ISyE/CS635 – Problem Set #4

Due Date: February 27, 2015. 11AM. (Marked out of 200)

Instructions for Handing In Homework

Formulate the following problems in GAMS and solve them. Please follow the instructions given in the problems closely. Submit this assignment electronically to the drop box, one zip file containing the files that are outlined below.

   You should hand in exactly 4 files with the following names: `hw4-1.gms`, `hw4-2.gms`, `hw4-3.gms`, `hw4-4.gms`

1   Malfoy Catering

Lucius and Narcissa Malfoy didn't get to be rich, pureblood, dark wizards without lots of galleons. Their family made their money in the Wizard catering business. As part of the business, their son Draco is responsible for ensuring that they have enough clean napkins to meet demand over a period of n days. The number d_j of napkins required on the jth day is known in advance. To satisfy this requirement, Draco can either buy new napkins for α galleons each, or he can have the napkins laundered. The laundry provides both fast and slow magical cleaning services, In fast service, napkins are returned q days later for a cost of β galleons per napkin, and in slow service, napkins are returned p days laters at a cost of γ dollars per napkin. Natrually, $p > q$, and $\alpha > \beta > \gamma$. Suppose that Draco must plan for a period of $n = 10$ days, and the number of required napkins is given as

```
set T /1*10/ ;
parameter d(T) / 1 50, 2 60, 3 80, 4 70, 5 50, 6 60, 7 90, 8 80, 9 50,
10 100 / ;
```

Finally, for Draco's instance, we have $p = 4, q = 2, \alpha = 200, \beta = 75$, and $\gamma = 25$.

1.1 Problem
Formulate Draco's problem as a min cost network flow problem and solve it using GAMS. In order to get full credit, you *must* model the problem as a min-cost network flow problem. **Hint:** Your network should have 22 nodes in it for Draco's instance. After solving the instance, you should display the following information, using GAMS parameters named as specified.

Display Item	GAMS Parameter Name
Minimum cost required to meet demand	Cost
Number of equations in your GAMS model	NumEqu
Total number of napkins purchased	NumBought

2   Least "Squares."

2.1   Dynamic Sets Background

This problem is also designed to test our ability to use dynamic sets. We will use the sets

```
sets
  ALLI /c1*c400/
  ALLJ /x1*x100/
;

sets
  I(ALLI)  /c1*c6/
  J(ALLJ)  /x1*x4/
;
```

You only need to *declare* your equations once. You will re-define the equations for the second model by dynamically adjusting the sets `I` and `J` to include all of the elements from `ALLI` and `ALLJ` for the second part of the problem. Remember, when writing your equations, you *declare* them over the full domain (larger set), but you *define* them (write them) only for those elements you want—the subset. All of this problem can be done in a single GAMS file.

2.2 The Small Problem

The set of six equations in four variables (1)—(6) does not have a unique solution.[1]

$$8x_1 - 2x_2 + 4x_3 - 9x_4 \;=\; 17 \tag{1}$$
$$x_1 + 6x_2 - x_3 - 5x_4 \;=\; -16 \tag{2}$$
$$x_1 - x_2 + x_3 \;=\; 7 \tag{3}$$
$$x_1 + 2x_2 - 7x_3 + 4x_4 \;=\; -15 \tag{4}$$
$$x_3 - x_4 \;=\; 6 \tag{5}$$
$$x_1 + x_3 - x_4 \;=\; 0 \tag{6}$$

For each equation i, and values of variables $x = (x_1, x_2, x_3, x_4)$, let e_i be the *absolute* difference (error) between the left hand side and the right hand side. For example, for $i = 2$ and $x = (-5, 3, 1, 4)$, the error is

$$e_2 = |(1)(-5) + 6(3) - (1)(1) - 5(4) - (-16)| = |8| = 8.$$

2.1 Problem

Write a linear programming instance that will minimize the total absolute error:

$$e_1 + e_2 + e_3 + e_4 + e_5 + e_6.$$

Solve this instance with GAMS. Display both the (minimum) total absolute deviation (in a parameter named `TotalDevSmall` and the values of x that achieve this, in a parameter named `xValSmall(ALLJ)`. For example, your code may look like this:

```
parameters
  TotalDevSmall,
  xValSmall(ALLJ)
;

TotalDevSmall = ztotdev.L ;
```

---

[1]Most six equations with four variables don't.

```
xValSmall(J) = x.L(J);

display TotalDevSmall;
display xValSmall;
```

2.2 Problem

For the same instance, write a *linear* programming instance that will minimize the *maximum* error in any one equation. Namely find values of x that will

$$\min \max\{e_1, e_2, e_3, e_4, e_5, e_6\}.$$

Create your instance in GAMS and solve it. What is the minimum max-error that can be achieved? Display this value in a parameter called `MinMaxDevSmall`. For example, your code may look like this:

```
parameters
  MinMaxDevSmall
;
MinMaxDevSmall = zminmax.L;
display MinMaxDevSmall;
```

2.3   General Least "Squares."

Let $J = \{1, 2, \ldots, |J|\}$. For each $j \in J$ you have a decision variable x_j. You are given a set of equations containing these decision variables as follows:

$$\sum_{j \in J} a_{ij} x_j = b_i \qquad \forall i \in I$$

Define the absolute error of each equation $i \in I$ as the quantity

$$e_i(x) = \left| \sum_{j \in J} (a_{ij} x_j) - b_i \right| \qquad \forall i \in I.$$

2.3 Problem

Write a *linear* program to solve the following optimization problem:

$$\min_{x \in \mathbb{R}^{|J|}} \sum_{i \in I} e_i(x).$$

Using the GAMS code below to create an instance with $|N| = 100, |M| = 400$ create and solve this large random instance. Note the use of `yes` make the sets `I` and `J` contain all of the elements in `ALLI` and `ALLJ`. Be sure to also include the `option seed=666` line so that all of us create the same random instance.

```
* Reset sets
I(ALLI) = yes;
J(ALLJ) = yes;

option seed = 666 ;
A(I,J) = uniform(-10,10) ;
b(I) = uniform(-100,100) ;
```

Solve your instance and display the final total deviation in a parameter named `TotalDevBig`. For example,

```
parameters TotalDevBig ;
TotalDevBig = ztotdev.L ;
display TotalDevBig;
```

2.4 Problem

Solve the same instance from Problem 2.3, but in this case to minimize the maximum error:

$$\min_{x \in \mathbb{R}^{|J|}} \max_{i \in I} e_i(x)$$

Solve your instance and display the final minimum maximum deviation in a parameter named `MinMaxDevBig`. For example,

```
parameters MinMaxDevBig;
MinMaxDevBig = zminmax.L;
display MinMaxDevBig;
```

3   Broom Rental

Oliver Wood retired from his career as keeper at Puddlemere United to start the Wood Broom Rental Company WBC, [2] and he needs your help. There is a fleet of 94 brooms that are distributed among 10 different locations. The (x, y) coordinates of each broom rental agency (in a grid based on kilometers[3] is given in Table 1, as are the number of brooms currently in each location and the number of brooms required for tomorrow's rentals. The cost of transporting a broom from one location to another is 0.5 galleon/km. Naturally brooms travel "as the crow flies," so distances between agencies are Euclidean distances.

Place	x Coor.	y Coor.	Required Brooms	Current Brooms
Hogwarts	0	0	10	8
Godric's Hollow	20	20	6	13
Little Whinging	18	10	8	4
Shell Cottage	30	12	11	8
The Leaky Cauldron	35	0	9	12
Ollivander's	33	25	7	2
Zonko's Joke Shop	5	27	15	14
Dervish and Banges	5	10	7	11
Little Hangleton	11	0	9	15
Weasley's Wizard Wheezes	2	15	12	7

Table 1: Information About WBC

3.1 Problem

Write a *linear program* in GAMS that will determine the movement of all brooms to establish the required number of brooms at all agencies in a minimum cost manner. Ensure your linear program is

---

[2]In the Wizarding World, brooms may be rented and used, much like cars, in our normal Muggle world.
[3]Wizards use the metric system

in the form of a minimum cost network flow problem, and set up the GAMS file to use the appropriate CPLEX options so it solves as such.

Wood would like to know two things. First, he would like to know the minimum broom tranportation cost (in a GAMS parameter transportCost), as follows:

```
parameter transportCost ;
transportCost = cost.L;
display transportCost;
```

Second, according to the optimal transportation plan, he would like to know the set of all locations requiring extra brooms that do *not* receive any brooms from their closest location (in a set not_from_closest).

```
set not_from_closest(P);
option not_from_closest:0:0:1;
display not_from_closest;
```

hints:

- Be sure to put quotation marks around elements of sets when defining them if there are spaces in the element names.

- GAMS has functions sqrt and sqr that may be useful.

- The second part will require a little bit of GAMS coding trickery.

4   Untied Airlines

Prof. Wright *hates* flying United airlines through O'Hare (ORD). This is a problem, as he is a sought-after lecturer who frequently must make trips from his home base in Madison (MSN) to San Francisco (SFO), Houston (IAH), Washington DC (DCA), and Orlando (MCO). If Prof. Wright flies United, he must travel through ORD, if he travels Delta he can choose to go via Detriot (DTW) or Minneapolis (MSP).

The travel times between various locations in minutes are:

```
MSN.ORD 22, MSN.DTW 65, MSN.MSP 46,
MSP.SFO 213, MSP.IAH 139, MSP.DCA 125, MSP.MCO 176,
ORD.SFO 247, ORD.IAH 124, ORD.DCA  82, ORD.MCO 135,
DTW.SFO 280, DTW.IAH 147, DTW.DCA  53, DTW.MCO 130
```

Delay times at ORD are approximately uniformly distributed between 0 and 3 hours, at DTW the delay time are are between 0 and 1.5 hours and between 0 and 2 hours at MSP.

We will assume that Prof. Wright makes 3 trips to each location every year. Prof. Wright is a notorious cheapskate, and lives for frequent flyer miles. So he would like to only fly one airline.

4.1 Problem
Should Prof. Wright switch to Delta? Justify your answer with a mathematical model and explain what your model does to deal with the uncertainty.

4.2 Problem
What if you add a constraint that he must always use the same hub?

4.3 Problem
What if he forgoes frequent flyer miles - which route should he then use for which flight?