CS635 – Problem Set #10

Due Date: (Friday): May 8, 2015

Instructions for Handing In Homework

Write up the solutions for these problems electronically and submit them as a single zip file into the dropbox.

1. The variable y represents the yield in a chemical process. There are n process variables $x_1, x_2, \ldots, x_n$ (such as temperature, flow rate, etc) which influence the yield. Data was collected to observe the yield y for various values of the process vector $x^t = (x_1^t, x_2^t, \ldots, x_n^t)$. It is believed that y can be reasonably approximated by a convex quadratic objective function. Formulate the problem of finding the best convex quadratic approximation $Q(x)$ for y using the available data as a nonlinear program, and discuss the important features of your formulation.

   To test your model, ensure that you write GAMS statements to generate random inputs that are consistent with the above hypothesis. Also, write statements to print out the results of the model to show how the solution relates to your random inputs. [Hint: you may want to use the fact that if $Q = R^T R$ then Q is guaranteed to be symmetric positive semidefinite.]

2. Consider a truss consisting of two steel tubes pinned together at one end and fixed at two pivot points at the other end. The span - the distance between the two pivots is fixed at $2s$. The problem is to choose the height of the truss and the thickness and average diameter of the steel tubes so that the truss will support a load of $2W$ and so that the total weight of the truss will be minimized. Denote the average tube diameter by d, the tube thickness t and the truss height by h. What is the weight of the truss? (assume density of steel is 1).

   The following constraints must be observed:

   - Because of space limitations the height of the truss must not exceed b_1.
   - The ratio of the diameter of the tube to the thickness of the tube must not exceed b_2.
   - The compression stress is limited by yield stress:

     $$W(s^2 + h^2)^{1/2} \leq b_3 dth$$

   - The tube must not buckle:

     $$W(s^2 + h^2)^{3/2} \leq b_4 dh(d^2 + t^2)$$

   (a) Formulate this problem as a standard nonlinear program and solve using GAMS. Experiment with various starting points or solvers to ensure you find a good solution.

   ```
   scalar W / 15 /, b1 /2/, b2 /3/, b3 /0.5/, b4 /1/;
   scalar s /1/;
   ```

3. Suppose you have a number of spheres of different radii with their centers given. Find the radius and center of the smallest sphere that encloses all these spheres. Ensure you use a SOCP formulation for this.

   Use the following GAMS code to generate a data set to test your formulation:

```
$if not set n $set n 2000
set i /1*%n%/;
set j /1*3/;

* generate random reproducable data
option seed = 101;

parameter r(i); r(i) = uniform(0,0.2);
parameter c(i,j); c(i,j) = uniform(0,1);
```

4. Find the area of the largest n-gon (polyhedron with n vertices) that can be inscribed within the unit circle. Note that the solution is know to be unsymmetric for n odd. Find the (approximately) best areas for $n = 15, 25, 35$.