Swarm Intelligence

Task Allocation

Akin Tatoglu

Objectives

1. Understanding the **task allocation problem**
2. Understanding the **threshold model**

References

- Theraulaz, G., Bonabeau, E., & Deneubourg, J.-L. (1998). Response threshold reinforcements and division of labour in insect societies. Proceedings of the Royal Society B. Biological Sciences, 265(1393), 327-332.
- Krieger, M. J. B., & Billeter, J.-B. (2000). The call of duty: self-organised task allocation in a population of up to twelve mobile robots. Robotics and Autonomous Systems, 30(1-2), 65-84.

Task Allocation

Problem Statement

Task allocation is the problem of establishing **who does what** in a swarm

- **No predefined roles** are assumed
- **Dynamic allocation**
- In general, **time** and **space constraints** might apply

The Threshold Model

Division of Labor in Insect Colonies

- Insect colonies are capable of **dynamically** reassign individuals to different tasks
- It is a form of **colony flexibility** that derives from individual flexibility
- What are the **individual** mechanisms that promote colony flexibility?

Tasks and Stimuli

Assume that m **tasks** must be performed by the colony

- Each task is associated to a **stimulus** that increases if the task is not satisfied
- Either because not enough individuals participate
- Or because the task is not performed fast enough

$\boxed{\text{task } 1} \quad \cdots \quad \boxed{\text{task } j} \quad \cdots \quad \boxed{\text{task } m}$

The Threshold Model

One of the best models of **division of labor** among social insects is due to Theraulaz et al.

The model is based on the concept of **response threshold**

- Response thresholds refer to **likelihood of reacting to task-associated stimuli**
- Low-threshold individuals perform tasks at a lower level of stimulus than high threshold individuals

The Threshold Model

The hypothesis before Theraulaz et al. was that the **thresholds** are different among individuals, but **static**

- However, these early results did not show **colony flexibility**
- **Idea (Theraulaz et al.)**: let the response **thresholds** vary over time

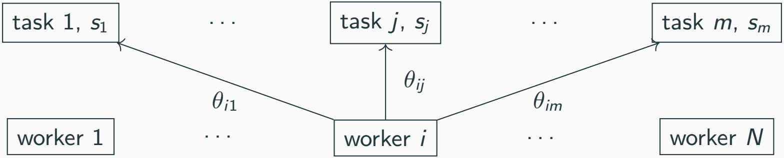
Workers

Assume there are N **workers**

- θ_{ij} is the **response threshold** of worker i to task j
- i.e. How likely worker i is to switch to j

And let's call s_j the **stimulus** to perform task j

- All individuals perceive the same stimulus

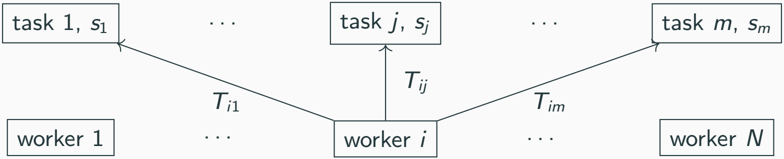


The Fixed Threshold Model

The **fixed threshold model** is

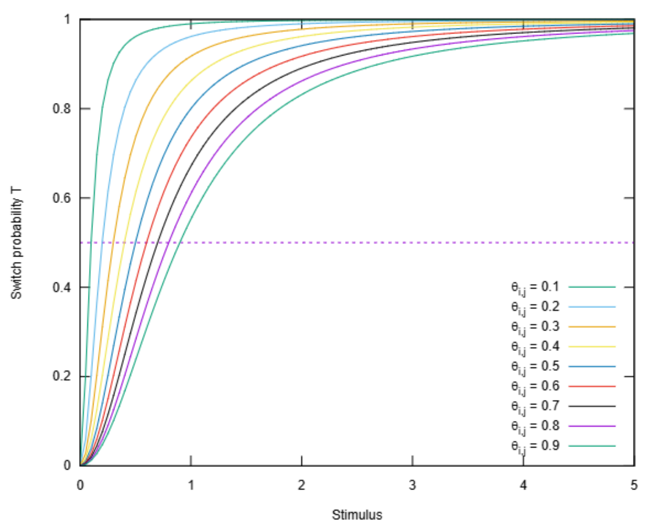
$$T_{\theta_{ij}}(s_j) = \frac{s_j^2}{s_j^2 + \theta_{ij}^2}$$

Every individual engages in task s_j with a probability $T_{\theta_{ij}}(s_j)$



The Fixed Threshold Model

The higher the threshold, the lower the switch probability



Learning and Forgetting

Theraulaz et al. improve this model by assuming that

- The more you perform a task, the more you are willing to continue (**lower** $\theta_{i,j}$)
- The less you perform a task, the less you want to do it (**higher** $\theta_{i,j}$)

Therefore, considering a period Δt, we update θ_{ij} using

$$\theta'_{ij} \leftarrow \theta_{ij} - \xi \, \Delta t \qquad i \text{ performs } j \text{ in } \Delta t \text{ (reinforce)}$$

$$\theta'_{ij} \leftarrow \theta_{ij} + \phi \, \Delta t \qquad i \text{ doesn't perform } j \text{ in } \Delta t \text{ (forget)}$$

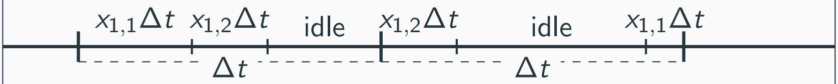
where ξ and ϕ are the **learning** and **forgetting coefficients**

Time Dynamics

Given a time period Δt, an individual i spends a fraction of time x_{ij} engaged in task j

- For a total time of $x_{ij}\Delta t$
- And a time $(1 - x_{ij})\Delta t$ doing other tasks

Example: $i = 1$, $m = 2$ tasks



Therefore, we can write

$$\theta'_{ij} \leftarrow \theta_{ij} - x_{ij}\xi\Delta t + (1 - x_{ij})\phi\Delta t$$

Imposing a Range on θ_{ij}

To write the final model, we need to impose that

$$\theta_{ij} \in [\theta_{\mathrm{min}}, \theta_{\mathrm{max}}]$$

Let's introduce the **indicator function**

$$\Theta(y) = \begin{cases} 0 & \text{if } y \leq 0 \\ 1 & \text{if } y > 0 \end{cases}$$

Then

$$\Theta(\theta_{ij} - \theta_{\mathrm{min}}) \cdot \Theta(\theta_{\mathrm{max}} - \theta_{ij})$$

is 0 when θ_{ij} is out of bounds, and 1 otherwise

Continuous-Time Model Formulation

The **continuous model** is then

$$\underbrace{\partial_t \theta_{ij}}_{\text{time derivative}} = \Big[\underbrace{(1 - x_{ij})\,\phi}_{\text{forget}} \underbrace{- x_{ij}\xi}_{\text{reinforce}}\Big] \underbrace{\Theta(\theta_{ij} - \theta_{\text{min}})\Theta(\theta_{\text{max}} - \theta_{ij})}_{\text{range indicator}}$$

Average Temporal Dynamics

Now we need to model the **average temporal dynamics**.

- $\sum_{k=1}^{m} x_{ik}$ is the **time fraction spent working on tasks**
- $(1 - \sum_{k=1}^{m} x_{ik})$ is the **idle time fraction**
- $T_{ij}(s_j)(1 - \sum_{k=1}^{m} x_{ik})$ says **how much of the idle time can be spent on task j**

$$\underbrace{\partial_t x_{ij}}_{\text{time derivative}} = T_{ij}(s_j)(1 - \sum_{k=1}^{m} x_{ik}) \underbrace{- p\, x_{ij}}_{\text{spontaneous switching}} + \underbrace{\psi(i, j, t)}_{\text{Gaussian noise}}$$

- $1/p$ is the **average time spent on a specific task**, after which an individual switches to idle

Stimulus Dynamics

Stimulus j depends on how much time the individuals spend on a task j:

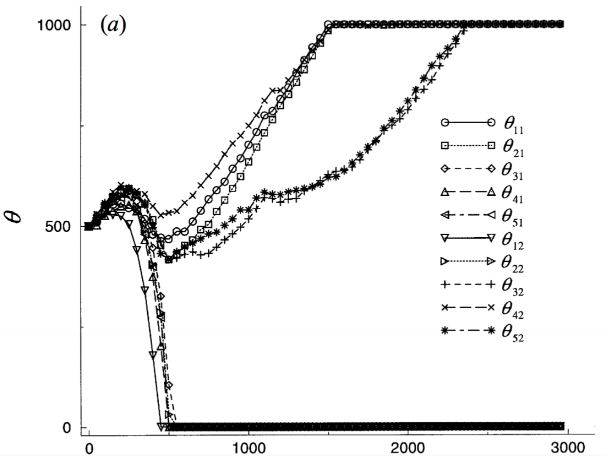
$$\partial_t s_j = \delta - \frac{\alpha}{N}\left(\sum_{i=1}^{N} x_{ij}\right)$$

- δ is the **increase per unit time**
- α is an **efficiency factor**

(Here α is a constant and is the same for all tasks and individuals, but in real life it can change wildly)

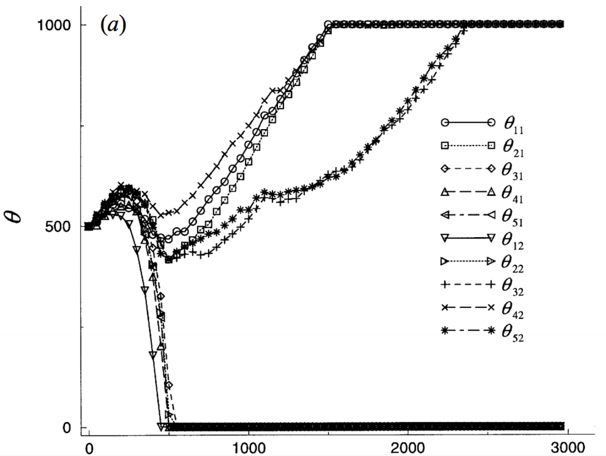
Results

$N = 5,\ m = 2,\ \alpha = 3,\ \delta = 1,\ p = .2,\ \xi = 10,\ \phi = 1$

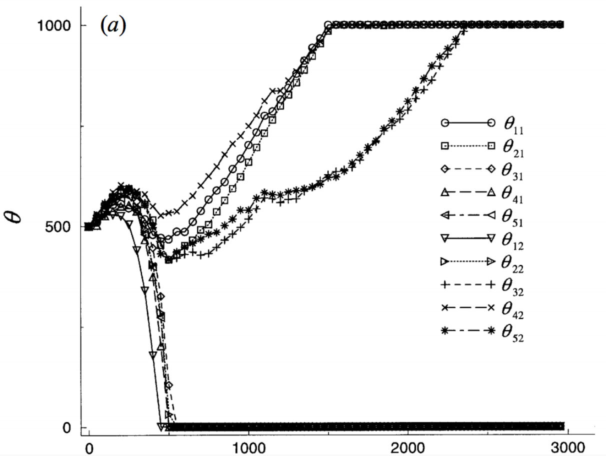


Results

$\theta_{ij}(t=0) = 500\ \forall i, j$: **specialization occurs**

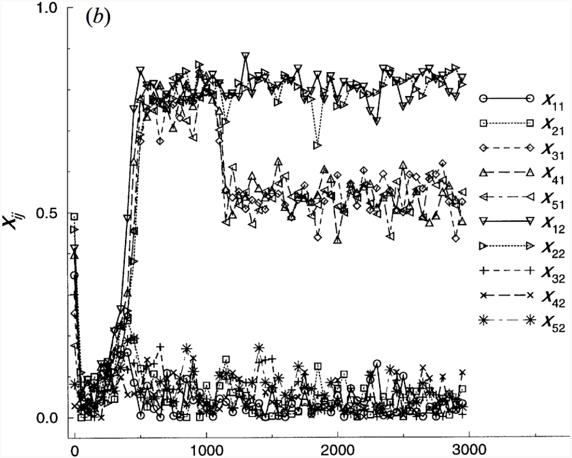


Here workers 3,4,5 are task-1 specialists and 1,2 are task-2 specialists



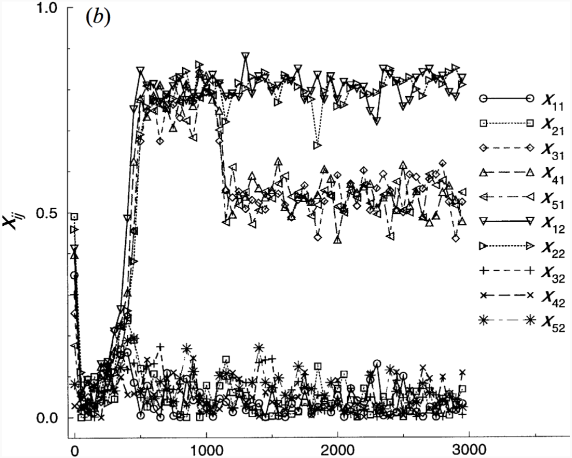
Results

Average task time dynamics



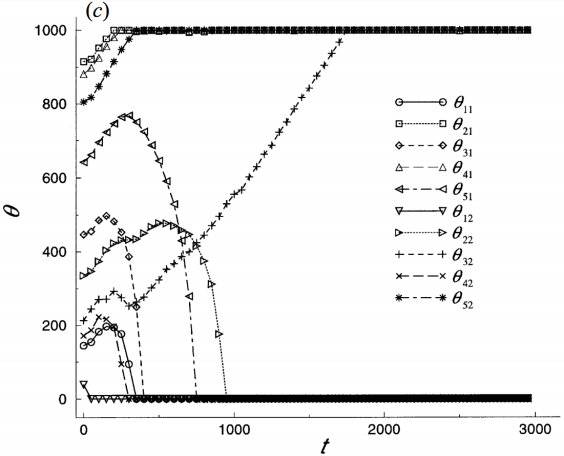
Results

Workers 3,4,5 spend most time on task 1 while 1,2 on task 2



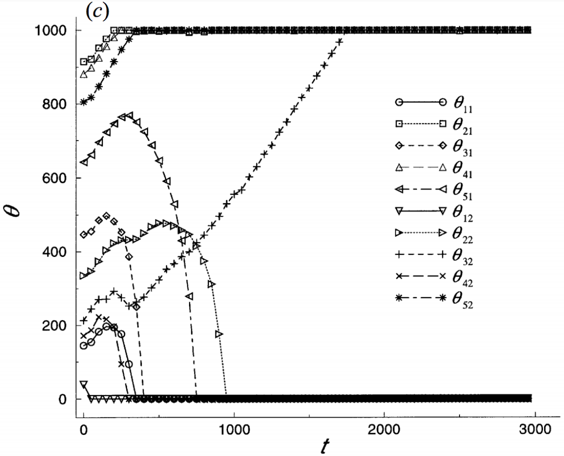
Results

$\theta_{ij}(t = 0)$ chosen uniformly $\in [\theta_{\min} = 1, \theta_{\max} = 1000]$



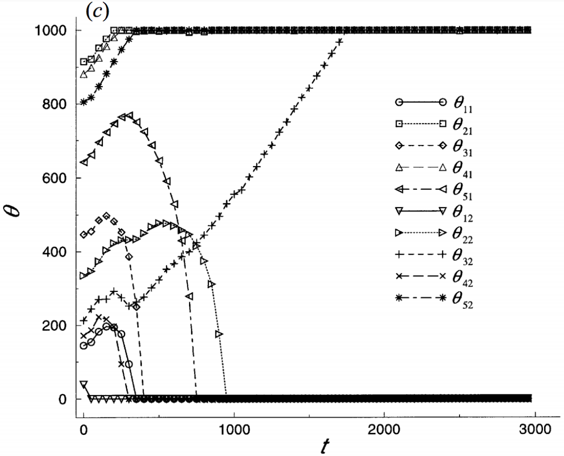
Results

1,3,5 are task-1 specialists, 1,2,4 are task-2 specialists



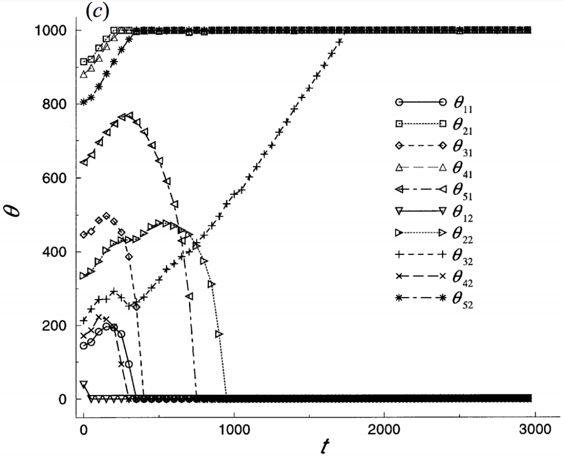
Results

individual 1 switches between tasks!

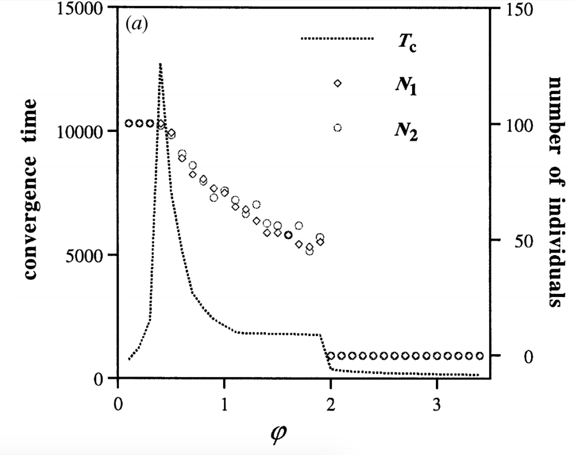


Results

only individuals with low initial $\theta_{i,j}$ tend to become specialists

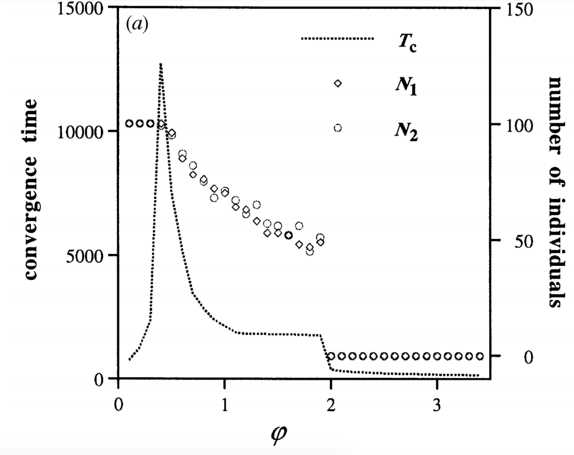


T_c (convergence time): how long it takes to have specialization



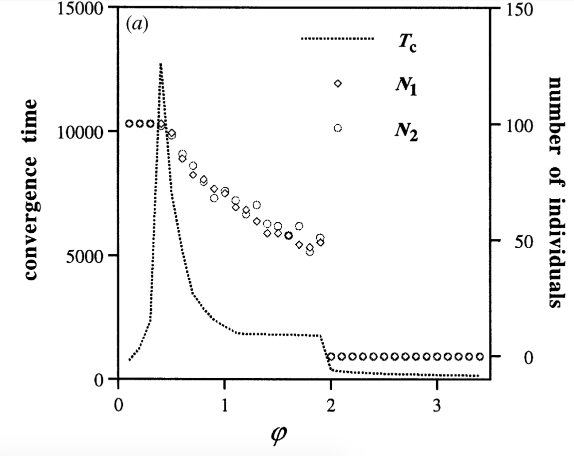
Results

Convergence is when $\theta_{ij} < 100$ or > 900



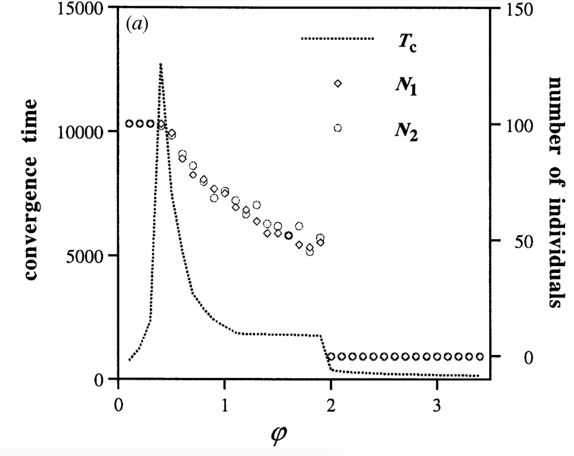
Results

N_1 and N_2 number of specialists for each task



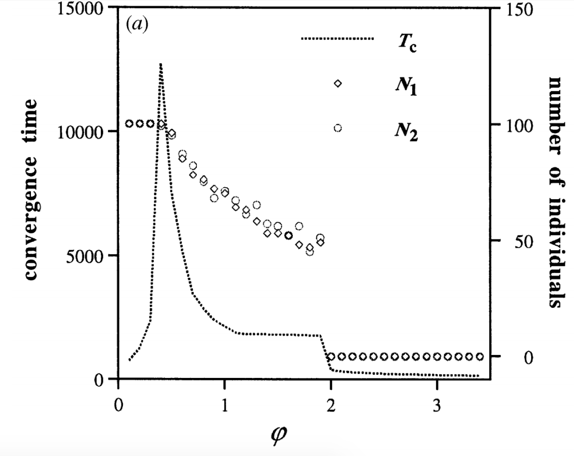
Results

Results for $N = 100$, $m = 2$, $\phi + \xi = 11$

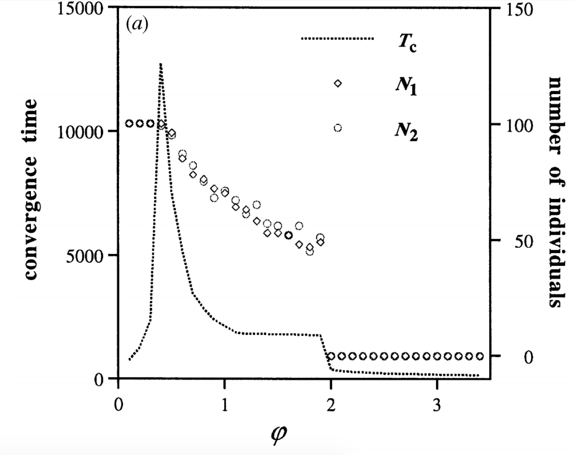


Results

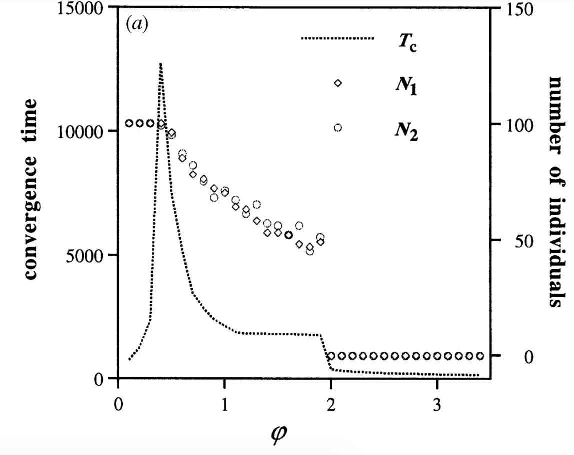
$\phi < 0.4$ lots of specialists because forgetting rate is small



T_c grows because the many specialists keep s_j low. . .

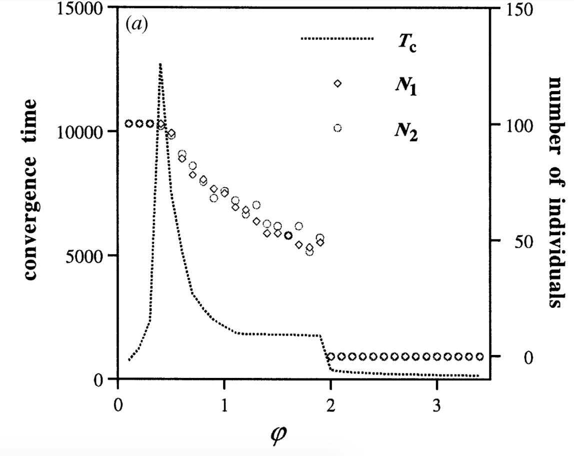


…and $\theta_{i,j}$ fluctuates a lot among idle workers



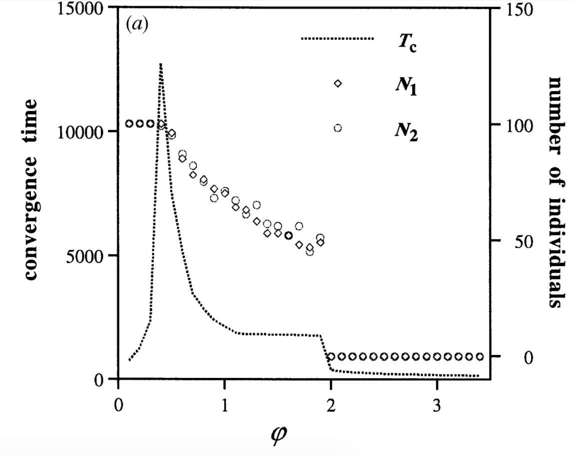
Results

$0.4 < \phi < 2$ number of specialists and T_c decrease



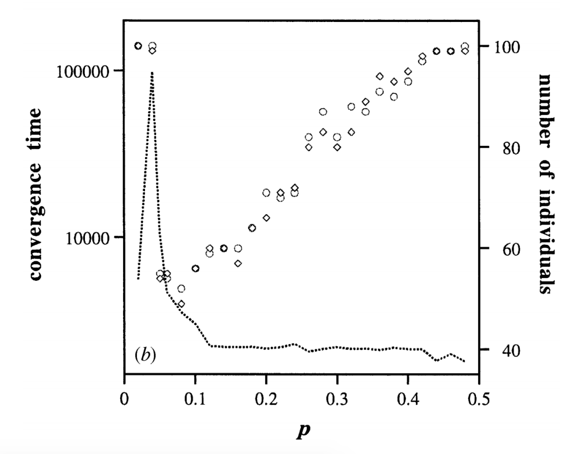
Results

$\phi > 2$ no specialization because forgetting rate pushes $\theta_{i,j}$ to high values



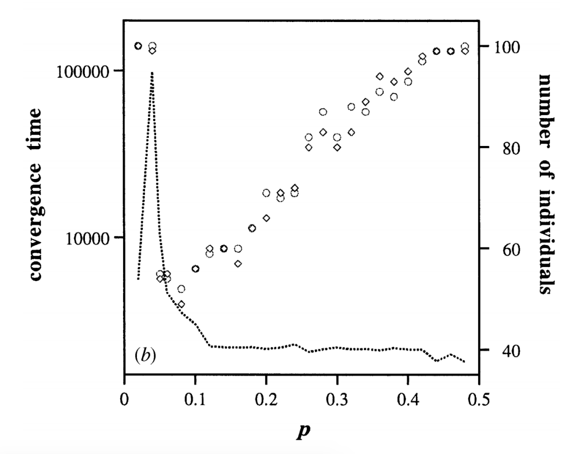
Results

Influence of spontaneous switching rate p



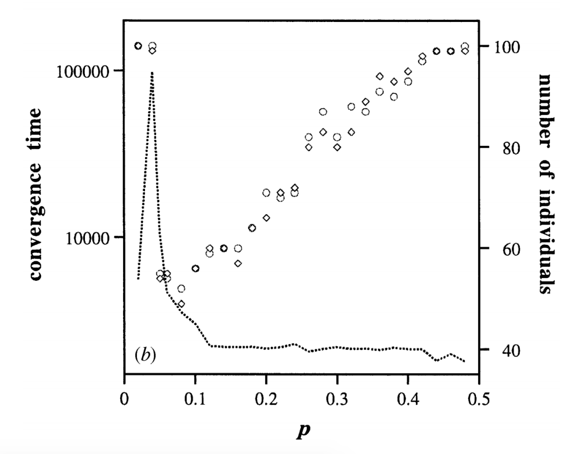
Results

$p < 0.04$: all specialists, because once they started a task, workers stay on it for a long time



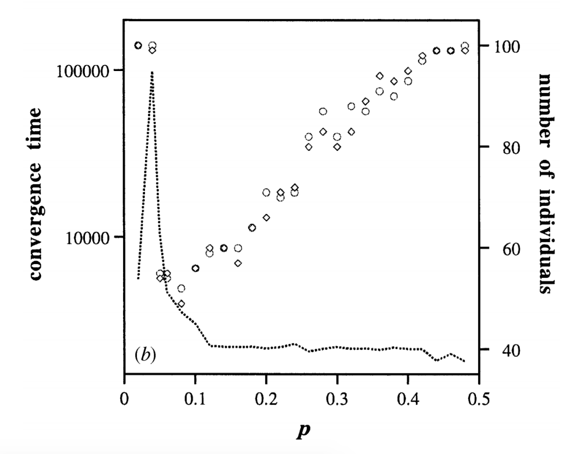
Results

$p > 0.04$: drop in the number of specialists



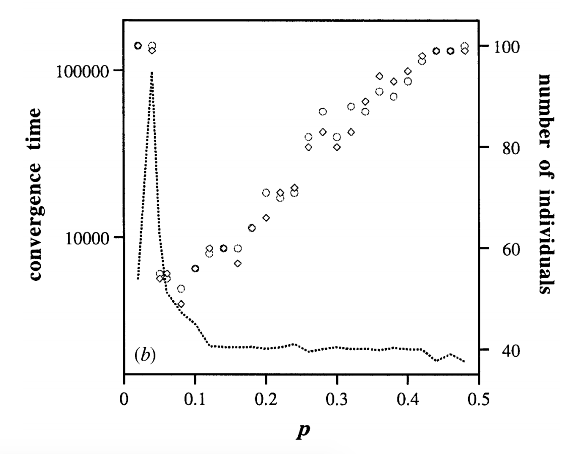
Results

$0.04 < p < 0.42$ number of specialists increases

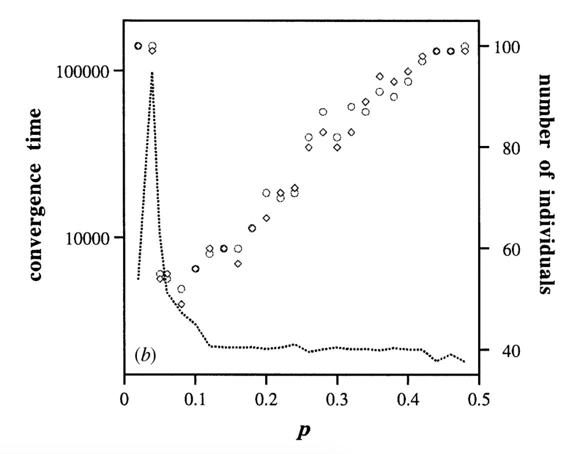


Results

$p > 0.42$ everybody is a specialist



…because they spend so little time working, it takes a lot of specialists to keep the stimulus low



Back to Robotics

Krieger and Billeter (2002)

Krieger and Billeter (2002) took inspiration from this model and applied it to a **robotic foraging scenario**

- Collecting **food items** to **store energy** at the nest
- Robots choose between **collecting** and **staying in the nest**
- Low **energy** $\Rightarrow$ higher **stimulus** to go collecting
- More people collecting $\Rightarrow$ more energy used

Fixed-Threshold Behavior

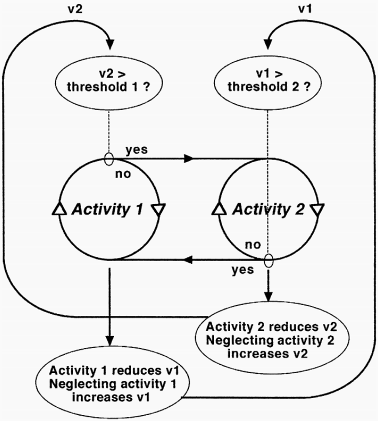


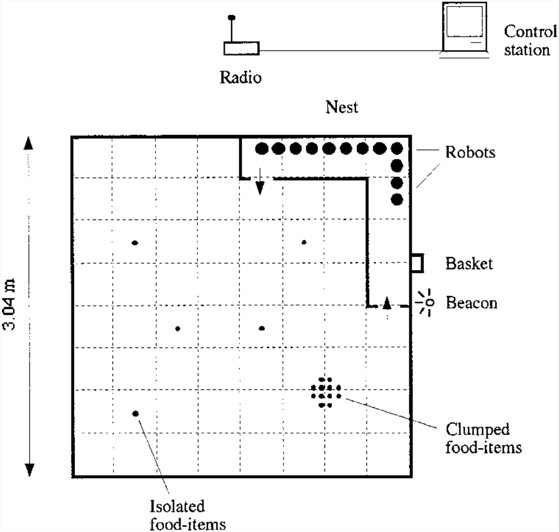
Fig. 2. Individual choice between two activities with a fixed activation-threshold. Neglecting activity 1 causes the stimulus for activity 1 to increase, prompting individuals to change from activity 2 to activity 1; and conversely.

Goal

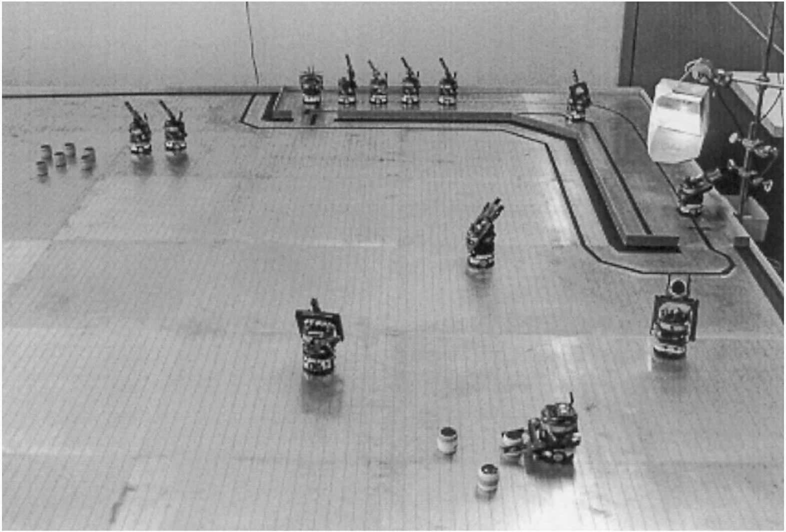
Taking inspiration from nature, Krieger and Billeter want to

1. Validate the threshold model using robots
2. Study what happens when the threshold is fixed but different among robots
    - So not the model we just saw
3. Test the effect of different food distributions on performance

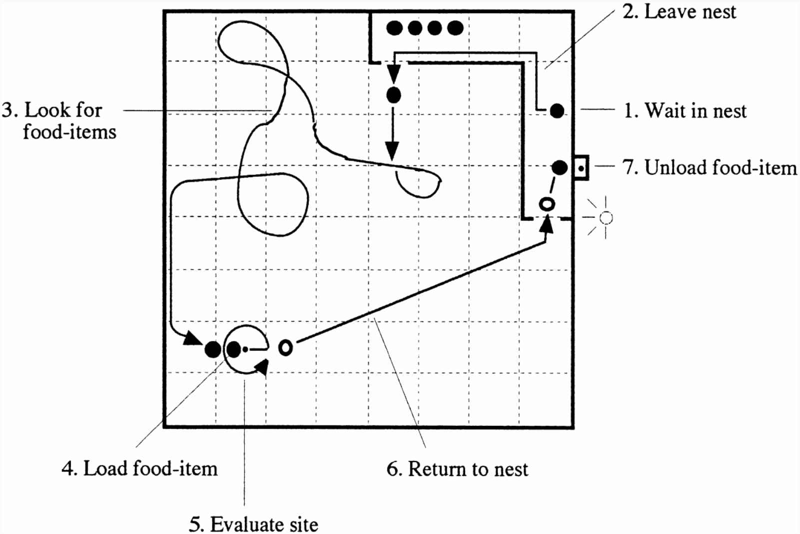
Experimental Setup



Experimental Setup



Behavior


2. Leave nest
3. Look for food-items
1. Wait in nest
7. Unload food-item
4. Load food-item
6. Return to nest
5. Evaluate site

Recruitment

Krieger and Billeter tested two group behaviors:

- Robots behaving **individually**
- Robot **recruiting** other robots
  - A robot just back from collecting remembers where food was and leads another robot to the food location

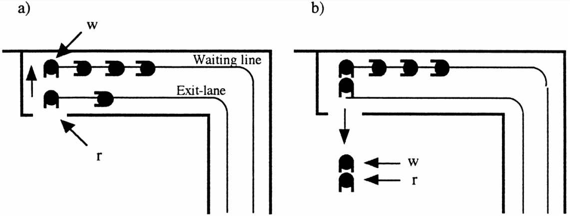


Fig. 6. Tandem recruitment. (a) Recruiting robot (r) backs up toward the waiting robot (w) at the head of the waiting line. When w's proximity sensors detect r, w goes into follower mode. (b) Recruited robot (w) follows recruiting robot (r) to the food patch.

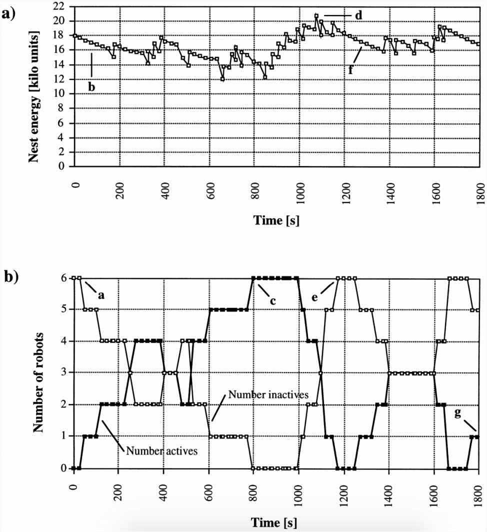
Experimental Scenarios

The initial response threshold was chosen at random

Each type of experiment was repeated eight times (the experiment with group size one was dropped in Series C for an obvious reason: a single robot cannot recruit another robot)

Recruitment		Food distribution	Group size				
Series A	No	Dispersed	1	3	6	9	12
Series B	No	Clumped	1	3	6	9	12
Series C	Yes	Clumped		3	6	9	12

Active Robots



Nest Energy

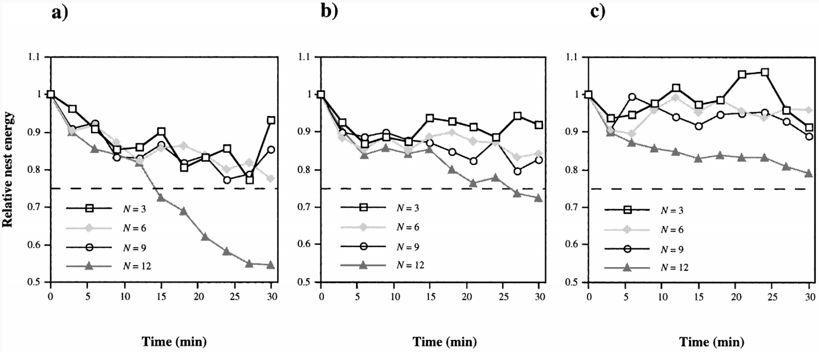


Fig. 11. Nest energy recorded during the experiments for the three, six, nine and twelve robot teams normalised for the number of robots in each team. (a) Dispersed food distribution with no information sharing, (b) clumped food distribution with no information sharing, (c) clumped food distribution with information sharing.

Performance

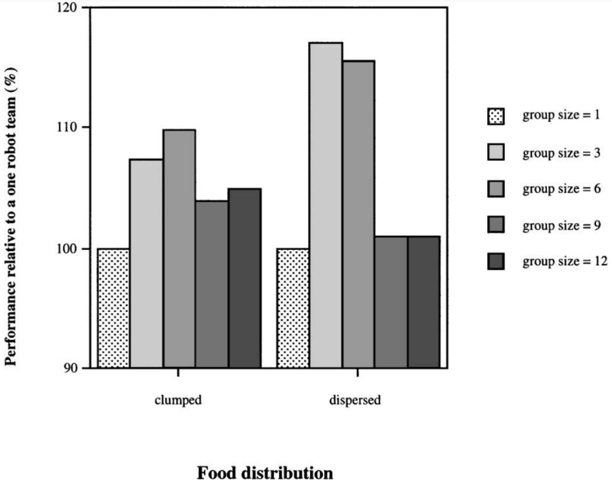


Fig. 12. Performance relative to the one robot teams in an environment with clumped and a dispersed food distribution with no information sharing. Performance was measured as the inverse of the total energy used and the normalised for the number of robots.

Minimal Nest Enegy, No Recruitment

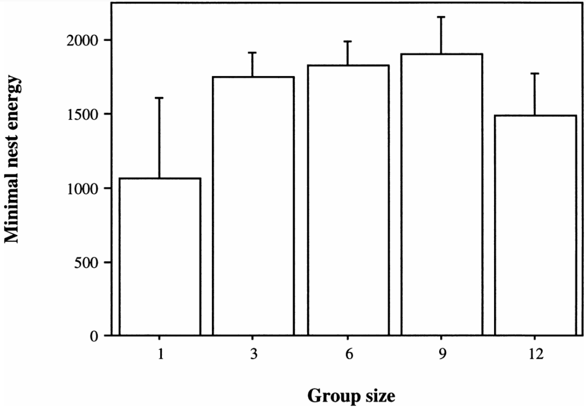


Fig. 13. Minimal nest-energy recorded during the experiments with no information sharing for the teams of different group size. The minimal nest-energy was normalised for the number of robots. Error bars indicate the 95% confidence interval.

Proportion of Time Active

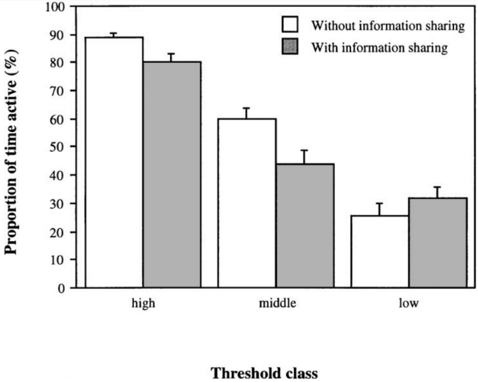


Fig. 14. Proportion of time spent in an active (working) state depending on the threshold class and information sharing. The robots were categorised according to their activation-threshold in one of the three classes (high, middle, low). Error bars indicate the 95% confidence interval.