CS635 – Problem Set #9

Due Date: (Friday): April 24, 2015

Instructions for Handing In Homework

Write up the solutions for these problems electronically and submit them as a single zip file into the dropbox.

1. When considering a second order cone constraint, a temptation might be to square it in order to obhain a classical convex quadratic constraint. This might not always work. Consider the constraint
$$2x_1 + x_2 \geq \|x\|_2 ,$$
and its squared conterpart:
$$(2x_1 + x_2)^2 \geq \|x\|_2^2 .$$
Is the set defined by the first inequality convex? Is the set defined by the second inequality convex? Draw them both and discuss.

2. We would like to minimize the function $f : \mathbb{R}^3 \mapsto \mathbb{R}$, with values:
$$f(x) = \max\left(x_1 + x_2 - \min(\min(x_1 + 2, x_2 + 2x_1 - 5), x_3 - 6), \frac{(x_1 - x_3)^2 + 2x_2^2}{1 - x_1}\right),$$
with the constraint $\|x\|_\infty < 1$. Explain precisely how to formulate the problem as an SOCP in standard form. Solve using GAMS.

3. The returns on $n = 4$ assets are described by a Gaussian (normal) random vector $r \in \mathbb{R}^4$, having the following expected value $\hat{r}$ and covariance matrix Σ:
$$\hat{r} = \begin{bmatrix} 0.12 \\ 0.10 \\ 0.07 \\ 0.03 \end{bmatrix}, \Sigma = \begin{bmatrix} 0.0064 & 0.0008 & -0.0011 & 0 \\ 0.0008 & 0.0025 & 0 & 0 \\ -0.0011 & 0 & 0.0004 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$
The last (fourth) asset corresponds to a risk-free investment. An investor wants to design a portfolio mix with weights $x \in \mathbb{R}^4$ (each weight x_i is non-negative , and the sum of the weights is one) so as to obtain the best expected return $\hat{r}^T x$, while guaranteeing that

   (a) no single asset weights more that 40%;

   (b) the risk-free assests should not weight more that 20%;

   (c) no asset should weight less than 5%;

   (d) the probability of experiencing a return lower than $q = -3\%$ should be no larger that $\epsilon = 10^{-4}$.

   What is the maximal achievable expected return, under the above constraints?