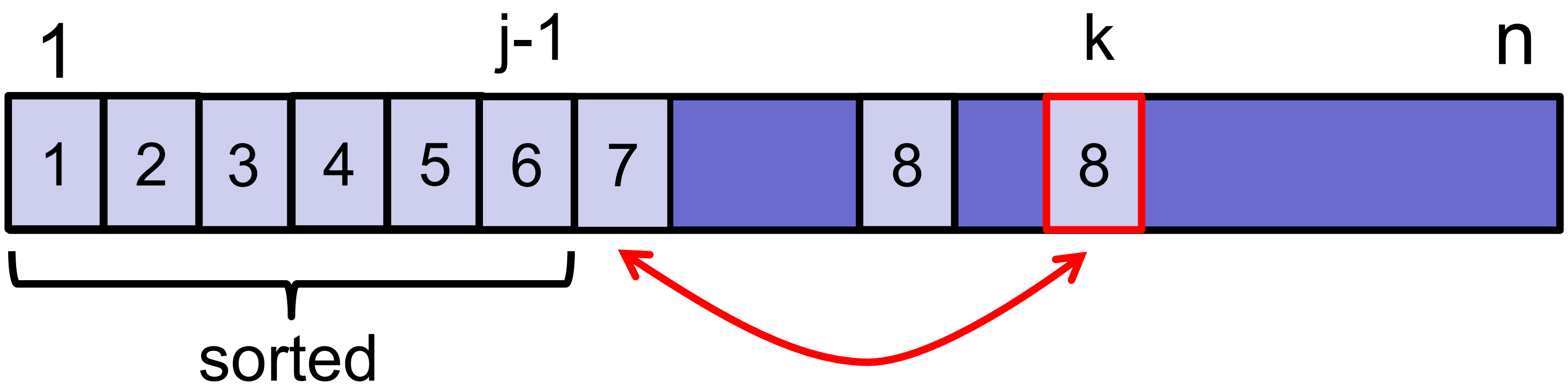
SelectionSort(A, n)

for j ← 1 **to** n-1**:**

find minimum element A[j] in A[j..n]

swap(A[j], A[k])

Not stable: swap changes order



Sorting Analysis

Summary:

- BubbleSort: $O(n^2)$
- SelectionSort: $O(n^2)$
- InsertionSort: $O(n^2)$

Properties: time, space, stability

For next time…

Monday lecture:

- More sorting

Problem Set 3:

- Released today.
- Some depends on Monday’s lecture.

Sorting and Java:

- See slides that follow for some Java issues.

Today: Sorting

Sorting algorithms
- BubbleSort
- SelectionSort
- InsertionSort
- MergeSort

Properties
- Running time
- Space usage
- Stability

MergeSort

Divide-and-Conquer

1. Divide problem into smaller sub-problems.
2. Recursively solve sub-problems.
3. Combine solutions.

MergeSort

Divide-and-Conquer Sorting

1. Divide: split array into two halves.
2. Recurse: sort the two halves.
3. Combine: merge the two sorted halves.

MergeSort

Divide-and-Conquer Sorting

1. Divide: split array into two halves.
2. Recurse: sort the two halves.
3. Combine: merge the two sorted halves.

Advice:

When thinking about recursion, do not “unroll” the recursion.
Treat the recursive call as a magic black box.
(But don’t forget the base case.)

MergeSort

Step 1:
Divide array into two pieces.

MergeSort(A, n)

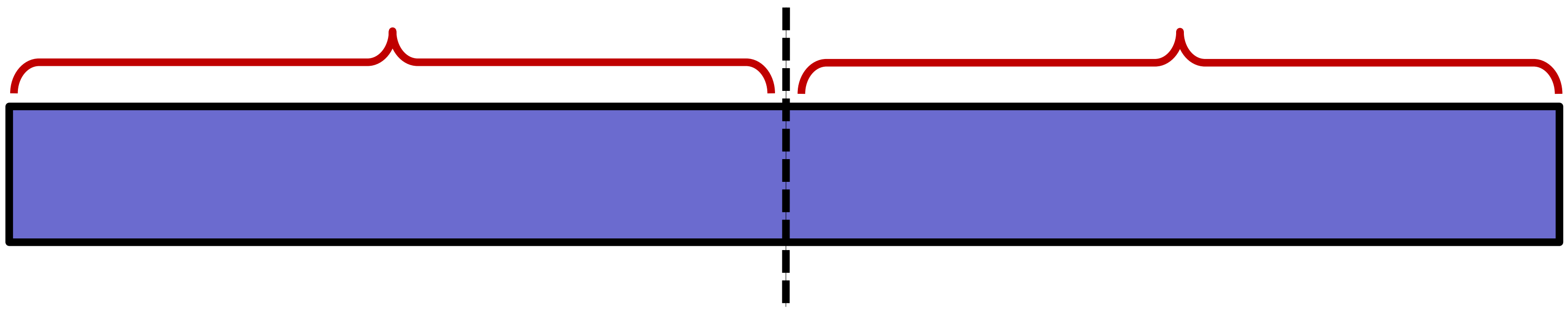
if (n=1) **then return;**

else:

X ←MergeSort**(**A[1..n/2], n/2**);**

Y ←MergeSort**(**A[n/2+1, n], n/2**);**

return Merge **(**X,Y, n/2**);**



MergeSort

Step 2:
Recursively sort the two halves.

MergeSort(A, n)

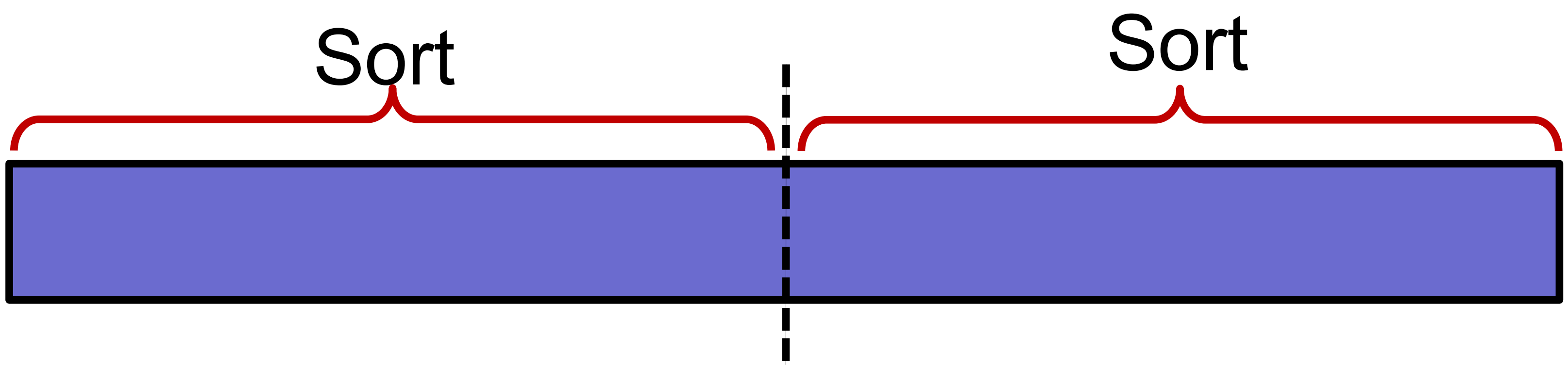
if (n=1) **then return;**

else:

X ←MergeSort**(**A[1..n/2], n/2**);**

Y ←MergeSort**(**A[n/2+1, n], n/2**);**

return Merge **(**X,Y, n/2**);**



MergeSort

Step 3:
Merge the two halves into one sorted array.

MergeSort(A, n)

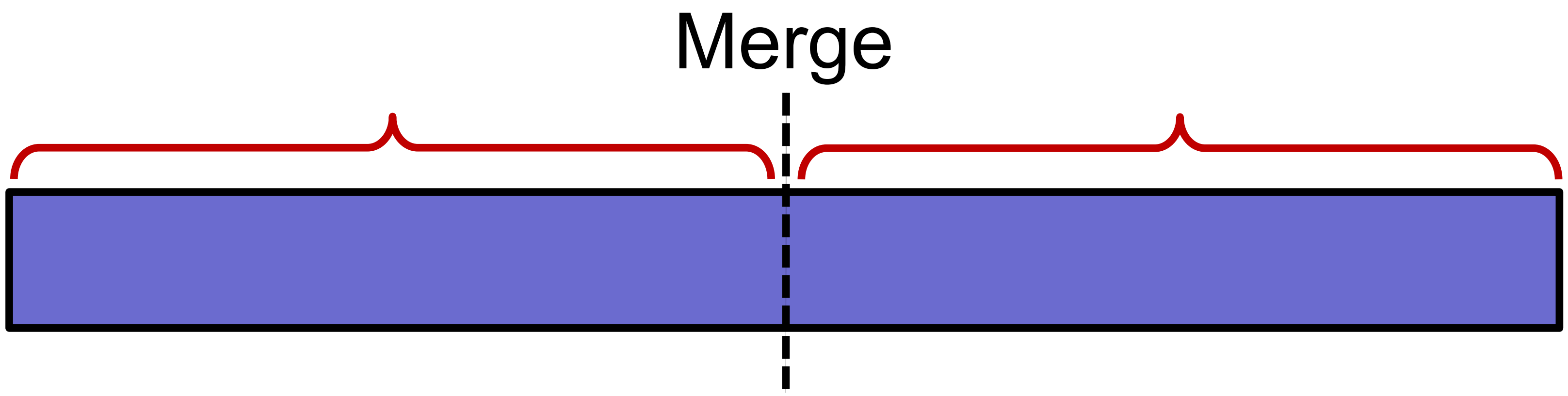
if (n=1) **then return;**

else:

X ←MergeSort**(**A[1..n/2], n/2**);**

Y ←MergeSort**(**A[n/2+1, n], n/2**);**

return Merge **(**X,Y, n/2**);**



MergeSort

```
MergeSort(A, n)
    if (n=1) then return;
    else:
        X ←MergeSort(A[1..n/2], n/2);
        Y ←MergeSort(A[n/2+1, n], n/2);
    return Merge (X,Y, n/2);
```

Base case

Recursive "conquer" step

Combine solutions

The only "interesting" part is merging!

MergeSort

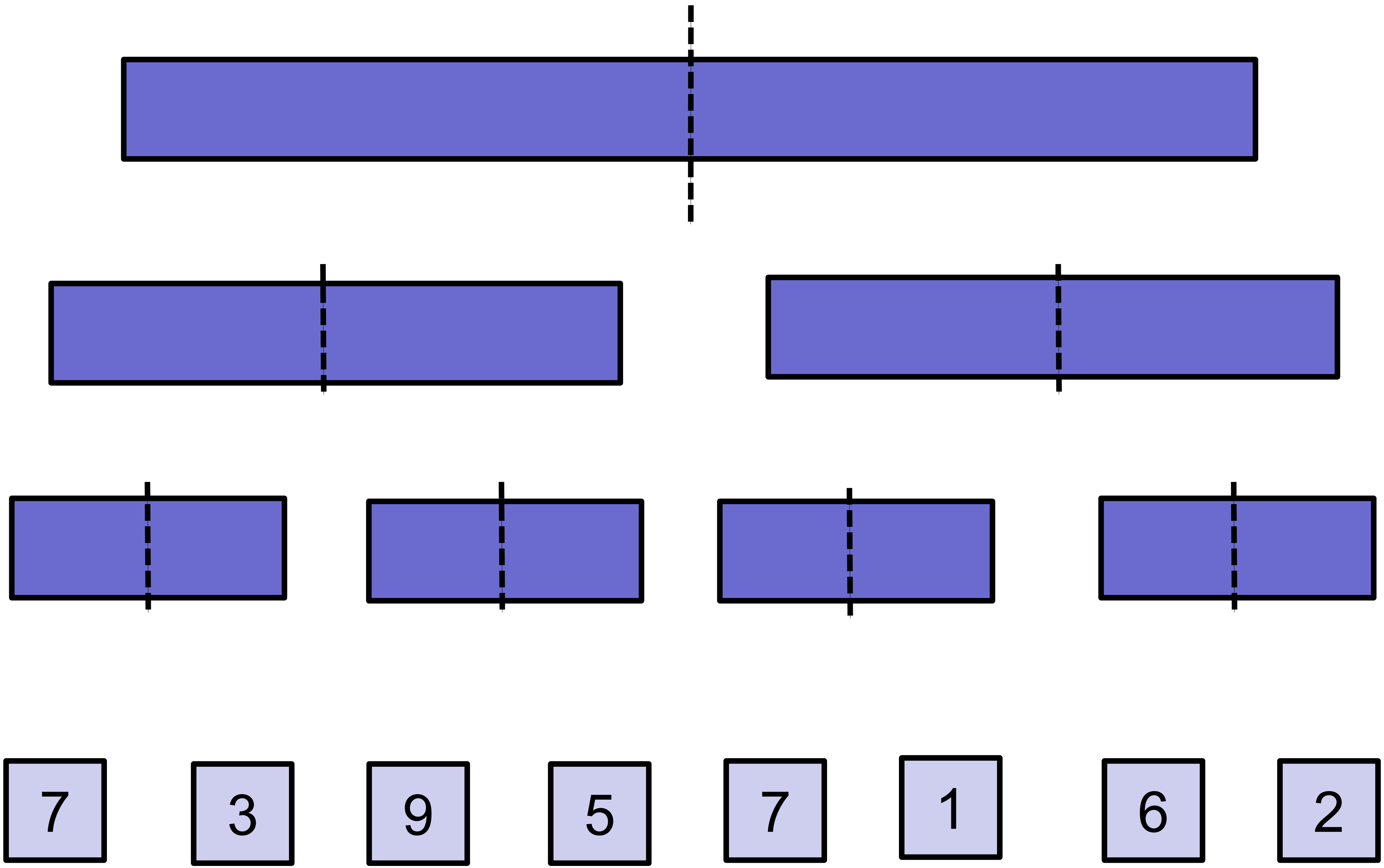
Divide-and-Conquer Sorting

1. Divide: split array into two halves.
2. Recurse: sort the two halves.
3. Combine: merge the two sorted halves.

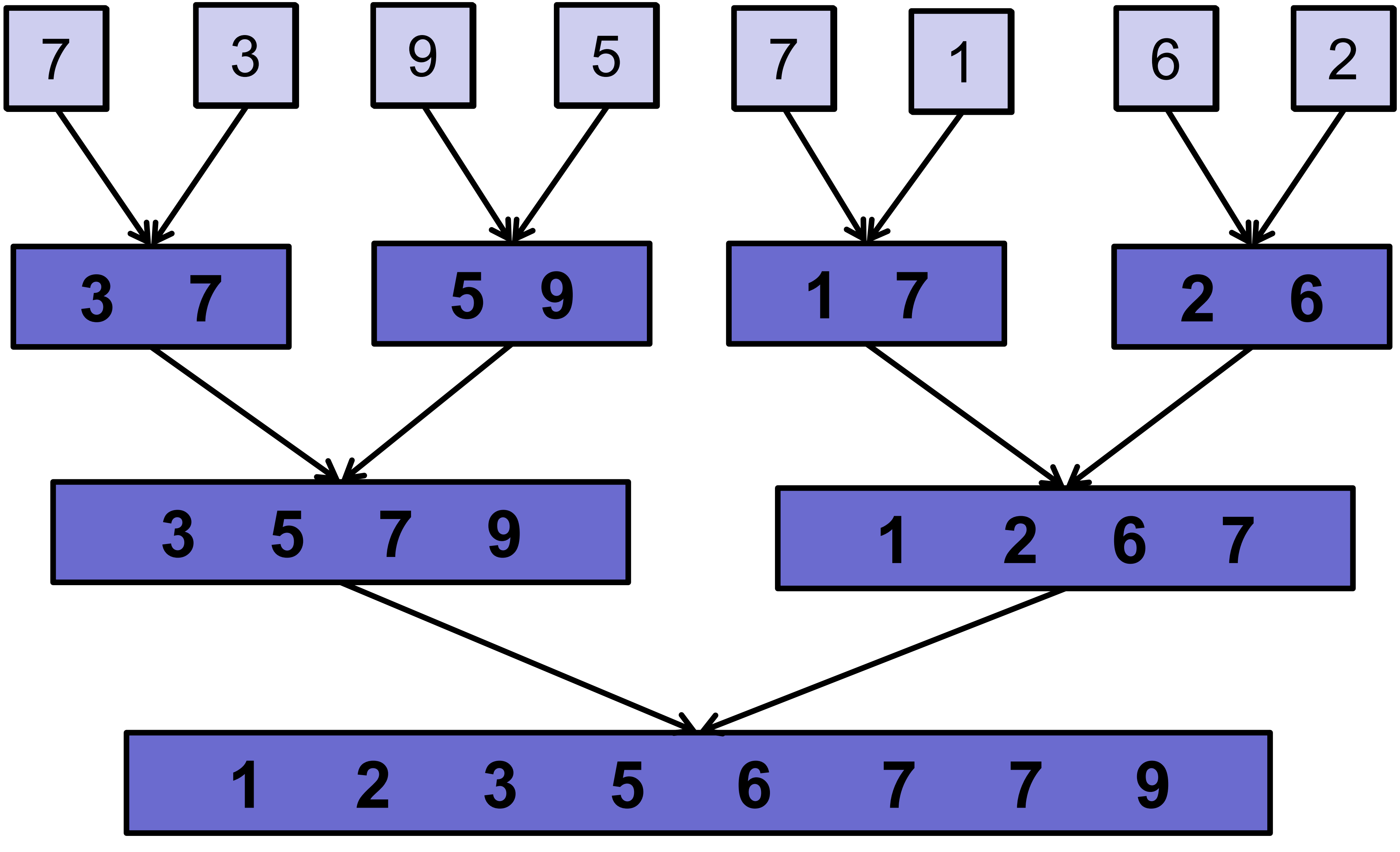
Advice:

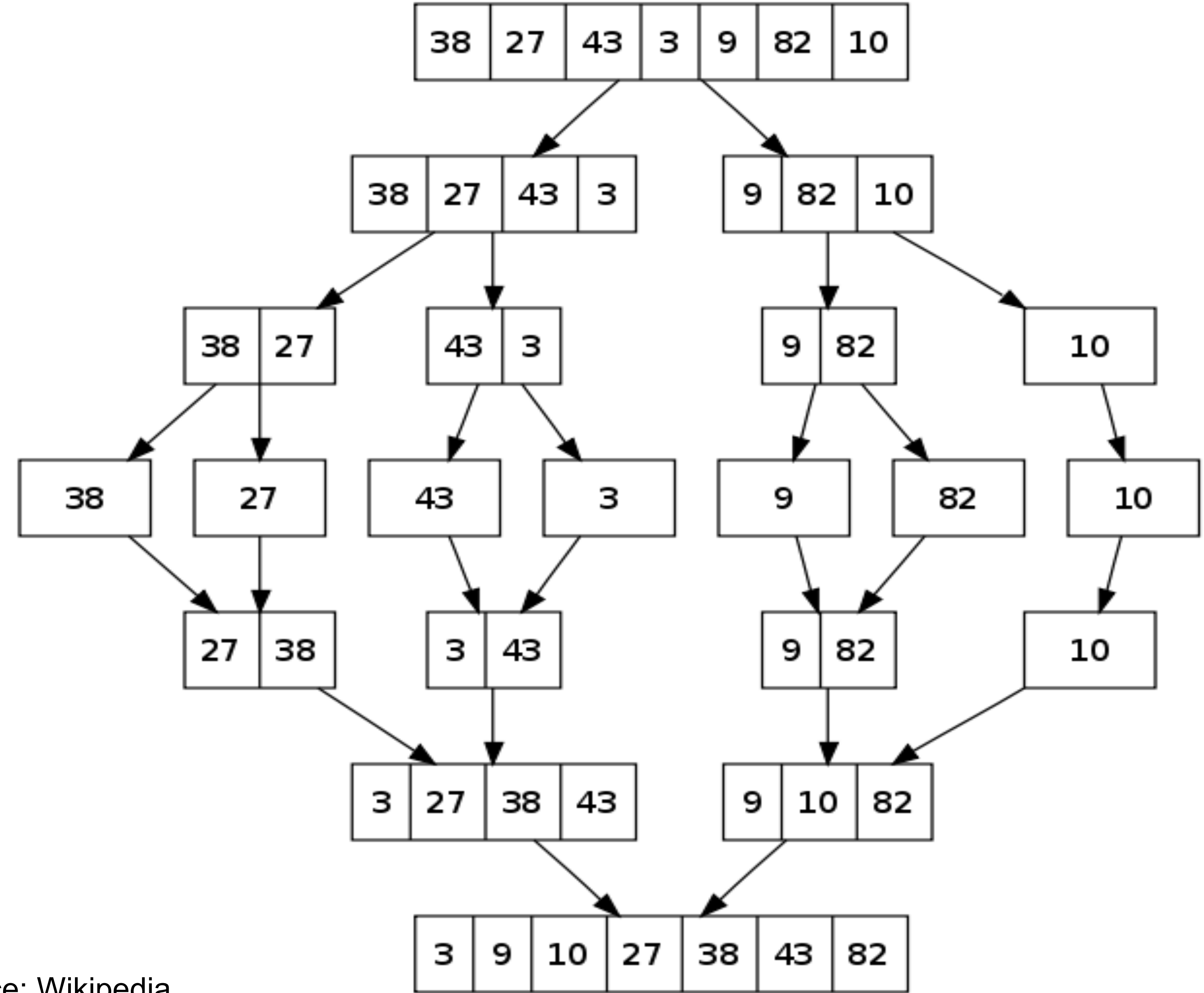
When thinking about recursion, do not “unroll” the recursion.
Treat the recursive call as a magic black box.
(But don’t forget the base case.)

Divide-and-Conquer



Merging





Source: Wikipedia

Merging Two Sorted Lists

Key subroutine: Merge

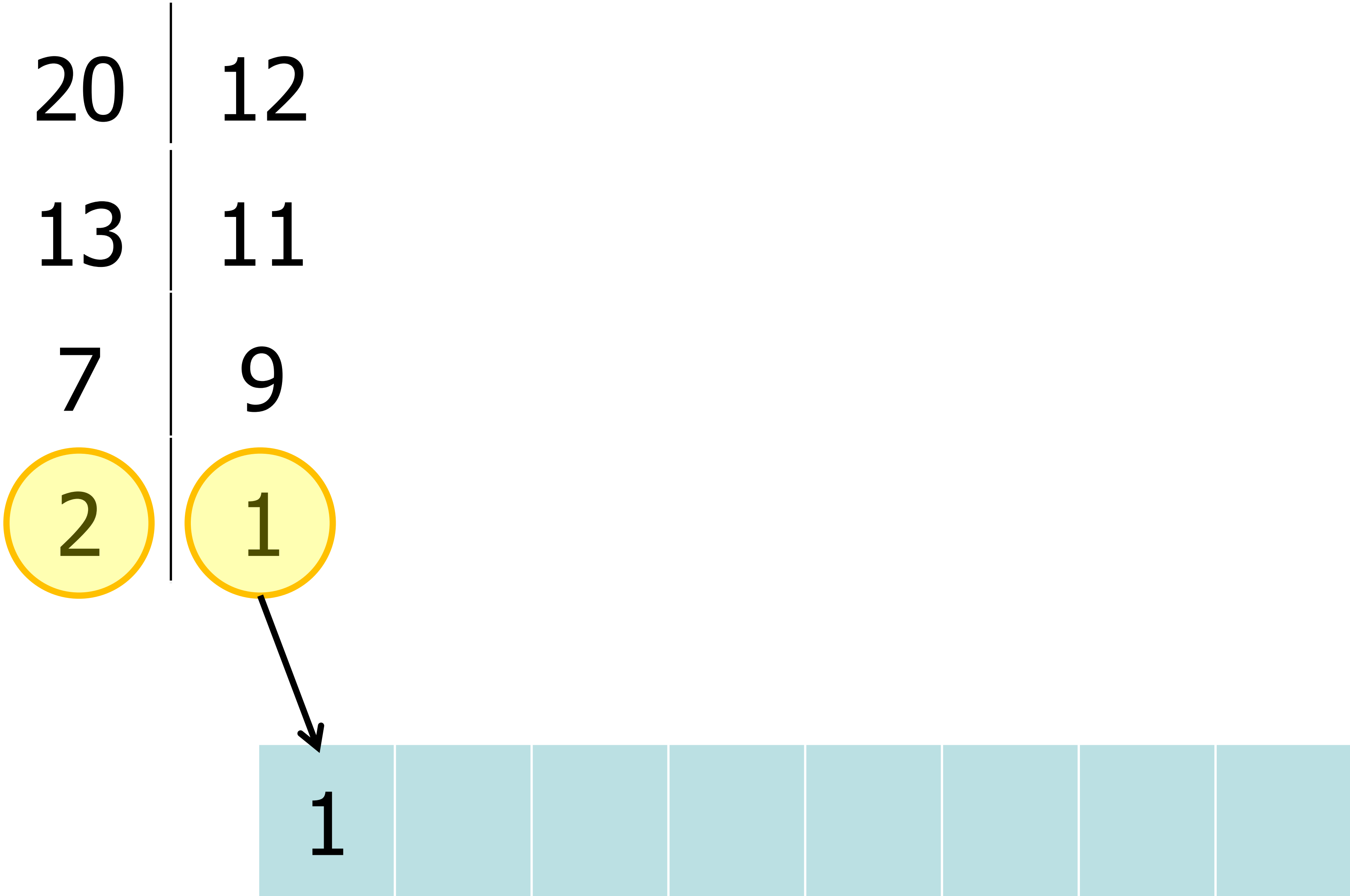
- How to merge?
- How fast can we merge?

Merging Two Sorted Lists

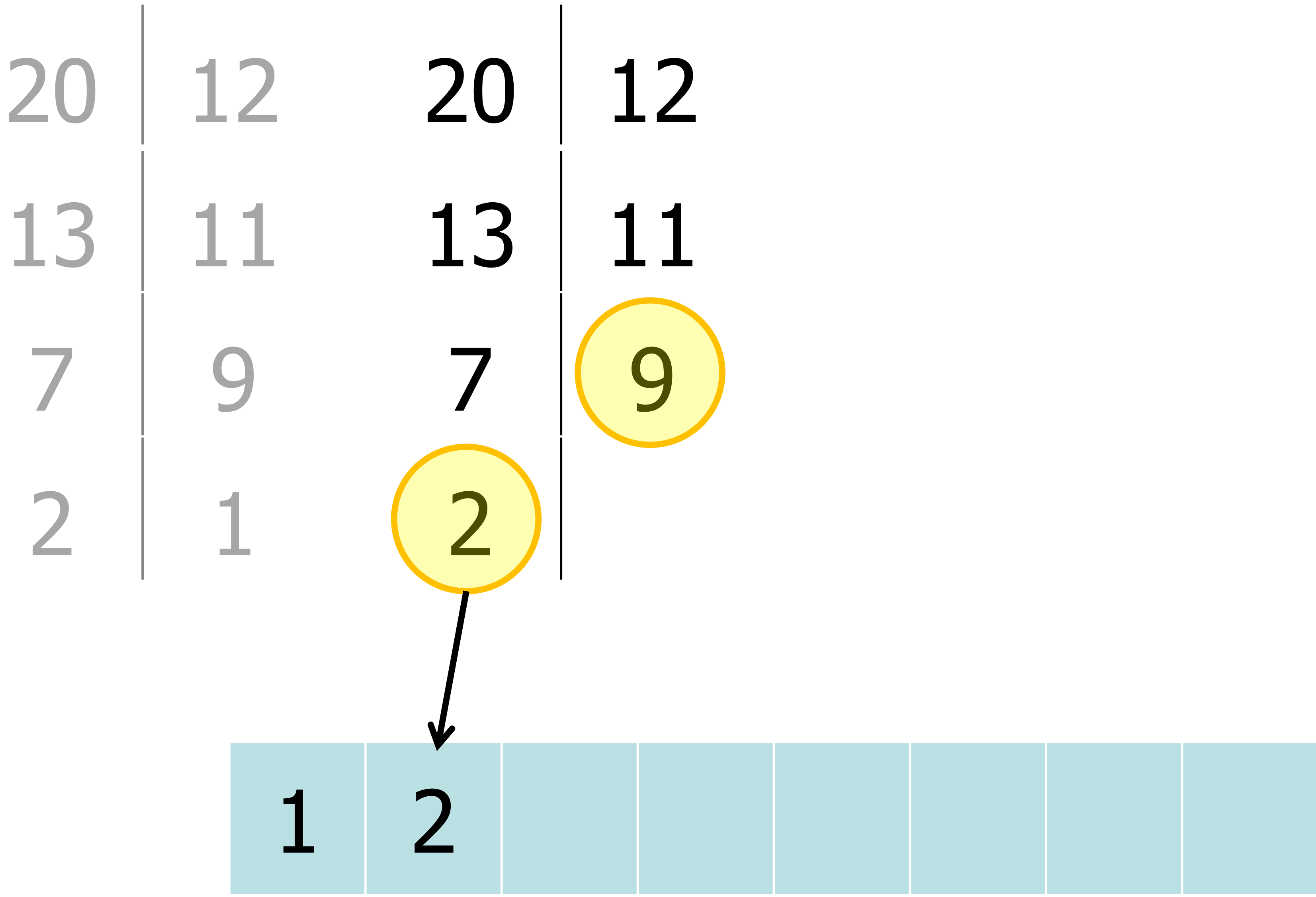
20	12
13	11
7	9
2	1

sorted
from
smallest
to
biggest

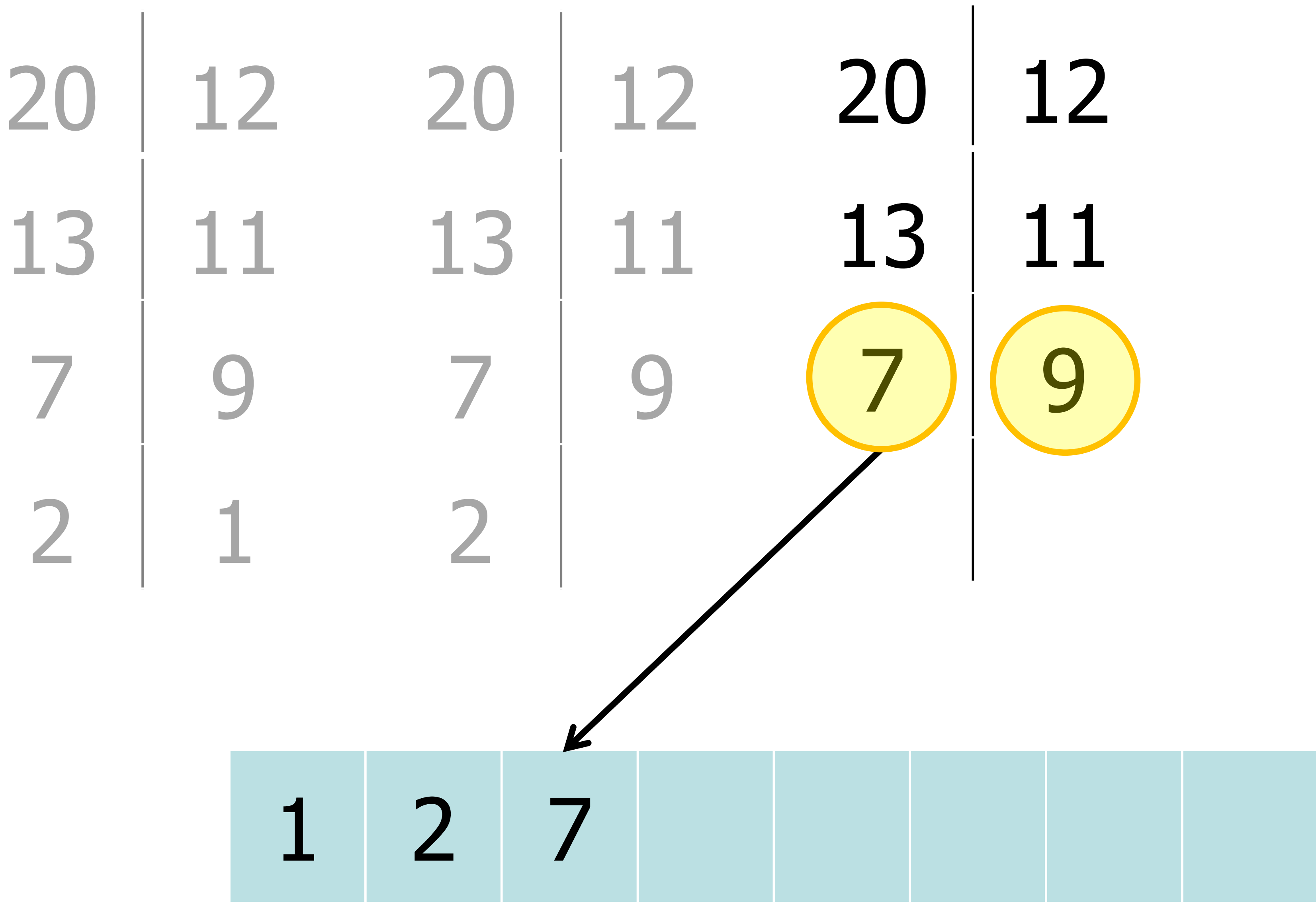
Merging Two Sorted Lists



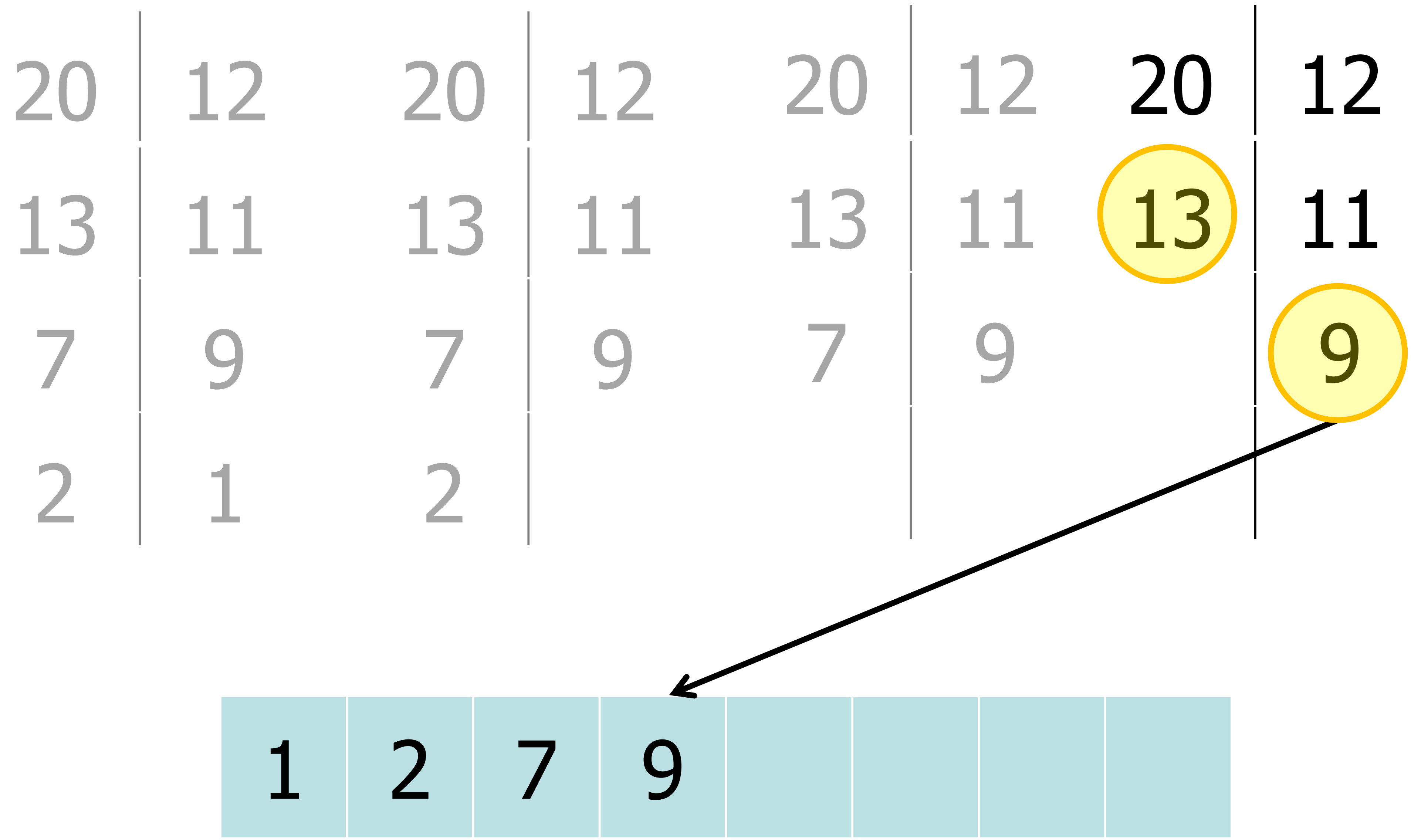
Merging Two Sorted Lists



Merging Two Sorted Lists



Merging Two Sorted Lists



Merging Two Sorted Lists

20	12	20	12	20	12	20	12
13	11	13	11	13	11	13	11
7	9	7	9	7	9		
2	1	2					

1	2	7	9	11	12	13	20

Merge: Running Time

Given two lists:

- A of size n/2
- B of size n/2

Total running time: ??

Merge: Running Time

Given two lists:

- A of size n/2
- B of size n/2

Total running time: $O(n) = cn$

- In each iteration, move *one* element to final list.
- After n iterations, all the items are in the final list.
- Each iteration takes $O(1)$ time to compare two elements and copy one.

Merge-Sort Analysis

Let $T(n)$ be the worst-case running time for an array of n elements.

MergeSort(A, n)

if $(n=1)$ **then return;** ← - - - - - - - - - - - - - - - - $\theta(1)$

else:

$X \leftarrow$ Merge-Sort**(**...**);** ← - - - - - - - - - - - - $T(n/2)$

$Y \leftarrow$ Merge-Sort**(**...**);** ← - - - - - - - - - - - - $T(n/2)$

return Merge **(**X,Y, n/2**);** ← - - - - - - - - - - - - - $\theta(n)$

MergeSort Analysis

Let $T(n)$ be the worst-case running time for an array of n elements.

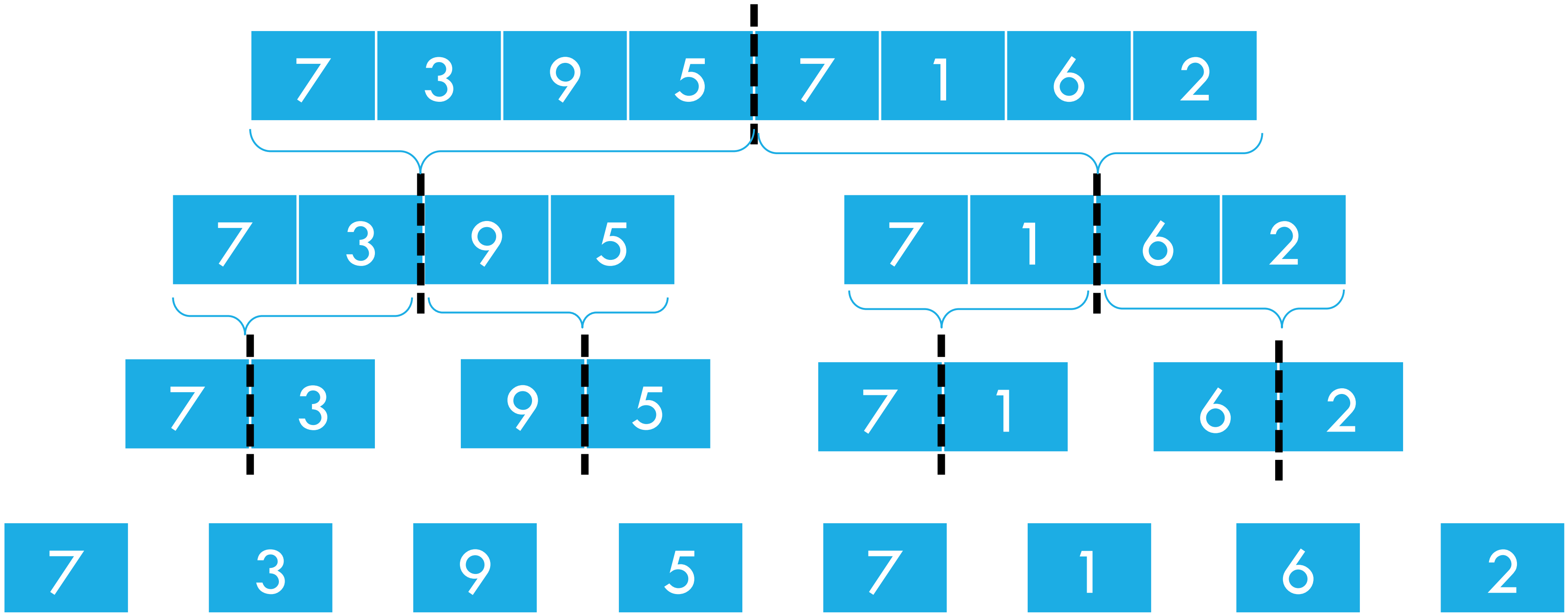
$$T(n) = \theta(1) \quad \textbf{if } (n=1)$$

$$= 2T(n/2) + cn \quad \textbf{if } (n>1)$$

Techniques for Solving Recurrences

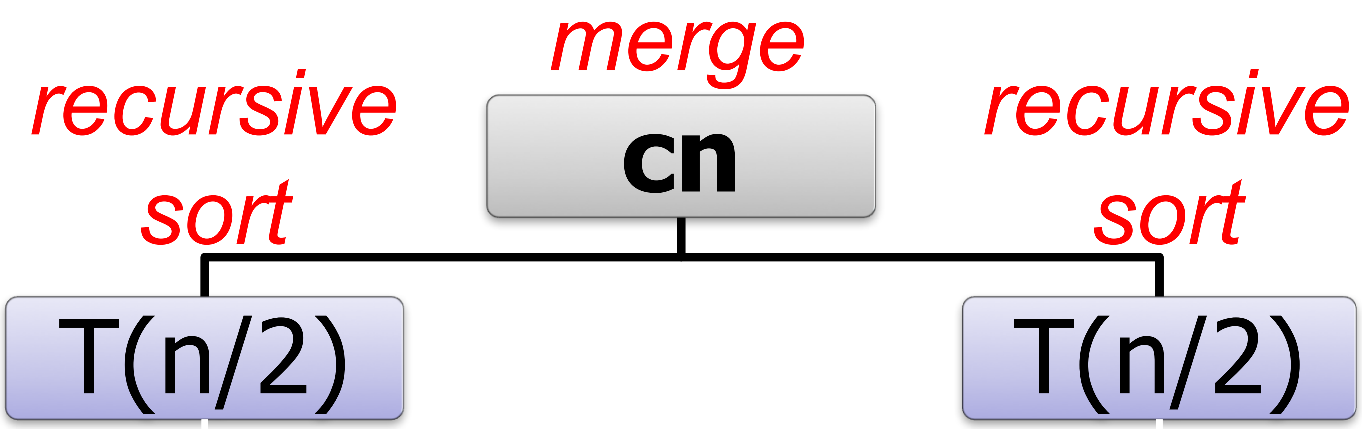
1. Guess and verify (via induction).
2. Draw the recursion tree.
3. Use the Master Theorem (see CS3230) or the Akra–Bazzi Method, or other advanced techniqus.

MergeSort: Recurse “downwards”



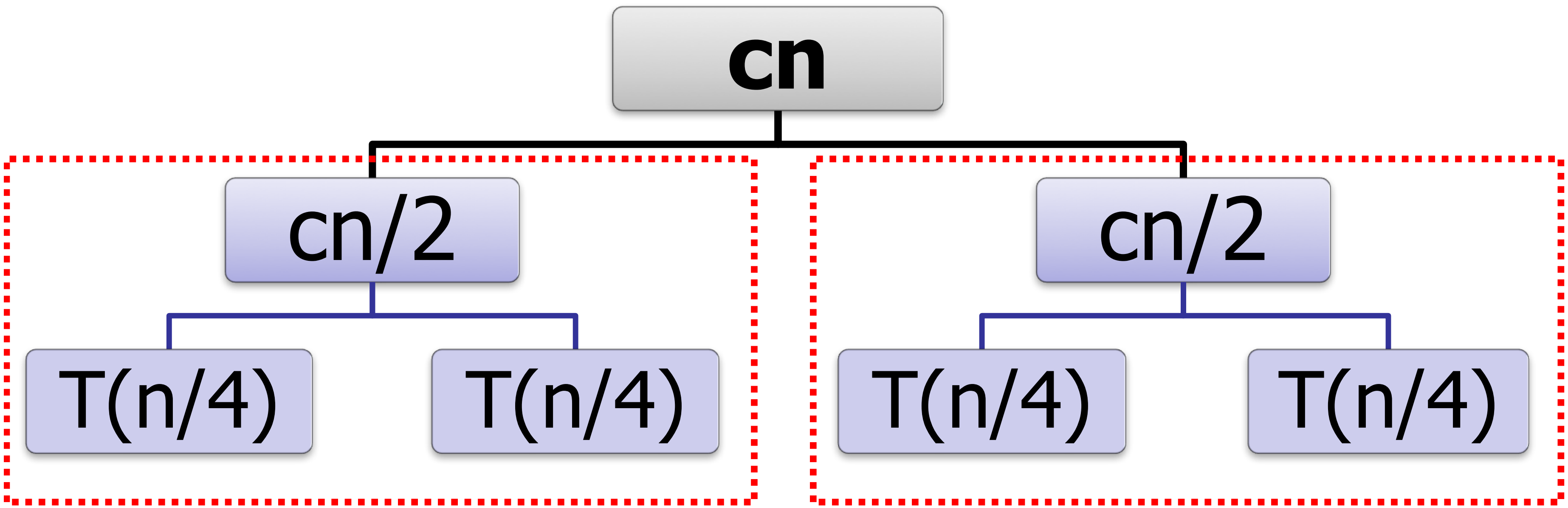
MergeSortAnalysis

$$T(n) = 2T(n/2) + cn$$



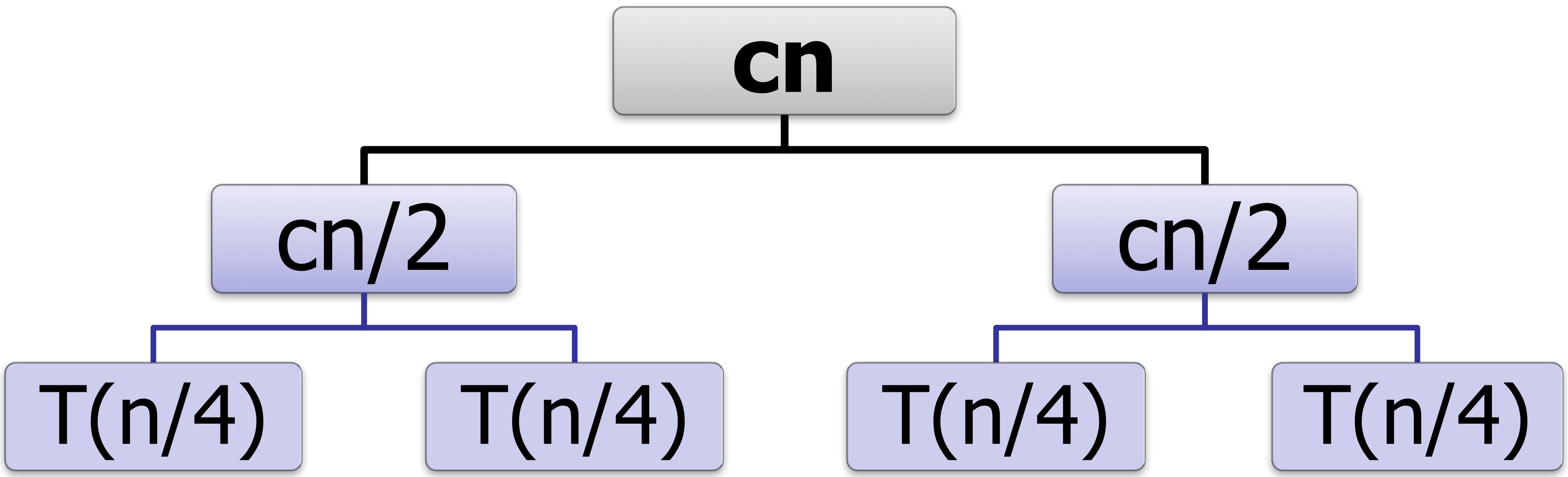
MergeSortAnalysis

$$T(n) = 2T(n/2) + cn$$



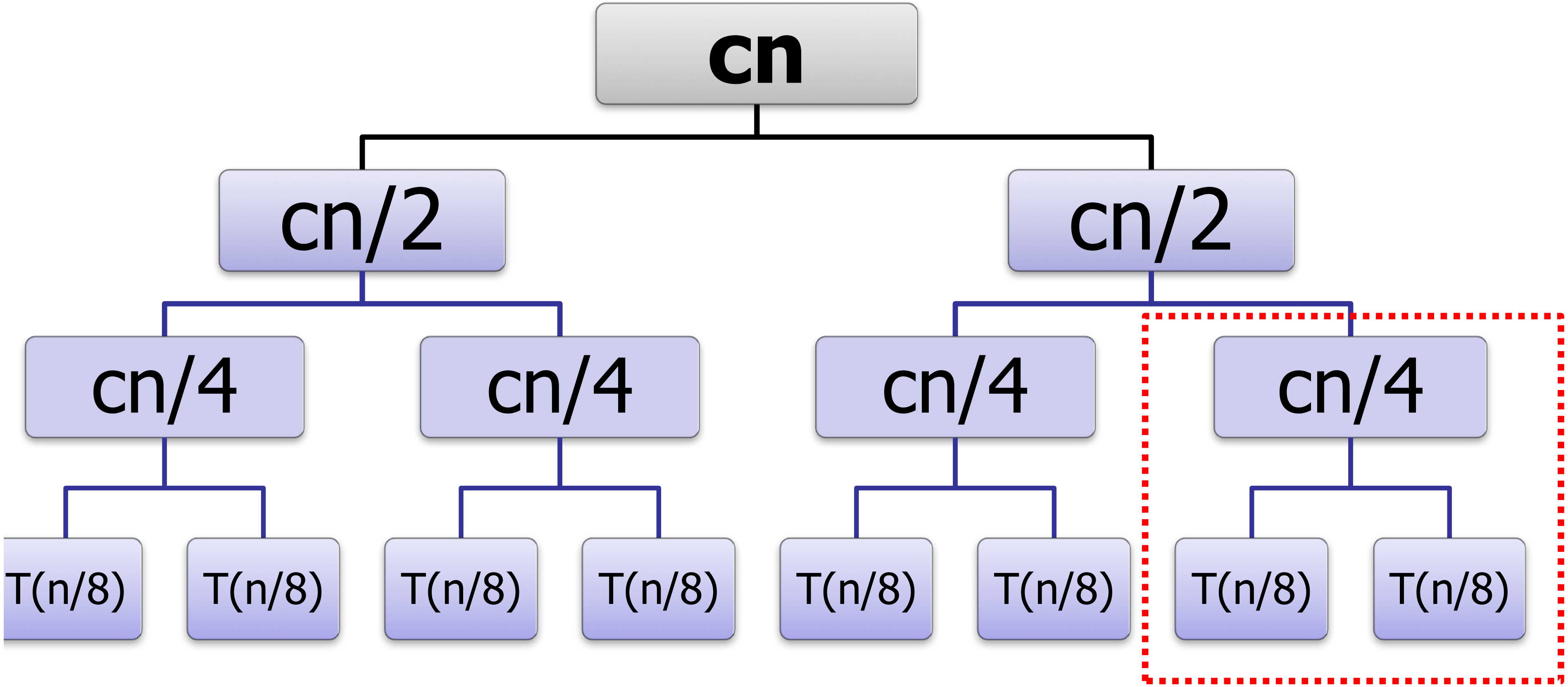
MergeSortAnalysis

$$T(n) = 2T(n/2) + cn$$



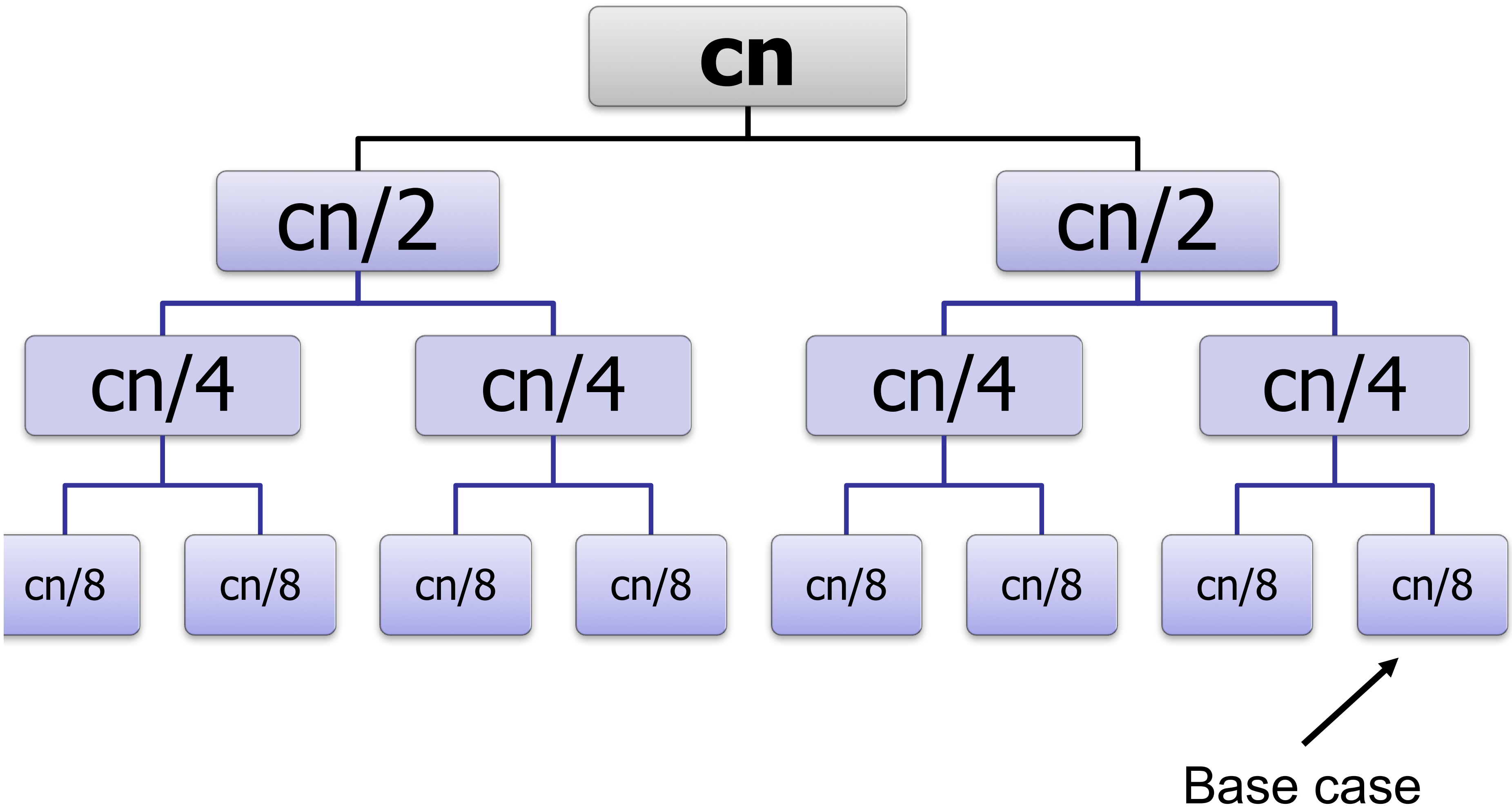
MergeSort Analysis

$$T(n) = 2T(n/2) + cn$$



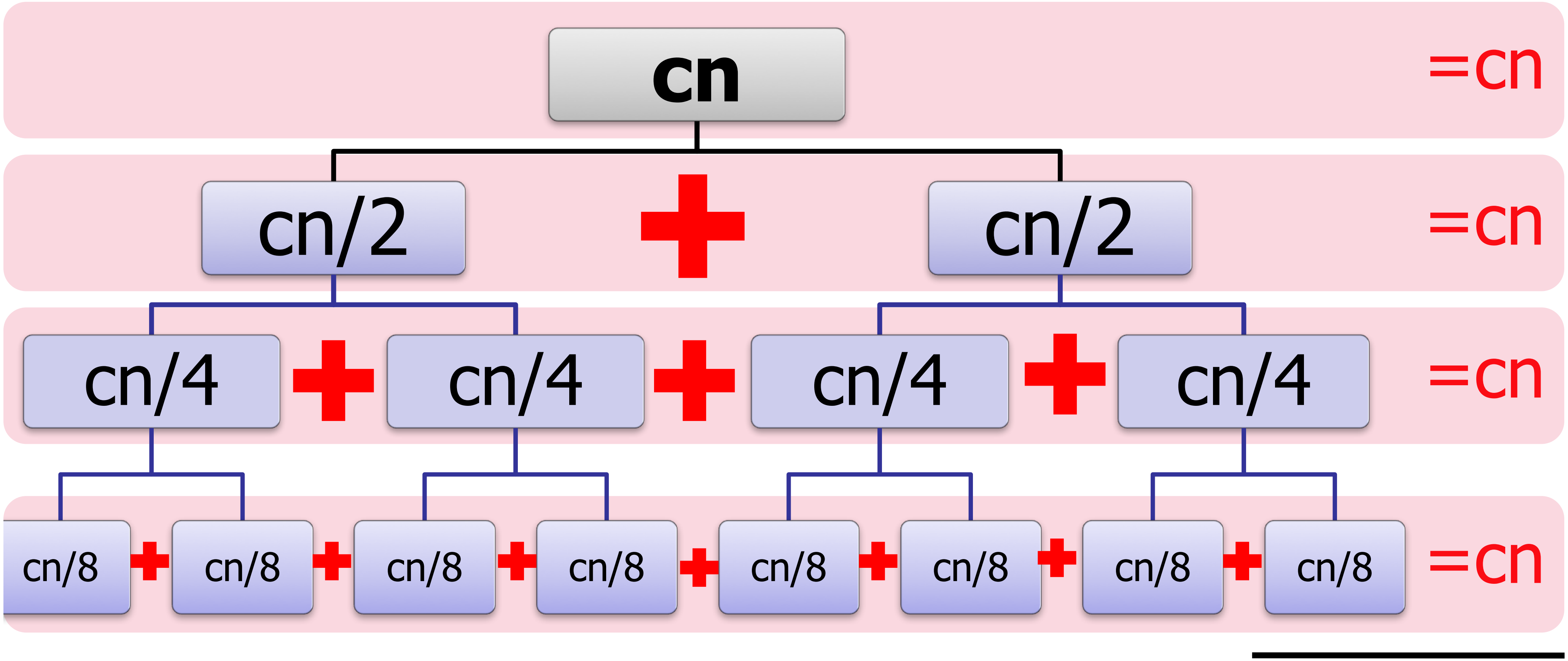
MergeSort Analysis

$T(n) = 2T(n/2) + cn$



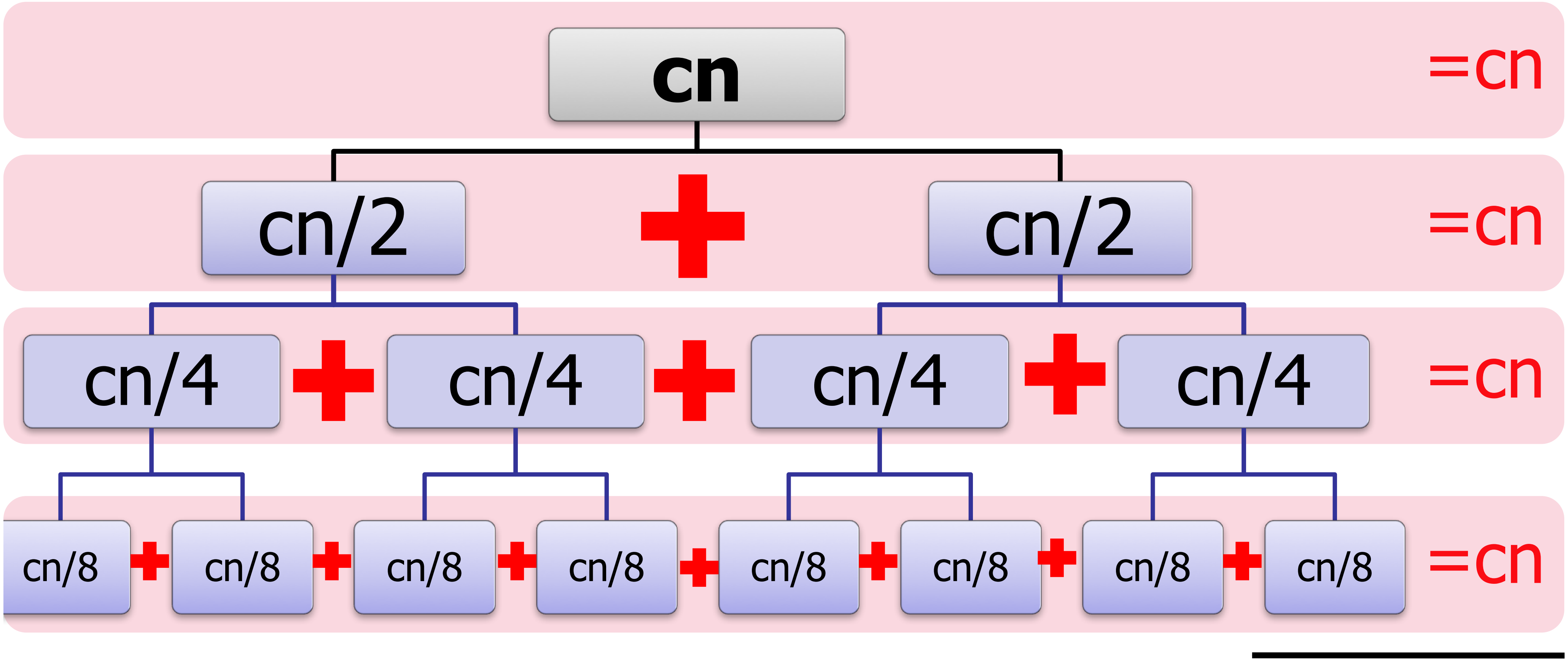
MergeSort Analysis

$$T(n) = 2T(n/2) + cn$$



MergeSort Analysis

$$T(n) = 2T(n/2) + cn$$



Key question: how many levels?

MergeSort Analysis

$$T(n) = 2T(n/2) + cn$$

level	number
0	1
1	2
2	4
3	8
4	16
…	…
h	??

$$\text{number} = 2^{\text{level}}$$

MergeSort Analysis

$$T(n) = 2T(n/2) + cn$$

Level	Number
0	1
1	2
2	4
3	8
4	16
…	…
h	*n*

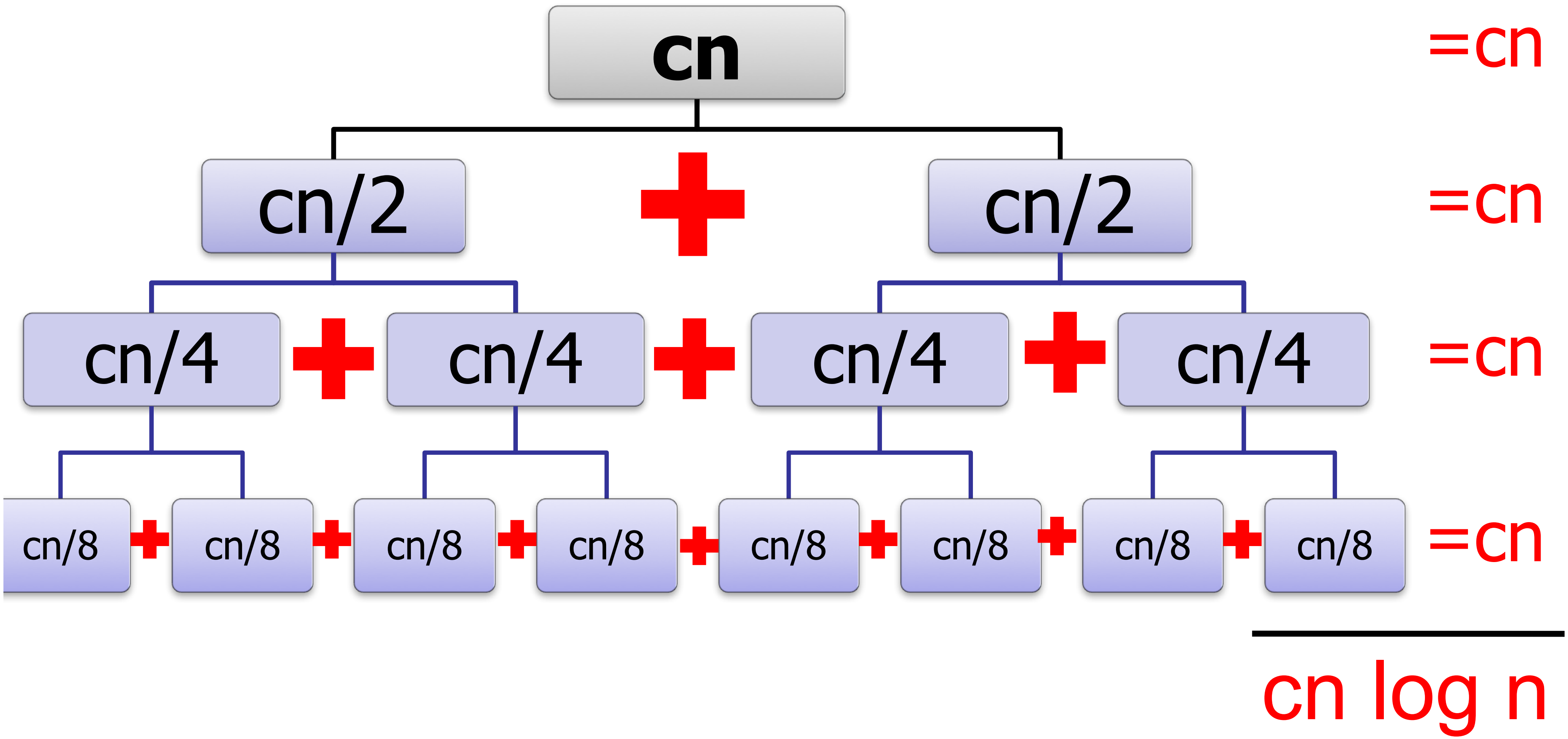
$$\text{number} = 2^{\text{level}}$$

$$n = 2^h$$

$$\log n = h$$

MergeSort Analysis

$$T(n) = 2T(n/2) + cn$$



MergeSortAnalysis

$T(n) = O(n \log n)$

```
MergeSort(A, n)
    if (n=1) then return;
    else:
        X ←MergeSort(...);
        Y ←MergeSort(...);
    return Merge (X,Y, n/2);
```

Techniques for Solving Recurrences

1. Guess and verify (via induction).

2. Draw the recursion tree.

3. Use the Master Theorem (see CS3230) or the Akra–Bazzi Method, or other advanced techniqus.

Guess: $T(n) = O(n \log n)$

<u>Recurrence being analyzed:</u>

$T(n) = 2T(n/2) + c \cdot n$

$T(1) = c$

Guess: $T(n) = c \cdot n \log n$

More precise guess:
Fix constant c.

Recurrence being analyzed:

$T(n) = 2T(n/2) + c \cdot n$

$T(1) = c$

Guess: $T(n) = c \cdot n \log n$

Induction:
Base case

$T(1) = c$

Recurrence being analyzed:

$T(n) = 2T(n/2) + c \cdot n$

$T(1) = c$

Guess: $T(n) = c \cdot n \log n$

Induction:
Assume true for all smaller values.

$T(1) = c$

$T(x) = c \cdot x \log x$ for all $x < n$.

Recurrence being analyzed:

$T(n) = 2T(n/2) + c \cdot n$

$T(1) = c$

Guess: T(n) = c·n log n

Induction:
Prove for n.

T(1) = c

T(x) = c·x log x for all x < n.

$$\begin{aligned} T(n) &= 2T(n/2) + cn \\ &= 2(c(n/2)\log(n/2)) + cn \\ &= cn\log(n/2) + cn \\ &= cn\log(n) - cn\log(2) + cn \\ &= cn\log(n) \end{aligned}$$

Recurrence being analyzed:

T(n) = 2T(n/2) + c·n

T(1) = c

Guess: T(n) = c·n log n

T(1) = c

T(x) = c·x log x for all x < n.

$$\begin{aligned} T(n) &= 2T(n/2) + cn \\ &= 2(c(n/2)\log(n/2)) + cn \\ &= cn\log(n/2) + cn \\ &= cn\log(n) - cn\log(2) + cn \\ &= cn\log(n) \end{aligned}$$

Induction:
It works!

Recurrence being analyzed:

T(n) = 2T(n/2) + c·n

T(1) = c

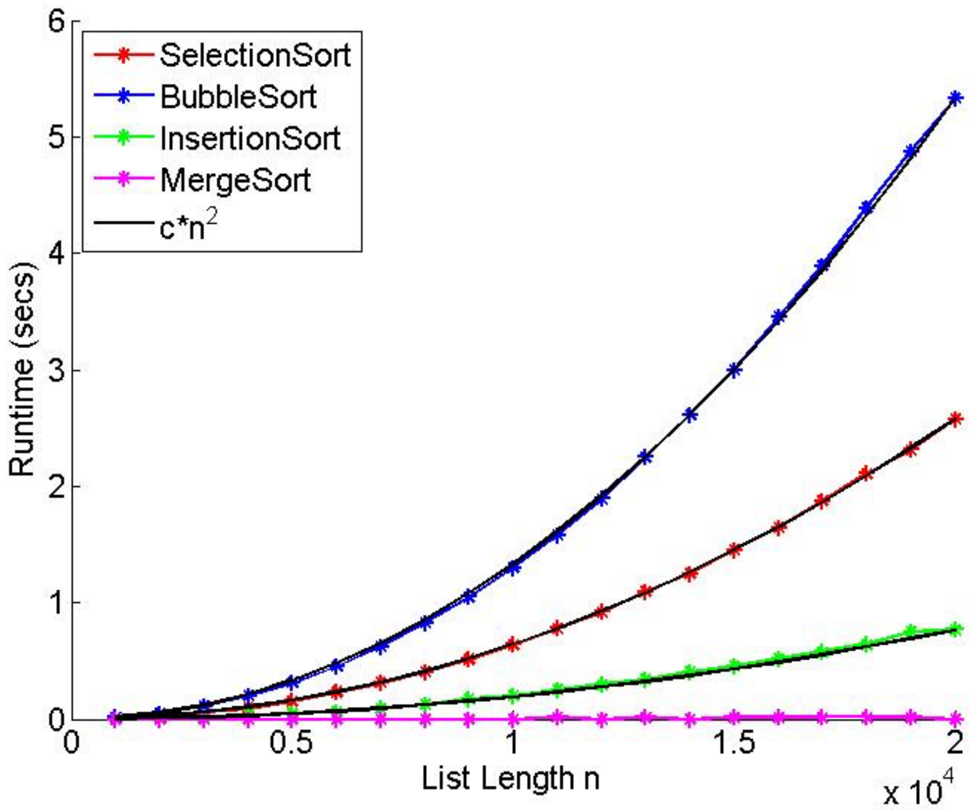
Performance Profiling

(*Dracula* vs. *Lewis & Clark*)

Version	Change	Running Time
Version 1		4,311.00s
Version 2	Better file handling	676.50s
Version 3	Faster sorting	6.59s
Version 4	No sorting!	2.35s

V.2 → V.3 was using MergeSort instead of SelectionSort.

real world performance



http://www.cs.toronto.edu/~jepson/csc148/2007F/notes/sorting.html]

When is it better to use InsertionSort instead of MergeSort?

A. When there is limited space?
B. When there are a lot of items to sort?
C. When there is a large memory cache?
D. When there are a small number of items?
E. When the list is mostly sorted?

MergeSort

When the list is mostly sorted:

- InsertionSort is fast!
- MergeSort is $O(n \log n)$

How "close to sorted" should a list be for InsertionSort to be faster?

MergeSort

Small number of items to sort:

- MergeSort is slow!
- Caching performance, branch prediction, etc.
- User InsertionSort for $n < 1024$, say.

Base case of recursion:

- Use slower sort.

Run an experiment
and post on the forum
what the best switch-over
point is for your machine.

MergeSort

Space usage:

- Need extra space to do merge.
- Merge copies data to new array.
- How much extra space?

Challenge of the Day 2:

How much space does MergeSort need to sort n items?

(Use the version presented today.)

Design a version of MergeSort that minimizes the amount of extra space needed.

MergeSort

Stability:

- MergeSort is stable if “merge” is stable.
- Merge is stable if carefully implemented.

Sorting Analysis

Summary:

- BubbleSort: $O(n^2)$
- SelectionSort: $O(n^2)$
- InsertionSort: $O(n^2)$
- MergeSort: $O(n \log n)$

Also:

The power of divide-and-conquer!

How to solve recurrences…

Properties: time, space, stability

For next time…

Monday lecture:

- More sorting

Problem Set 3:

- Released today.
- Some may depend on Monday’s lecture.

Sorting and Java:

- See slides that follow for some Java issues.

Sorting

```
public interface ISort{

    public void sort(int[] dataArray);

}
```

Sorting Widgets

```
public interface ISortWidgets{

    public void sort(Widget[] dataArray);

}
```

```
public void sort(Widget[] dataArray) {

   Widget x = dataArray[0];
   Widget y = dataArray[1];
   if (x < y) {
```

Generic Types

```
public interface ISort<TypeA>{

    public void sort(TypeA[] dataArray);

}
```

What goes wrong?

```
public void sort(TypeA[] dataArray) {

  TypeA x = dataArray[0];
  TypeA y = dataArray[1];

  if (x < y) {
     …
     …
  }
```

What goes wrong?

```
public void sort(TypeA[] dataArray) {

   TypeA x = dataArray[0];
   TypeA y = dataArray[1];

   if (x < y) {
      …
      …
   }
```

Illegal comparison!

What if: TypeA == Student?

```
class Student {
   double m_CAP;
   String m_name;
   Matric m_id;
}
```

Comparable Interface

```
class Student implements Comparable<Student> {
...
...
}
```

```
interface Comparable<TypeA> {

        int compareTo(TypeA other);

}
```

Comparable Interface

x.compareTo(y) :

-1: if (x<y)

0: if (x == y)

1: if (x>y)

Must define a total ordering
Must be transitive.

```
interface Comparable<TypeA> {

    int compareTo(TypeA other);

}
```

Sorting Students, again

```
class Student implements Comparable<Students> {
...
...
}
```

```
public void sort(Student[] dataArray) {

   Student x = dataArray[0];
   Student y = dataArray[1];

   if (x.compareTo(y) < 0) {  // if (x<y)
```

Implementing Comparable

```
class Student implements Comparable<Student> {
// compare students by CAP
int compareTo(Student other){
   if (this.getCAP() < other.getCAP())
         return -1;
   else if (this.getCAP() > other.getCAP())
         return 1;
   else // equal CAP
         return 0;
   }
}
```

Almost works…

```
public interface ISort{

    public void sort(Comparable[] dataArray);

}
```

Comparable to what?

```
public interface ISort{

    public void sort(Comparable<ZZZ>[] dataArray);

}
```

Generic Sorting

```
public interface ISort<TypeA extends Comparable<TypeA>>
{
    public void sort(TypeA[] dataArray);
}
```

Generic Sorting

```
public interface ISort<TypeA extends Comparable<TypeA>>
{
    public void sort(TypeA[] dataArray);
}
```

extends, not **implements**!!

weird… no good reason... a mystery…

Generic Sorting

```
public interface ISort<TypeA extends Comparable<TypeA>>
{
    public void sort(TypeA[] dataArray);
}
```

Generic Sorting

```
public interface ISort{

    public <TypeA extends Comparable<TypeA>>
    void sort(TypeA[] dataArray);

}
```

Sorting

```
public <TypeA extends Comparable<TypeA>>
void sort(TypeA[] dataArray) {
    for (int i=0; i<dataArray.length; i++){
        for (int j=0; j<dataArray.length-1; j++){
            TypeA first = dataArray[j];
            TypeA second = dataArray[j+1];
            if (first.compareTo(second) > 0)
                swap(dataArray, j, j+1);
        }
    }
}
```

Generic Sorting

```
Student[] dataArray = new Student[100];
sort(dataArray);
```

```
class Student implements Comparable<Student> {
    int compareTo(Student other) {
        …
    }
}
```

Generic Sorting

```
Emotion[] dataArray = new Emotion[100];
sort(dataArray);
```

Error!

```
class Emotion {
    int compareTo(Emotion other) {
        …
    }
}
```

Comparable Interface

Most Java classes support Comparable

- Integer, Float, etc.
- BigInteger
- String
- Date
- Time
- …

Generic Array

Problem:

```
class Widget<TypeA> {
    void buildArray(int size){
        TypeA[] array = new TypeA[size];
        ...
    }
}
```

Cannot instantiate generic arrays!

(How big should it be? Without knowing sizeof(TypeA), Java cannot decide.)

Generic Array

Solution: use ArrayList

```
class Widget<TypeA> {
    void buildArray(int size){
        ArrayList<TypeA> array = new ArrayList<TypeA>(size);
        …
    }
}
```

Comparing Students

```
class Student implements Comparable<Student> {
…
…
}
```

```
interface Comparable<TypeA> {

        int compareTo(TypeA other);

}
```

Generic Sorting

```
public interface ISort{

    public <TypeA extends Comparable<TypeA>>
    void sort(TypeA[] dataArray);

}
```