CS635 – Problem Set #8

Due Date: April 11, 2015

Instructions for Handing In Homework

Formulate the following problems in GAMS and solve them. Submit this assignment electronically using the instructions on the course web page. You should hand in a single zip file containing exactly 3 files with the following names: `hw8-1.gms, hw8-2.gms, hw8-3.gms`

1 Support Vector Machines

Support vector machines are a popular method for classification within the machine learning community. Essentially, a linear classifier f is generated as $f(x) = w'x + \gamma$ such that if $f(x) > 0$ then x is in class P_+ whereas if $f(x) < 0$ then $x \in P_-$.

To generate this classifier a quadratic optimization model is solved:

$$\begin{array}{lc} \min_{w,\gamma,\psi} & \frac{1}{2}w'w + C\sum_i \psi_i \\ \text{s.t.} & y_i[w'x_i + \gamma] \geq 1 - \psi_i, i = 1, \ldots, m \\ & \psi_i \geq 0 \end{array}$$

Here m runs over a set of training examples, and y_i is 1 if $i \in P_+$ and -1 if $i \in P_-$. x_i are the values of the predictor variables.

The dual problem of this quadratic program is:

$$\begin{array}{lc} \max_{\alpha} & \sum_i \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i\alpha_j y_i y_j x_i' x_j \\ \text{s.t.} & \sum_i y_i\alpha_i = 0 \\ & C \geq \alpha_i \geq 0 \end{array}$$

1.1 Problem

You should formulate both these problems in GAMS. Note that for the dual problem, you may want to introduce some intermediate variables

$$v_k = \sum_i \alpha_i y_i x_{ik}$$

and express the objective function as

$$\sum_i \alpha_i - \frac{1}{2}\sum_k v_k^2$$

where k runs over the predictor features. Solve (the easiest one) using the qcp solver of your choice, and the use the multiplier information from this first solve to set the starting values for the second solve. Try to make the second solve take as little time as possible. You should use the data that is provided in the abalone.gdx file and choose an appropriate value of C. What you should attempt to predict is whether the "number of rings" is greater than 10 or not. You should allow the value of C to be changed at the command line as shown in class. For your information the data fields are:

Name	Data Type	Meas.	Description
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Sex	nominal		M, F, and I (infant)
Length	continuous	mm	Longest shell measurement
Diameter	continuous	mm	perpendicular to length
Height	continuous	mm	with meat in shell
Whole weight	continuous	grams	whole abalone
Shucked weight	continuous	grams	weight of meat
Viscera weight	continuous	grams	gut weight (after bleeding)
Shell weight	continuous	grams	after being dried
Rings	integer		+1.5 gives the age in years

Note that the first nominal value has been converted to 3 binary values and can be treated as separate data for this assignment. Use the first 4000 data points for training your classifier, and report the number of errors that you make with your classifier to a file 'error.txt' on the remaining 177 samples.

2 Points in Polyhedra

In this problem, you are given a "random" polyhedron

$$P(\xi) = \{x \in \mathbb{R}^n \mid Ax \sim b\},$$

where $\sim$ is one of $\leq, \geq$, and $b \in \mathbb{R}^m$. We will create each $P(\xi)$ by choosing its parameters uniformly and randomly in the intervals:

- $b_i \in [-10, 10] \forall i = 1, \ldots, m$
- $a_{ij} \in [-2, 2] \forall i = 1, \ldots, m, j = 1, \ldots, n$.

To ensure that $P \neq \emptyset$, we will ensure that $0 \in P$ by letting the inequalities defining P be

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i$$

if $b_i \geq 0$ and

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i$$

if $b_i < 0$.

You are also given a (random) point $\hat{x}(\omega) \in \mathbb{R}^n$, where each component of $\hat{x}$ is chosen randomly and uniformly in the interval $\hat{x}_j \in [-20, 20]$.

2.1 Problem

Let $n = 400, m = 100$. Use multiple solves in GAMS to estimate the probability that $\hat{x}(\omega) \in P(\xi)$.

- If $\hat{x} \notin P$, then estimate the following quantities:
  1. $z_2 \stackrel{\text{def}}{=} \min_{x \in P} \|x - \hat{x}\|_2$
  2. $z_1 \stackrel{\text{def}}{=} \min_{x \in P} \|x - \hat{x}\|_1$
  3. $z_\infty \stackrel{\text{def}}{=} \min_{x \in P} \|x - \hat{x}\|_\infty$

Recall that

$$\|x\|_2 = \sqrt{\sum_{j=1}^{n} x_j^2}$$
$$\|x\|_1 = \sum_{j=1}^{n} |x_j|$$
$$\|x\|_\infty = \max_{j=1,\ldots,n} \{x_j\}$$

Note that the first problem can be solved with QCP, and the second and third minimum distance problems can be solved with LP.

3 Double pendulum

Consider the double inverted pendulum system shown in Figure 1. The objective of the system is

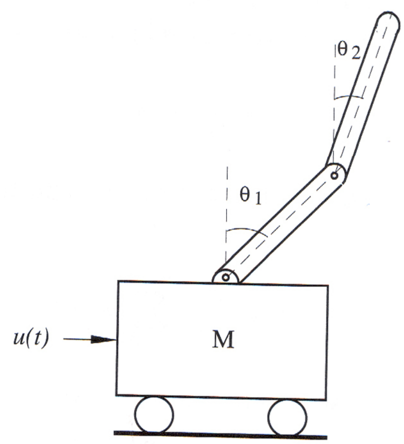

Figure 1: Double pendulum

to maintain the pendulum in the upright position using the minimum amount of energy. This is achieved by applying an appropriate control force to the car to damp out any displacements $\theta_1(t)$ and $\theta_2(t)$.

3.1 Problem

Formulate the problem as an optimal control problem within GAMS and solve for the given data.

Detail: The dynamics of the system are nonlinear but can be approximated by

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where

$$x(t) = \begin{bmatrix} \theta_1(t) \\ \dot{\theta_1}(t) \\ \theta_2(t) \\ \dot{\theta_2}(t) \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \alpha & 0 & -\beta & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha & 0 & \alpha & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

with $\alpha > 0$, $\beta > 0$ and $\alpha \neq \beta$. The "dot" notation represents first derivatives with respect to time and the parameters α and β depend on the length and weight of each pendulum and the mass of the car, etc. Suppose at $t = 0$ small nonzero displacements of $\theta_1(t)$ and $\theta_2(t)$ occur. What control $u(t)$ is needed to steer the system back into the equilibrium state $x(t) = 0$ at $t = T_0$?

Discretize over time to generate:

$$x(k+1) = \phi x(k) + gu(k)$$

where $T_0 = K\Delta t$. The energy consumed by these control actions is

$$J = \sum_{k=0}^{K-1} u^2(k)$$

which needs to be minimized. You should assume that $|u(k)| \leq U$.

Use the above facts and the model *lqr.gms* given in class to generate the model. Data is: $K = 100$, $\alpha = 10$, $\beta = 12$, $U = 10$, $\theta_1(0) = 0.4$, $\theta_2(0) = 0.2$, $T_0 = 1$. Comment on the solution.